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Abstract. - Accurate measurement of the polarization of the extracted beam at different energies has revealed small decreases between 280 and 310 MeV and between 460 and 470 MeV . These locations coincide with the depolarizing resonances predicted at $298 \mathrm{MeV}(3.796 \gamma=5)$ and $466 \mathrm{MeV}\left(3.796 \gamma=6-v_{z}\right)$. A third major resonance at $51 \mathrm{MeV}(3.796 y=4)$ lies below the extractable energy range. The $3.796 y=5$ resonance has been excited directly by adjusting fifth harmonic horizontal field components. The initial depolarization of $3 \%$ could be reduced to zero or increased to $15 \%$, with the expected quadratic field dependence in between. The results of experiments and computations on these resonances are reported, as well as an attempt to accelerate a horizontally polarized beam.

1. Introduction.- Depolarization processes in circular accelerators have been studied by Froissart and Stora l) and by many subsequent authors, notably Kim and Burcham 2) for the case of sector focusing cyclotrons. Depolarization may occur wherever there is resonance between the oscillating magnetic fields experienced by the particle and its spin precession frequency. For a relativistic particle of mass $m$ and charge e in a purely magnetic field B Bargmann, Michel and Telegdi 3) (BMT) have shown that the angular
frequency of precession of the spin (in the particle's rest frame) is given by

$$
\begin{equation*}
\Omega=\gamma G \omega_{c} \tag{1}
\end{equation*}
$$

Here $\gamma$ is the relativistic energy factor, $\omega_{c}$ is the cyclotron frequency $e B / \gamma m$, and $G$ stands for $(g / 2-1)$, the $g$-factor giving the magnetic moment $\mu=g(e \hbar / 2 \mathrm{~m}) \mathrm{s}$ in terms of the spin quantum number $s$. The condition for a depolarizing resonance is thus

$$
\begin{equation*}
\gamma G=n+\ell v_{z}+m v_{r}, \tag{2}
\end{equation*}
$$

where $v_{z}$ and $v_{r}$ are the vertical and radial betatron tunes, respectively, and $n, \ell$ and m may take any integer value, positive or negative. These integers ( $n$, l, m) will be used to identify the resonance. They also determine its strength, which depends on the $\ell$ th and $m^{\text {th }}$ powers of the dimensionless betatron oscillation amplitudes $a_{z} / r, a_{x} / r-s o$ that the resonance is known as of $(\ell+m)$ th order-and involves the $n$th harmonic field component. "Intrinsic" resonances, where $n=k N$, an integer multiple of the sector periodicity $N$, are likely to be more serious than the other "imperfection' resonances.

The approximate magnitude of the depolarization $\Delta P$ for a beam of initial polarization $P_{0}$ is given by ${ }^{l, 2}$ )

$$
\begin{equation*}
\Delta P=2 P_{0}\left[1-e^{-\pi \omega^{2} / 2 \Gamma}\right] \tag{3}
\end{equation*}
$$

Here $\Gamma$ is the rate of passage through the resonance (assumed linear) and $\omega=\omega_{0} B_{h} / B_{V}$ where $\omega_{0}=\Omega+\gamma \omega_{c}$ is the precession frequency in a non-rotating frame, $B_{V}$ is the average vertical magnetic field and $B_{h}$ the horizontal perturbing field (both taken in the rest frame). For small $\triangle P$

$$
\begin{equation*}
\frac{\Delta P}{P_{0}}=\frac{\pi}{\Gamma}\left(\frac{\omega_{0} B_{h}}{B_{V}}\right)^{2} \tag{4}
\end{equation*}
$$

and the depolarization is proportional to $B_{h}^{2}$. As mentioned above $\mathrm{B}_{\mathrm{h}}$ is proportional to $\left(\mathrm{a}_{\mathrm{z}} / \mathrm{r}\right)^{\ell}\left(\mathrm{a}_{\mathrm{x}} / \mathrm{r}\right)^{\mathrm{m}}$.

For protons $g / 2=2.7928$ and $G=1.7928$. For deuterons $G=-0.143$. Since the number of magnet sectors is chosen so that $N>2 \gamma_{\text {max }}$, intrinsic resonances never occur with zero order for these particles, only with higher orders. Depolarization effects for protons and deuterons have been predicted-and found-to be negligible in sector focusing cyclotrons.
2. Polarized negative ions.- For $\mathrm{H}^{-}$ions the situation is different-and not so favourable. In a given magnetic field an $\mathrm{H}^{-}$ion and a proton will orbit in opposite directions, but their proton spins will precess in the same direction. The precession frequencies $\gamma G \omega_{c}$ observed in their rest frames will therefore differ, the G-value of 2.79-1 for the proton being replaced by approximately $-2.79-1$ for the $H^{-}$ion.

More precisely, in applying Bargmann et $\alpha Z .{ }^{1}$ s analysis to a composite system, the charge and mass of the spinning particle (e,m) must be distinguished from those of the complete system ( $e^{\prime}, m^{\prime}$ ). The BMT equation of motion for the spin four-vector $s$ in proper time $\tau$ therefore becomes

$$
\begin{equation*}
\frac{d s}{d \tau}=\frac{e}{m}\left[\frac{g}{2} F \cdot s+\left(\frac{g}{2}-\frac{m e^{\prime}}{e m^{\prime}}\right)(s \cdot F \cdot u) u\right] \tag{5}
\end{equation*}
$$

where $u$ is the velocity four-vector and $F$ is the electromagnetic field tensor. Equations (1) and (2) above remain valid provided we define

$$
\begin{equation*}
G=\frac{e^{\prime}}{m e^{\prime}}\left(\frac{g}{2}\right)-1 \tag{6}
\end{equation*}
$$

For the $H^{-}$ion $G=-3.7959$, and for $D^{-} G=-1.857$. The large numerical value of $G$ for $H^{-}$ions raises the possibility of intrinsic resonances of low or even zero order. In the case of the TRIUMF 520 MeV 6 -sector $\mathrm{H}^{-}$ cyclotron $\gamma G$ ranges from -3.796 to -5.90 , so that the latter possibility is just excluded. However, a number of first- and second-order intrinsic resonances and zero-order imperfection resonances are crossed (see also figure 1):

[^0]

Fig. 1 : Deviations from various depolarizing resonances. Relative strengths are indicated by line thicknesses.
$3.796 \gamma=4$
$3.796 \gamma=5$
$3.796 \gamma=6-v_{z}-v_{r}$
$3.796 \gamma=6+v_{z}-v_{r}$
$3.796 \gamma=6-2 v_{z}$
$3.796 \gamma=6-v_{z}$

$$
\text { at } \begin{gathered}
51 \mathrm{MeV} \\
298 \mathrm{MeV} \\
200 \mathrm{MeV} \\
300 \mathrm{MeV} \\
390,420,440 \mathrm{MeV} \\
466 \mathrm{MeV}
\end{gathered}
$$



Fig. 2 : Measured polarization of the accelerating and decelerating beams. The lines are to guide the eye.
throughout the near resonance region out to 500 MeV (cf. figure 1). There is experimental confirmation of this extended effect in the spin down data (figure 2).
3. Depolarization near 300 MeV . - Direct evidence has been obtained linking the drop in polarization observed near 300 MeV to the 3.796 y $=5$ resonance, and corrective action has resulted in recovery of the lost polarization. The $(6,1,-1)$ and $(5,0,0)$ resonances are both possible candidates. The former depends on the intrinsic sixth harmonic of the magnetic field ( 2400 G ), but only through its radial derivatives, and it is further weakened by being of second order in the betatron oscillation amplitudes $\left(B_{h} \sim a_{z} a_{X} / r^{2}\right)$; the resulting depolarization is expected to be $\ll 1 \%$. The latter is an imperfection resonance driven by fifth harmonic horizontal field components, but is of zero order. Numerical integration of the spin precession between 280 and 330 MeV indicates that the fifth harmonic $\mathrm{Br}_{\mathrm{r}}$ and B 9 components measured during the 1974 field survey ( $\sim 2 \mathrm{G}$ ) would produce $4 \%$ depolarization; in fact a $3 \%$ loss is


Fig. 3 : Change in the vertical spin component for two rays with vertical betatron amplitudes of 3 and 6 mm tracked through the $3.796 \gamma=6-v_{z}$ resonance.


Fig. 4 : Contribution to $\mathrm{Br}_{\mathrm{r}}$ from each of the 13 harmonic coil sets. The arrows indicate the radii of the $(4,0,0)$ and ( $5,0,0$ ) resonances.
observed. To correct this, harmonic coil set No. 12 was powered in "first harmonic" $\mathrm{Br}_{\mathrm{r}}$ mode. Each set consists of six $60^{\circ}$ wide coils, No. 12 giving a maximum $\mathrm{Br}_{\mathrm{r}}$ near 300 MeV (figure 4). In this mode the currents in the six coils $(j=1 . .6)$ are given by

$$
\begin{equation*}
I_{j}=I_{0} \cos (j \pi / 3-\phi) \tag{7}
\end{equation*}
$$

This maximizes the first harmonic field at a phase angle $\phi$ ahead of the $\mathrm{j}=\mathrm{l}$ coil, and zeroes harmonics 3 n and $6 n \pm 2$; however, the $(6 n \pm 1)$ th higher harmonics-including the fifth-remain non-zero. Scans were made for different current amplitudes (from - 400 to +400 At ) at fixed phase $\left(140^{\circ}\right)$, and for different phases (from $200^{\circ}$ to $350^{\circ}$ ) at fixed amplitude. It was found possible to increase the polarization P at 400 MeV by $3 \%$ to its 200 MeV value or to lower it by as much as $13 \%$-a much larger effect than has been observed with any other cyclotron control parameter. Moreover, the form of the variation (figure 5) agrees well with that expected theoretically for small changes from (6) above:

$$
\Delta P=A\left|I-\underline{I}_{0}\right|^{2}
$$

where I stands for the vector current in H.C. 12 (assumed proportional to the field it produces) and $I_{0}$ for that needed to correct the existing imperfection. The curve is a quadratic fit to the data with $A$ and $I_{0}$ as free parameters. The optimum value for $\underline{I}_{0}$ was 225 At at $270^{\circ}$.

A similar resonance (3.796y $=4$ ) is expected at 51 MeV , driven by fourth harmonic imperfection fields. Particles tracked through 51 MeV show no significant depolarization. Although the field survey showed 1.6 G horizontal imperfections the apparent direction of their rotation is opposite to the sense of spin. 51 MeV , however, is below the extractable energy range so it is not possible to observe a loss in polarization directly as the resonance is crossed. Therefore, attempts have been made to drive the resonance using harmonic coil set H.C. 7 (see figure 5) and observe polarization changes at higher energy. For this purpose H.C. 7 was wired in "second harmonic" $\mathrm{B}_{\mathrm{r}}$ mode, where

$$
\begin{equation*}
I_{j}=I_{0} \cos 2(j \pi / 3-\phi) . \tag{8}
\end{equation*}
$$

This zeroes its contributions to harmonics $3 n$ and $6 \mathrm{n} \pm 1$ and maximizes those to harmonics $6 \mathrm{n} \pm 2$, particularly harmonics 2 and 4. So far, however, our attempts have not succeeded in provoking any significant change in polarization, either in the normal vertical component (at 400 MeV ), or in the normally zero horizontal component (at 200 MeV ).


Fig. 5 : Polarization of a 400 MeV extracted beam as a function of the deviation of the harmonic coil No. 12 current from its optimum value $I_{0}$. The current $I$ was scanned in amplitude at fixed phase (triangles) and vice versa (circles).
4. Horizontal polarization in the cyclotron.- Recently Hatanaka et al. B) have reported the acceleration of horizontally polarized protons to 65 MeV . The provision of extracted beams from a cyclotron with polarization available in any direction is of interest, as it would obviate the need for precession magnets and solenoids upstream of the experimental targets. Initial attempts have therefore been made to accelerate horizontally polarized $\mathrm{H}^{-}$ions through the TRIUMF cyclotron. The horizontal polarization was produced by adjusting the fields in the Wien filter in the injection line, which normally precesses the spin to maximize the vertical polarization ( $E=240$ in figure 6), to zero it instead $(E=-1000)$. The Wien filter could also be physically rotated to rotate the spin in the horizontal plane. To keep the spin precession as coherent as possible the cyclotron was run in separated turn mode and beam was extracted at 200 MeV . Using a vertically scattering polarimeter (provided by the BASQUE group) neighbouring turns were extracted and analysed, but no significant polarization could be observed. The coherency requirement for $\mathrm{H}^{-}$ions is twice as hard to satisfy as for protons, and for both, of course, the difficulty increases with kinetic energy. Further trials are planned.

With separated turns the passage of a beam with a vertical spin component through a depolarization resonance will yield a coherent horizontal component of spin. The direction of this component, when extracted, will depend on the number of turns made from the resonance. To maximize the intensity of a polarized beam TRIUMF normally operates with a wide, $45^{\circ}$ phase acceptance and a radial amplitude equivalent to several


Fig. 6 : Vertical polarization as a function of Wien filter electric field for spin up $\left(P_{+}\right)$and down ( $P_{-}$). The curves are sinusoidal fits to the data.
times the radius gain per turn from the rf voltage. Consequently at extraction there will be a spread in the number of turns made from the resonance and a spread in the direction of the horizontal spin vectors.

Some experimental groups ${ }^{9}$ ) have had indications of a coherent spin component in the horizontal plane at 300 MeV and 500 MeV . Several particles were tracked by GOBLIN through the $3.796 y=5$ resonance. The starting phases were spread over $40^{\circ}$ and the maximum radial amplitude was 6 mm . The spin vectors in the horizontal plane had a rms spread in angle of $6^{\circ}, 30^{\circ}$ and $50^{\circ}$ at simulated extractions 10,20 and 30 MeV beyond the resonance. Calculations made from 450 to 515 MeV show a correlation between horizontal spin direction and particle height above the beam plane. A stripping foil lowered partially into the beam will perform a vertical selection and some coherence in the horizontal plane would be expected. This is a common mode of operation which facilitates a change in the beam current shared between two beam lines by altering the relative heights of the foils. This coherence should be reduced as the foil is lowered further into the beam. The fairly strong correlation predicted between height and horizontal direction is probably due to the extended nature of the resonance (figure l).

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