Resonant proton beam depolarisation in a synchrocyclotron

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ABSTRACT

The main reasons which prevent the conservation of horizontal beam polarisation in a synchrocyclotron are pointed out.

With a vertically polarised beam, one finds that the horizontal field perturbations should be <5 G in the CERN synchrocyclotron in order to avoid 20% loss in polarisation. Phase incompatibilities between the horizontal field and polarisation may, however, reduce this figure.

1. INTRODUCTION

Resonant beam depolarisation has been studied in almost every kind of cyclic particle accelerator[1-5] but the problem does not seem to be completely solved in the medium energy range and for the moderately inhomogeneous field encountered in synchrocyclotrons. Nevertheless, serious doubts as to the conservation of polarisation in the accelerated beam were expressed at Rochester[5] on account of the small azimuthally varying displacements of the magnetic median plane involving horizontal depolarising fields. In this paper some requirements about other field imperfections encountered in the CERN SC are pointed out and it turns out that the depolarising effects are then reduced by phase incompatibilities between a circularly polarised horizontal field and a precessional polarisation rotating in relative opposition.

2. DESCRIPTION OF RESONANCES

The relativistic spin Eqn (6) for particles that perform vertical betatron oscillations or encounter horizontal field imperfections $\delta\vec{B}$ with frequency ω can be reduced to a single perturbation $2\vec{\Gamma}$ parallel to $\delta\vec{B}$ which yields the following spin rotation

$$\frac{d\vec{S}}{dt} = \vec{S} \left[\vec{\Omega}_p + 2\vec{\Gamma}(\omega t) \right] \tag{1}$$

Ω_p: angular frequency of the spin precession around the main vertical field, connected to the cyclotron frequency Ω by

$$\Omega_p = \gamma(g/2 - 1)\,\Omega \tag{2}$$

γ: relative mass m/m_o of the proton
g: gyromagnetic ratio
t: laboratory time.

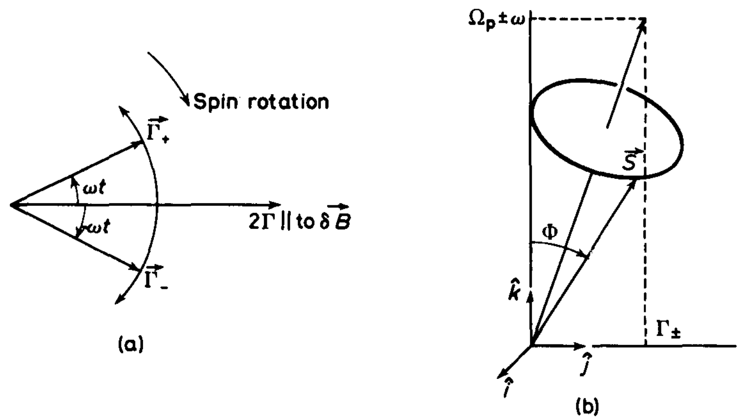

Fig. 1. (a) Horizontal plane; (b) spin precession in the rotating frames

The excitation experienced by the spin gives rise to a significant change in the spin orientation if the frequencies Ω_p and ω get in tune. To investigate this situation it is useful to transform[7] the oscillatory rotation vector $2\vec{\Gamma}$ into two rotating vectors $\vec{\Gamma}_\pm$ with angular frequencies $\pm\omega$, so that with respect to the rotating frames where $\vec{\Gamma}_\pm$ are fixed, the spin Eqn (1) is transformed into

$$\frac{d\vec{S}}{dt} = \vec{S} \left[(\Omega_p \pm \omega)\hat{k} + \vec{\Gamma}_\pm \right] \tag{3}$$

(1) The Γ_+ component rotating in the direction opposite to the spin precession (with respect to the initial frame) still results in a nearly vertical spin precession in the rotating frame on account of

$$\frac{\Gamma_+}{(\Omega_p + \omega)} \ll 1 \tag{4}$$

This is a case of phase incompatibility between a rotating perturbation and the spin precession, with no significant change in the spin orientation; this will be used in section 4 in connection with circularly polarised field harmonics.

(2) The Γ_- component rotating in phase with the spin precession causes a reversal of the spin rotation vector if the time varying vertical component goes through the resonance

$$\Omega_p - \omega = 0 \tag{5}$$

at a certain rate depending on the energy increase in the beam.

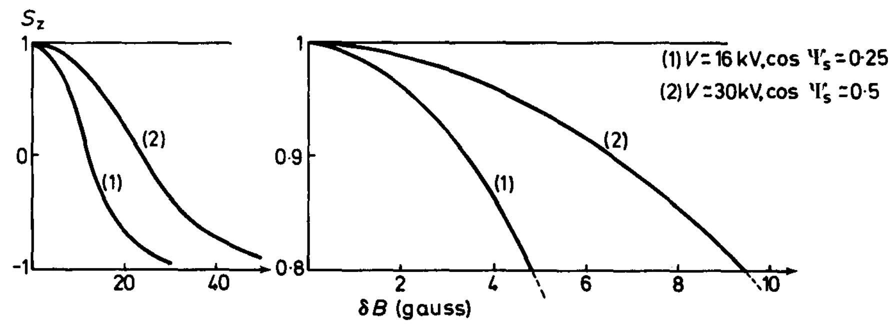

Fig. 2. Depolarisation curves in the CERN SC

Using the calculations made by Froissart and Stora[1] for the resonant decrease of the vertical spin component S_z, one finds that a tolerable limit for the horizontal oscillatory field at resonance ranges from ~5 G (existing acceleration programme) to ~10 G (new programme[8]) if one wants to prevent more than 20% loss in the vertical polarisation (Fig. 2).

3. SELECTING A DIRECTION FOR THE BEAM POLARISATION

The elementary polarisation $\vec{S}$ of a particle consists in a vertical component (angle φ defined in Fig. 1)

$$S_z = \cos\varphi, \; (|\vec{S}| = 1) \tag{6}$$

and a horizontal rotational component

$$S_+ = S_x + jS_y = \sin\varphi \, e^{j\alpha}, \tag{7}$$

with

$$\alpha = \gamma(g/2 - 1)\theta,$$

$$\theta = -\int \Omega \, dt.$$

A resonant spin rotation (angular spread φ) results in an exchange between the two polarisations in Eqns (6) and (7), without any loss of magnitude for $\vec{S}$, so that one cannot actually speak of one particle depolarisation.[3] Building up the total beam polarisation with such elementary polarisations, the model of an exchange between two directions holds as long as the spread of spin orientations α is kept small for every particle in the beam. A decay of the horizontal polarisation would occur in synchrocyclotrons mainly for the two following reasons.

3.1. *Synchrotron oscillations of particles*

The spread of the accelerated beam inside the separatrix of the phase-energy plane results in a radial and azimuthal spread of the beam in the real median plane (beam 'sausage'). According to Eqn (7) the dependence of α with both azimuth θ and radius $r(\gamma)$ is generating a spread of spin orientations α in the beam. The calculations[9] show that the horizontal polarisation subsisting for small synchrotron oscillations vanishes for large oscillations. On the contrary, the resonant loss of vertical polarisation is found roughly independent of the amplitude of the oscillations, the only effect being then a broadening of the resonances.

3.2. *Regenerative extraction of the beam*

In order to avoid the horizontal spread of spin orientations in the beam, all particles would have to perform the same number of cyclotron turns. According to orbit calculations[10] in the regenerator field, this is a basic requirement which is not fulfilled by the extraction process.

Therefore one concludes in favour of a vertically polarised beam so that any resonant increase of the horizontal spin component of a particle can be considered as a true depolarisation.

4. RESONANT PERTURBING FIELDS IN THE CERN SC

4.1. *The radial focusing field*

The resonant terms in a series expansion of the field in the vicinity of the equilibrium orbit are at least of third order with respect to the radial and vertical displacements x, z from the reference orbit. They are felt as an oscillatory radial field by particles that perform vertical betatron oscillations. One finds that they are $<0{\cdot}5$ G in magnitude which causes no significant depolarisation.

4.2. *Azimuthally varying fields*

We have attempted to establish a relation between the phases of radial and azimuthal field harmonics in order to deal with the resulting horizontal field which interacts as a whole with the polarisation.

An increase of the flux density in the SC field takes place in the vicinity of the yokes and the field at right angles is ~ 20 G lower; this results in a second field harmonic b_{z2} (r, z) increasing with radius. The model of square-wave pole tips with radially increasing depth is illustrative of this imperfection. Along a circular path displaced from the median plane (Fig. 3) the radial and azimuthal field components are always seen rotating in opposition with the beam. On the contrary a radially decreasing depth would reverse the radial field harmonic and keep the azimuthal field unchanged, so that the horizontal field would then rotate in the direction of the beam.

This figure has been completed[9] by a Fourier analysis of the field imperfections with the result that the following requirements define a non-resonant structure on account of the phase incompatibility described in

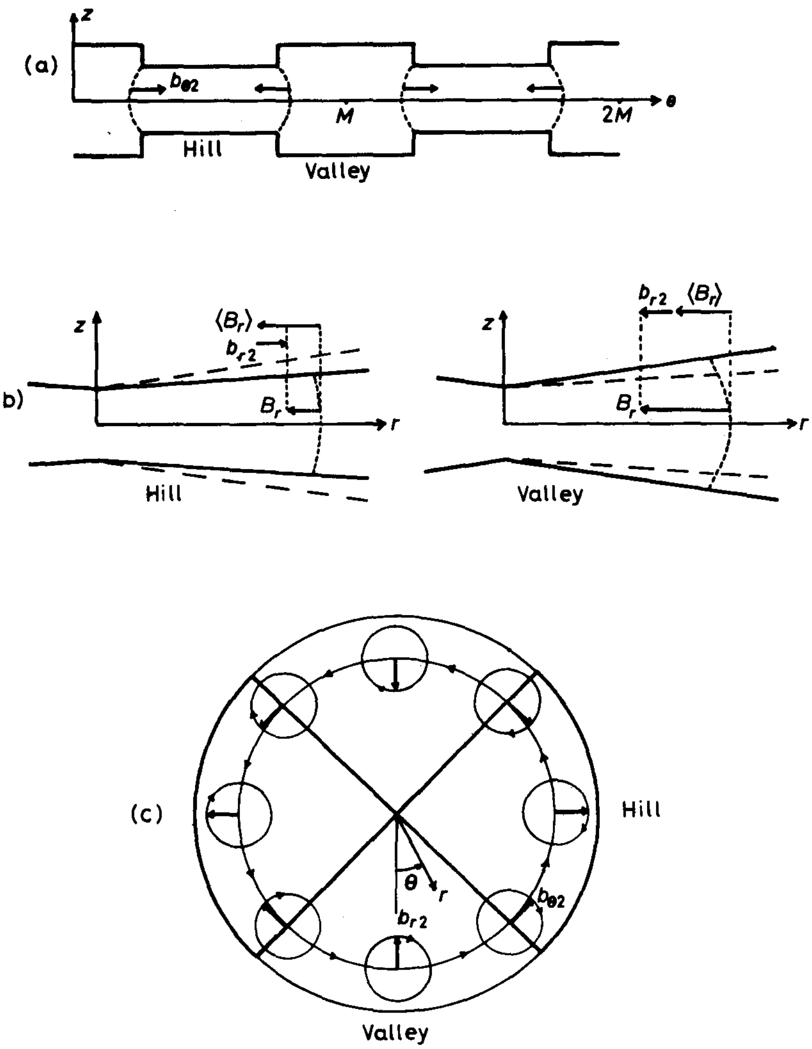

Fig. 3. (a) Azimuthal field; (b) radial field; (c) rotating horizontal field along a circular path

Section 2: the field harmonic should be:

(1) radially increasing [field measurements give approximatively b_{z2} $(r, z = 0) \sim r^2$]

(2) vertically uniform, $\partial b_{z2}/\partial z = 0$ and $\partial^2 b_{z2}/\partial z^2 = 0$ (the second derivative is theoretically zero with $b_{z2} \sim r^2$).

In practice the magnetic median plane is 15-20 mm below the mechanical median plane in the CERN SC, and this would be caused by the sinking of the horizontal yoke members under the weight of the poles.[11] Assuming that a parabolic distortion of the median plane is generating a second radial field harmonic of

8 G at the edge of the poles, the field differs from the non-resonant structure by a very small factor

$$\frac{\partial b_{z2}}{\partial z} \simeq 0{\cdot}1 \text{ G/cm at a radius of 2 m.}$$

This is enough to perturb the circularity of the horizontal field which contains now one-third of oscillatory component. The resonance occurs at 110 MeV with a field of 0·5 G inducing <1% loss in polarisation. The depolarisation is square field dependent and therefore is reduced by a factor of ~10 with regard to a full oscillatory field harmonic.

4. CONCLUSION

When explained by the vicinity of the yoke members, the second field harmonics encountered in the CERN SC compare almost favourably with a non-resonant structure. Consequently no significant loss of polarisation should occur in the CERN SC provided that the order of magnitude for the second radial field harmonic is not greater than assumed (8 G at a radius of 2 m). To complete this work calculations dealing with the regenerative extraction process are still in progress; they will be published as a CERN report.

ACKNOWLEDGEMENTS

The author is indebted to Prof. E. Regenstreif (Faculté des Sciences, Rennes) and to Dr N. Vogt-Nilsen (CERN) for many helpful discussions.

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