NECESSARY CONDITION FOR BEAM ORDERING

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Abstract

The very low momentum spread for small number of particle was reached on different storage rings. When the sudden reduction of the momentum spread ("phase transition") was observed during decreasing of the particle number it was interpreted as ordered state of ion beams. The most extensive study of ordered ion beams was done on storage rings ESR (GSI, Darmstadt) [1] and CRYRING (MSL, Stockholm) [2]. Recently, for the first time, the ordered proton beam has been observed on S-LSR (ICR, Kyoto University) [3].

This article presents the experimental investigation of low intensity proton beams on COSY (IKP, Juelich) and S-LSR which have the aim to formulate the necessary conditions for the achievement of the ordering state. The experimental studies on S-LSR and numerical simulation with the BETACOOL code [4] were done for the dependence of the momentum spread and transverse emittances on particle number with different misalignments of the magnetic field at the cooler section.

INTRODUCTION

Since very low momentum spread of proton beam was obtained with a help of electron cooling in NAP-M experiments [5] (BINP, Novosibirsk) the deep cooling of low intensity ion beams was studied in a few scientific centres. Essence of the experiments is a measurement of an ion beam momentum spread under cooling during long period of time when the beam intensity slowly decreases. At given value of the particle number the momentum spread is determined by equilibrium between the electron cooling and heating effects, main of which is an intrabeam scattering in the ion beam. The intrabeam heating rates decrease with the particle number that leads to decrease of the equilibrium momentum spread. At large intensity the momentum spread $\Delta p/p$ is scaled with the particle number N in accordance with a power law $\Delta p/p \sim N^{\xi}$, where ξ is some constant depending on settings of a storage ring and cooling system. When the particle number becomes less than certain threshold value the momentum spread can saturate (like in NAP-M, COSY experiments) or suddenly drop down by about one order of magnitude (ESR, SIS, CRYRING, S-LSR), which was interpreted as a phase transition to the ordered state.

Initially the ordered state was observed at ESR for heavy ions only [6], for light ions C^{6+}, Ne^{10+}, Ti^{22+} (except protons) the ordering was reached much later [7]. A few attempts for ordering of the proton beam were made at COSY (FZJ, Juelich), however the sudden reduction of the momentum spread was not observed [8]. Firstly the proton beam ordering was reached at S-LSR (Kyoto University) [3].

From analysis of the ESR experimental results one can conclude that the ordered state was reached when the dependence of momentum spread on particle number had a power coefficient $\xi \leq 0.3$ [6, 7]. In the first experiments at ESR with the light ions this condition was not satisfied and the beam ordering did not occur. This condition can explain why in experiments at COSY (where ξ was larger than 0.5) and NAP-M (where ξ was about 1) a sudden reduction of the proton beam momentum spread was not observed. First attempt to reach ordered proton beam at S-LSR in 2006 was not successive also, the ξ value in this experiment was about 0.4 [10].

EXPERIMENTS AT COSY

A few attempts for searching of the ordered proton beams were made at COSY ring [8]. The Schottky spectrum of proton beam was measured at injection energy (45 MeV) at different electron current values for different proton number in the beam. After injection the proton number was being reduced with introducing of the horizontal scraper that decreased the ring aperture and led to a fast proton loss.

The last measurements have shown that minimum value of the proton momentum spread can be reached at proton number below $1 \cdot 10^{6}$ protons and does not decrease below $2 \cdot 10^{-6}$ (Figure 1). The result does not depend actually on the feedback of high voltage power supply. The value ξ was about 0.5 in these experiments and the sudden reduction of the momentum spread was not observed.

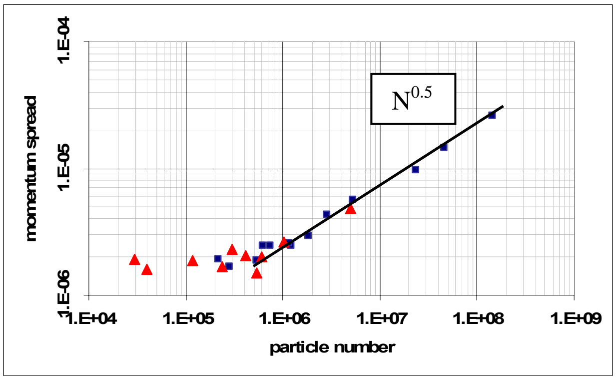

Figure 1: Momentum spread vs particle number: squares - fedback OFF, triangles – feedback ON. I_e=70 mA.

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OREDERED PROTON BEAMS AT S-LSR

The successful achievement of the ordered state of the proton beam on S-LSR [3] permitted to start careful studies of the low temperature ion beam at different experimental settings. The describing experiments were performed with 7 MeV proton beam. The measured beam parameters was a momentum spread and a horizontal beam profile. They were measured by a Schottky monitor and a residual gas ionization monitor, respectively. The particle number in the ring was measured by the residual gas ionization monitor and a beam intensity monitor. At first, an electron beam alignment was done to maximize the longitudinal cooling force measured by the induction accelerator. This setting was defined as “0 mrad”. Then, the misalignment between the electron and proton beams was created by the Helmholtz coils in the horizontal and vertical directions.

In the experiments the variation of the ξ value was realized by introduction of well-controlled angular deviation of the electron beam in respect to the proton beam equilibrium orbit. At small angles the electron beam misalignment leads to decrease of the cooling efficiency only. When the misalignment reaches a certain threshold value a qualitatively different situation is obtained. The ions start to oscillate with a certain value of betatron amplitude. The amplitude of the oscillations depends on the misalignment angle and the beam emittance can not be less than the value corresponding to the oscillation amplitude. In absence of another effects leading to heating of the beam, the beam profile has specific double-peak structure, and sudden appearance of this structure at variation of misalignment angle is called “chromatic instability”.

Intrabeam scattering heating rate linearly scales with the particle number and inversely proportional to the beam phase volume $\varepsilon_x\varepsilon_y(\Delta p/p)$. Electron cooling rates have constant values at small temperature. At perfectly aligned electron beam a thermal equilibrium between degrees of freedom corresponds to $\varepsilon_x \sim \varepsilon_y \sim (\Delta p/p)^2$. In the case of equilibrium between the electron cooling and intrabeam scattering rates and after substitution $(\Delta p/p)^2$ instead ε_x and ε_y we have $(\Delta p/p) \sim N^{0.2}$.

At large misalignment in one transverse plane (for instance in the horizontal) we have ε_x=const, $\varepsilon_y \sim (\Delta p/p)^2$ and $(\Delta p/p) \sim N^{0.33}$. The large misalignment in both transverse planes leads to $(\Delta p/p) \sim N$. At small misalignment angle one can expect some intermediate value of the power coefficient. Thus the electron beam misalignment in one transverse plane permits to vary the ξ value from about 0.2 to 0.33, two dimensional misalignments can lead to variation of the ξ value from about 0.33 to 1.

Evolution of the momentum spread with decrease of the proton number was measured for different misalignment angles in the horizontal direction (Figure 2) and in both (horizontal and vertical) directions (Figure 3) with electron current 25.5 mA. For small misalignment (<0.5 mrad), the momentum spread was proportional to $N^{0.29}$.

When the misalignment was larger (>0.5 mrad), the momentum spread scaled with $N^{0.44}$ at large particle number and saturated (or changed the slope) below 3×10^5 particles. Especially, the behaviours at the horizontal and vertical misalignment were similar. A misalignment in both directions leads to saturation of the momentum spread at the larger value (Figure 3). In this case the dependence of the momentum spread on proton number is similar to the results obtained at COSY and NAP-M.

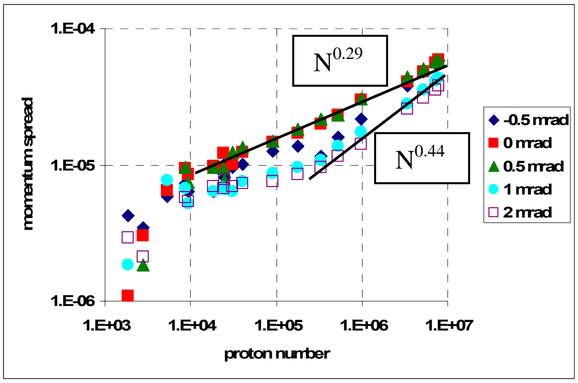

Figure 2: The dependence of momentum spread on the particle number for different horizontal misalignment angles (-0.5, 0, 0.5, 1, 2 mrad), I_e=25.5 mA.

The drop of the momentum spread, which indicates the ordered state, was observed at different horizontal or vertical misalignments (Figure 2), but it was not noticed at large misalignments (>0.5 mrad) in both directions (Figure 3).

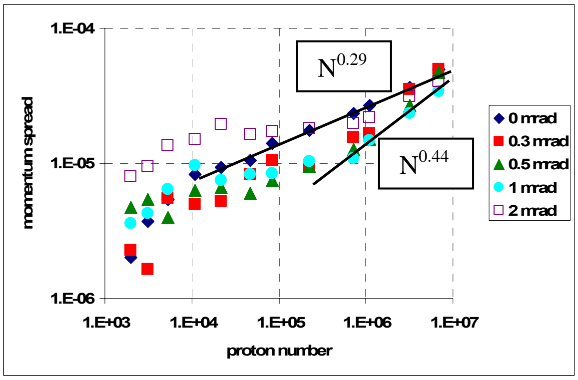

Figure 3: The dependence of momentum spread on the particle number for different angles in both directions (0, 0.3, 0.5, 1, 2 mrad), I_e=25.5 mA.

NUMERICAL SIMULATION

The dependence of the momentum spread on the particle number for different values of the misalignment angle was simulated with BETACOOL code. The friction force in the electron beam was calculated using Parkhomchuk’s formula [11] with the effective electron velocity spread determined by the magnetic field imperfection. Intrabeam scattering was simulated in accordance with Martini model [12]. This model presumes Gaussian distribution of the ions that can lead to some mistake at large misalignment angles.

The simulations show that a transverse misalignment can change the power coefficient ξ in the range from 0.21 up to 0.53 (Figure 4). In the case of a large horizontal misalignment only (horizontal/vertical = 1/0, 1/0.2 mrad) the power coefficient ξ can be changed up to values of about 0.45, however the low temperature can be reached.

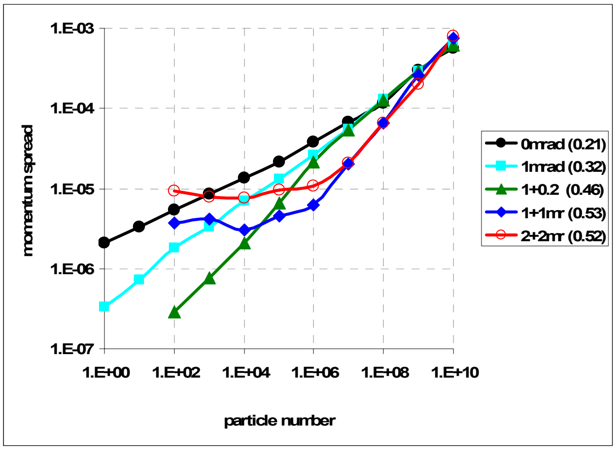

Figure 4: The dependence of the momentum spread on the particle number for different misalignments. I_e=25 mA, (0/0, 1/0, 1/0.2, 1/1 mrad, ξ=0.21, 0.32, 0.46, 0.53, 0.52).

In the case of a large misalignment in both transverse directions (Figure 5, 1/1, 2/2 mrad) the saturation of the momentum spread exists due to large influence of the space charge of the electron beam [10]. Misalignments in both transverse directions leads to the large transverse size of proton beam even at small number of particles. Protons with large amplitude of the transverse oscillation have the momentum deviation which is comparable with the momentum spread. In this case the momentum spread does not decrease with particle number and its sudden reduction can not take a place.

The use of an additional field misalignment can change the power coefficient ξ but this behaviour can not fully explain the variation of the ξ value measured in experiments and predicted by the theory. Beam parameters for different particle number are shown on the phase diagram (Figure 5) which presents the sum of the transverse and longitudinal components of the intrabeam scattering and electron cooling rates. Different colours of the diagram correspond to the values of growth rates from 10^{-5} to 10^{5} sec^{-1} at the particle number $N_p = 10^3$. Boundary between white and colour regions means the equilibrium between intrabeam scattering and electron cooling. Gray straight line means the equilibrium of transverse and longitudinal temperatures of the proton beam, black straight line – ordering criterion Γ_2 [13].

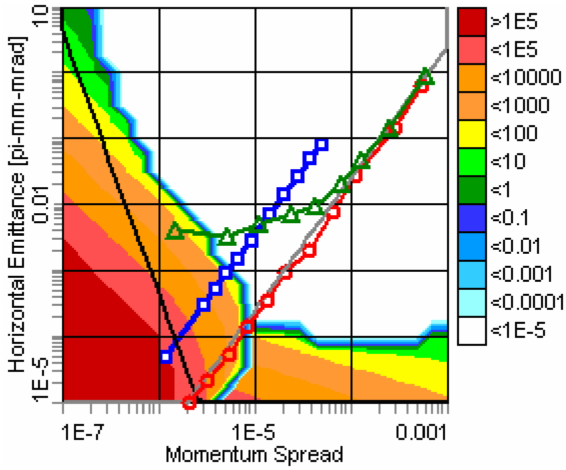

Figure 5: The dependence of horizontal emittance and momentum spread on the particle number. I_e = 25 mA. Blue square – experiment, red circles – simulation with zero misalignment, green triangle – simulation with 0.2/0.2 mrad misalignment.

The simulation results without any misalignment shows that the beam parameters are cool down along the thermal equilibrium between transverse and longitudinal degrees of freedom of the proton beam (Figure 5, red circles) and the power coefficient is ξ=0.21. The experimental results when the ordered state was observed (Figure 5, blue squares) show that the transverse temperature is much higher then the longitudinal one and the power coefficient is ξ=0.29.

Application of some misalignment in the transverse direction increases the power coefficient ξ=0.3. But this additional transverse heating changes the behaviour of the beam parameters (Figure 5, green triangles). It means that the experimental behaviour of beam parameters can not be explained by the misalignment only.

Martini model of intrabeam scattering can not describe the ordered state of the ion beam. For the simulation of the crystalline beam the Molecular Dynamics method [14] was used in BETACOOL code. Simulation results show that the longitudinal component of the intrabeam scattering heating rates has a specific behaviour for small number of particles (Figure 6).

Left part of the IBS longitudinal component is described by the shear force which always exists in bend magnets. The boundary of this part is dependent on the particle number (dashed black-gray line on Figure 6, 7) and region, where crystalline state is prohibited, is:

$$\frac{\Delta p}{p} < \frac{2N^3 r_{ion}\sigma_\perp}{\pi Q^2 \gamma_0^3 \beta_0^2 C^2}, \tag{1}$$

where N – particle number, r_{ion} – classical ion radius, $\sigma_\perp = \sqrt{\varepsilon_\perp \beta_\perp}$ - transverse beam size, Q – betatron value, β_0, γ_0 – relativistic factors, C – ring circumference.

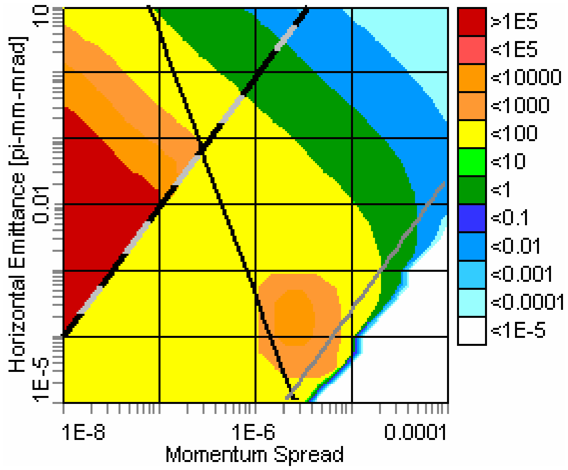

Figure 6: Intrabeam scattering rates for S-LSR ring (Molecular Dynamics, $N = 10^3$). Black line – ordering criteria (4), gray line – temperature equilibrium, dashed black-gray line – shear force boundary (5).

Ordering criteria Γ_2 [13] describes the transition to the ordered state when a relaxation between transverse and longitudinal degrees of freedom is switched off. This region is placed below the following condition (black line on Figure 6, 7):

$$\left(\frac{\Delta p}{p}\right)^2 < \frac{r_{ion}}{\pi\beta_0^2\sigma_\perp}, \qquad (2)$$

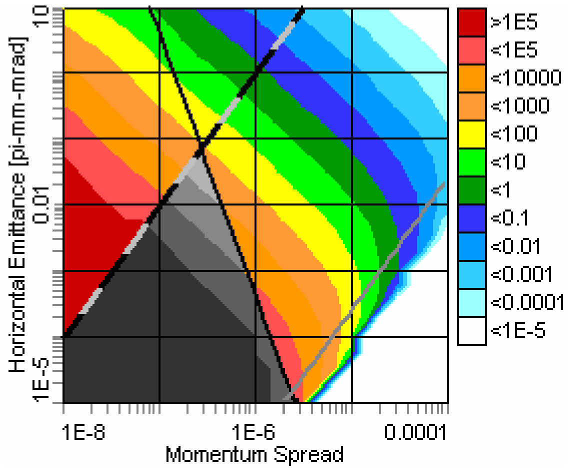

Figure 7: Intrabeam scattering rates for S-LSR ring (Martini model, $N = 10^3$).

Phase diagram (Figure 6) has a specific peculiarity: there is a region of high heating rate value surrounded by regions where the heating rate is sufficiently less. Origin of this "island" can be explained from the phase diagram calculated with Martini model (Figure 7) if we assume that in the region between conditions (1) and (2) the IBS process is suppressed due to the beam ordering (black-gray region on Figure 7). IBS island takes a place due to different angles on the phase diagram between ordering criteria (2) and the lines of equal heating rates.

CONCLUSION

The theoretical and experimental investigation of the ordered ion beam shows that the ordered state can be reached for a power coefficient $\xi \le 0.3$, describing the dependence of the momentum spread on the particle number. A large misalignment of the magnetic field at electron cooling section increases the power coefficient up to 0.5 and the ordered state of the ion beam can not be reached due to influence of the electron beam space charge.

The specific island of the IBS longitudinal component is described by different angles between lines of equal heating rates and ordered criteria Γ_2. In real experiments when the transverse temperature of the ion beams much higher then longitudinal one the cooling "around the IBS island" can help to reach the ordered state.

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