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HIGH ENERGY SUPERCONDUCTING CYCLOTRONS

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Abstract.- A two-stage superconducting ring cyclotron could accelerate $400 \mu \mathrm{~A}$ proton beams from TRIUMF to $\bar{\sim} 9 \mathrm{GeV}$ for a kaon/neutrino factory. The field of the superconducting magnets has been accurately computed and the orbit properties investigated. Studies are reported of beam behaviour near major radial resonances, of the tolerances these require, and of techniques for pulsed operation for neutrino physics.

1. Introduction.- In a previous paper l) it was shown how a two-stage superconducting ring cyclotron could be used to accelerate the full current available from TRIUMF $(400 \mu \mathrm{~A})$ to 8.5 GeV . Such a machine would be a powerful source of kaons, pions, nucleons and neutrinos for nuclear and particle physics research. The physics which would become accessible has been discussed at a number of recent meetings in Vancouver 2,3) and Los Alamos ${ }^{4,5)}$, from which a strong case for such an accelerator appears to be emerging. The prime areas for study appear to be rare decay processes involving CP and other symmetry violations, medium energy neutrino interactions, K-nucleon interactions and hypernuclear physics.

The first stage of this Canadian University Cyclotron for Kaons (CANUCK) would be a 15-sector ring accelerating protons from 400 MeV at 7.5 m radius to 3 GeV at 10.1 m . The second stage would take the protons from 3 GeV at 20.2 m to 8.5 GeV at 20.6 m . Using SIN-style rf cavities the energy gain per turn would be up to 5 MeV in the first stage and up to 13 MeV in the second. The beam would normally be cw at 23 MHz , but could be pulsed (e.g. for neutrino experiments) with a duty factor of $1 / 15000$ (see section 4 below). The previous orbit studies relied on a simulated field with an artificial Woods-Saxon edge shape. In the present work the field has been calculated directly from the proposed pole and coil shape. The results confirm the previous studies.
2. Magnetic field design. - In order to calculate the field produced by the superconducting magnets we have used a modified version of the COILS code developed by Heighway ${ }^{6)}$. Like Westcott's code MAGHILL ${ }^{7}$ ) and that of Gordon and Johnson 8), COILS computes the field on the horizontal mid-plane of a symmetric magnet on the assumption that the iron is fully saturated and can be replaced by a current sheet around its vertical sides. These codes have been very successful in reproducing the fields of the Chalk River and MSU superconducting cyclotron magnets. COILS is restricted to magnets shaped from arcs of circles, for which the field on the mid-plane can be computed directly in terms of elliptic integrals, without explicit integration along the arc or up the side of the current sheet. In order to deal accurately with the non-circular coils of the high energy cyclotrons a modified version (COILX) has been developed in which integration along the coil is carried out numerically (although the analytic vertical integration is retained). The circular arc frame-
work of the code nevertheless remains useful, for instance for the geometrical problem of sectioning a thick coil into thin current sheets.

COILX has been implemented on the TRIUMF VAX computer operating under VMS. For a grid of $120 \times 120$ points on the mid-plane the computing time is 1 hour. The fields produced are then run on the equilibrium orbit code CYCLOP to determine the orbit properties. The optimum coil and pole shape is determined by an iterative process, starting from the shape found for the Woods-Saxon field edge ${ }^{1)}$. Figure 1 shows the field contours for one of the second-stage magnets together with selected orbits. To keep the field edge sharp a polegap of 2.5 cm is used. The coil width is 8 cm , the coil current 2.1 MA and the maximum field $\sim 5 \mathrm{~T}$. As mentioned above, the focusing properties of this field were found to be very similar to those previously reported for a simulated field 1 ).


Fig. 1 : Magnetic field contours and proton orbits computed for the second stage ring cyclotron. The field contours are plotted at 1 T intervals from 0 to 5 T and the orbits at 0.5 GeV intervals from 3 to 8.5 GeV .


Fig. 2 : The position of the unstable fixed points in the vicinity of the $v_{r}=30 / 3$ resonance which occurs near 7.45 GeV . The dashed triangles connect the 7.42 and 7.48 GeV fixed points, about 2.3 turns on either side of the resonance. Also shown is the anticipated size of the beam ellipse; some particles will experience non-linear or unstable motion for 5 turns.

## 3. Dynamics of the second stage ring cyclotron.- A

 study of the beam stability and field tolerance associated with the 3 to 8.5 GeV ring has been begun using our equilibrium orbit code CYCLOP. Initial results of this study are presented here. The field used was that giving the betatron tunes reported at the Bloomington Conference l). It was seen immediately that the results are strongly affected by the field description and computational procedures used, and our results must be considered to be approximate. This sensitivity also undoubtedly reflects the precision of manufacture, assembly and trimming that will be required.3.1 Intrinsic resonances. - The field is 30 fold symmetric and the intrinsic resonances $30 / 3$ and $30 / 4$ will be met at 7.45 and 5.68 GeV ; the former is of lower order. Figure 2 shows how the unstable fixed points converge on and diverge from the equilibrium orbit fixed point near 7.45 GeV . Also shown is the area of phase space expected to be occupied by the beam. The shape was obtained from the Twiss parameters calculated by CYCLOP, the size by scaling the measured TRIUMF extracted emittance of $2 \pi$ mm-mrad at 0.5 GeV to account for adiabatic damping. Figure 3 shows a static phase space plot near the $30 / 3$ resonance for a field computed for a superconducting magnet (though not fully optimized) and shows the degree of nonlinearity expected for amplitudes approaching the fixed points; note that not all turns are plotted on the curves. From figures 2 and 3 we expect that for the particles of larger amplitude the radial motion will be non-linear or unstable for 4 or 5 turns. Even fewer turns are spent traversing the $30 / 4$ resonance.
3.2 Imperfection tolerances.- As an example, we chose to make calculations at $\nu_{r}=8$. The motion appears to be non-adiabatic for a few turns in the integer resonance for an 8 th harmonic field imperfection $\Delta B_{8} / \bar{B}$ of $\sim 10^{-5}$. The half-integer resonance, gradient of the $16^{\text {th }}$ harmonic, gives unstable motion over one turn with a gradient of $\mathrm{d}\left(\Delta \mathrm{B}_{16} / \bar{B}\right) / \mathrm{dR}$ of $3 \times 10^{-5} / \mathrm{cm}$ (see figure 4). Although the amplitude increase is very small for this imperfection, perhaps $3 \%$, the gradient tolerances will be of this order since there


Fig. 3 : Static phase space $\left(x, p_{x}\right)$ figure at 7.40 GeV in the vicinity of the $v_{r}=30 / 3$ resonance. Not all turns are shown. $=$ fixed point.
are 7 such resonance crossings in this ring and we wish to keep the overall growth to less than $10 \%$.
4. Pulsed operation. - An important area of research open to a cyclotron facility in the multi-GeV energy range is that of neutrino physics. Because of the necessity of discriminating against the ever-present cosmic rays it is essential for this area of research that the facility be capable of operation in a pulsed mode. One possible way of achieving this type of operation in the cyclotron facility is to stack approximately one hundred turns of beam near the extraction radius of TRIUMF and then to apply an axial electric field so that the hundred turns are intercepted by a stripper foil and thus extracted. If TRIUMF is also operated so that only one of the five radial bunches rotating around the cyclotron's centre is filled with beam, the result would be a 7 ns pulse of extracted beam every $22 \mu s$. While the beam pulse is being accelerated through the 3 GeV booster cyclotron, the dee voltage rising with radius would compress the pulse to a width of some 2 ns . Thus the net duty factor from the cyclotron facility would be about $1 / 10^{4}$, which is suitable for many neutrino experiments.


Fig. 4 : The half-trace of the horizontal transfer matrix in the vicinity of the $v_{r}=16 / 2$ resonance when a $16^{\text {th }}$ harmonic field gradient $d\left(\Delta B_{16} / \bar{B}\right) / d R$ of $3 \times 10^{-5} / \mathrm{cm}$ is superimposed. The motion is unstable for 1 turn.

# Proceedings of the 9th International Conference on Cyclotrons and their Applications September 1981, Caen, France 



Fig. 5 : The phase history of the ions starting at $57.6^{\circ}$ and $68.5^{\circ}$ for beam stacking between 7600 mm and 7625 mm radius. All the ions in the cross-hatched area are collected in one packet. The lower curves give the turn density for the two extreme values of the phase.

There are two techniques which have been investigated for stacking the beam in TRIUMF. One is magnetic ${ }^{9)}$, requiring a field which falls below isochronism so the ions get out of phase and start back towards the centre (figure 5). The phase change per radial interval is

$$
\frac{\delta \alpha}{\delta R}=\frac{2 \pi h}{\gamma} \frac{\Delta B}{B_{i}} \cdot \frac{1}{4 V_{D} \cos \alpha} \cdot \frac{E_{0}}{R_{C}} \beta\left(1+\mu^{\prime}\right)
$$

where $h=$ harmonic ( $h=5$ for TRIUMF), $\Delta B$ is the deviation from the isochronous field $B_{i}, V_{D}$ is the dee voltage, $E_{0}$ is the rest energy, $R_{C}$ is the cyclotron radius and $\mu^{\prime}=R / B d B / d R$. Also the radial number density of turns is

$$
\frac{\delta n}{\delta R}=\frac{E_{0}}{R_{C}} \frac{\beta \gamma\left(1+\mu^{\prime}\right)}{4 V_{D} \cos \alpha} .
$$

Consider a field which is 1.9 G below isochronism at a radius of 7600 mm and drops to 2.1 G below isochronism at 7625 mm . Ions which start out at the centre with a phase spread of $5^{\circ}$ can be spread over the phase interval of $57.6^{\circ}$ to $68.5^{\circ}$ at $R=7600 \mathrm{~mm}$ by tailoring the magnetic field. Approximately 100 turns of the accelerated and decelerated beam will be involved in the above radial interval and can be extracted as one packet.

The second technique of stacking the beam depends upon the expansion of the phase of the beam with reduction in the accelerating voltage according to the relation $V \Delta \sin \alpha=$ constant ${ }^{10)}$. The effective dee voltage could be reduced by a ground plane starting at $r=0$ (which could correspond to $R=7500 \mathrm{~mm}$ ), according to some relations such as

$$
\frac{V}{V_{D}}=\frac{1}{2}\left(1+\cos \frac{\pi r}{d}\right) .
$$

Now the radial number density of turns becomes

$$
\begin{aligned}
\frac{\delta n}{\delta r} & =\frac{E_{0}}{R_{C}} \cdot \frac{B_{\gamma}\left(1+\mu^{\prime}\right)}{4 V_{D}} \cdot \frac{1}{V / V_{D} \cos \alpha} \\
& \sim \frac{1}{V / V_{D} \cos \alpha}
\end{aligned}
$$

if $B$ is held isochronous. For the TRIUMF example with $V_{D}=80 \mathrm{kV}$ and $\mathrm{d}=150 \mathrm{~mm}$ and the ions spread in phase from $\alpha=5^{\circ}$ to $\alpha=10^{\circ}$ at $r=0$, there is a compaction of 150 turns in the radial interval of 25 mm at $R=7600 \mathrm{~mm}$.

According to our preliminary investigations it appears to be possible to produce a density of turns which is somewhat higher with the falling rf voltage technique, but the magnetic stacking appears to have some advantages in ease of application.
5. Conclusions. - The magnetic field distribution for the superconducting magnets for the proposed kaon factory cyclotron has been computed and shown to give very similar orbit properties to the artificial fields previously used. The beam behaviour has been studied in the neighbourhood of a number of radial resonances. For the most important intrinsic resonances $\nu_{r}=30 / 3$ and $30 / 4$ it appears that the non-linear region can be crossed in no more than 5 turns. The imperfection resonance $\nu_{r}=8$ (or $16 / 2$ ) has also been examined. The tolerances on the 8th harmonic field and 16th harmonic gradient are no worse than those on similar imperfections in the TRIUMF 520 MeV cyclotron.

For pulsed operation for neutrino physics, duty factors $1 / 10000$ at 3 GeV or $1 / 15000$ at 9 GeV could be obtained by modifying the magnetic or rf fields in the TRIUMF cyclotron to produce high turn compaction and extracting with the aid of a pulsed deflector.

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## " dISCUSSION "

F.G. RESMINI : Have you given any thought to use high field magnets, like 8 T instead of the more customary 5 T ?
M.K. CRADDOCK : The optimum field from the point of view of performance and economy will be considered during the forthcoming design study.
W. SCHOTT : What is your maximum spiral angle ?
M.K. CRADDOCK : About $75^{\circ}$, a few degrees more than in the TRIUMF cyclotron - but with a much better defined edge (the pole gap being $2-5 \mathrm{~cm}$ rather than 50.8 cm ).

# Proceedings of the 9th International Conference on Cyclotrons and their Applications 

 September 1981, Caen, France" DISCUSSION " (continued)
H. BLOSSER : I wanted to ask you wether you are doing work on building a superconducting coil ?
M.K. CRADDOCK : At the moment no. I should say, our plans in the next year are to work intensively on the
design of both dynamics and engineering, to be in a position to complete a proposal for funding by about the end of 1982. At this stage, if things look favorable, we would be pushing hard to build a test coil.

