Conversion studies for the SREL 600-MeV synchrocyclotron *

M. Reiser, H. Kim, and D.LeVine
University of Maryland, College Park, Maryland, U.S.A.

Presented by M. Reiser

ABSTRACT

A study of the feasibility of improving the 600-MeV synchrocyclotron at the Space Radiation Effects Laboratory has been carried out at the University of Maryland. To achieve the desired beam currents in the range of 20–50μA a change of the main magnetic field is required as well as an increase of dee voltage and repetition rate. Conversion into an isochronous cyclotron is considered inpractical due to severe difficulties in magnetic field design. The ion capture process in various other non-isochronous field shapes of a frequency-modulated conversion scheme was investigated with the aid of a special computer program. Use of a magnetic cone to achieve adequate vertical focusing (ν_z values between 0·1 and 0·15) was found to be feasible without loss of capture efficiency.

The computer studies revealed the existence of trapped particles which oscillate about an equilibrium radius throughout the entire fm cycle without returning to the centre. A very attractive solution of improving the SREL machine appears to be a magnetic field which is isochronised for deuterons (360 MeV) permitting operation either in a fixed-frequency mode (for deuterons and alphas) or as a fm sector-focused cyclotron for 600-MeV protons.

1. INTRODUCTION

Improving some of the existing synchrocyclotrons to obtain substantially higher beam currents than are presently available appears to be a promising and less expensive alternative to developing new accelerators of the sector-focusing type in the range of several hundred MeV of proton energy.

Two conversion projects have already been funded. One is the 600-MeV synchrocyclotron at CERN[1] which will be modified to increase internal beam

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currents from presently 1 μA to ~10–15 μA. This increase in intensity will be achieved by building a new rf system with higher dee voltage and repetition rate and by improving the central region while the magnetic field remains essentially unchanged.

The other project under way is the NEVIS synchrocyclotron at Columbia University.[2] This machine will be converted into a (non-isochronous) sector-focused synchrocyclotron whereby the beam current will be raised from presently 2 μA to an estimated 5–40 μA while simultaneously the energy will be increased from 385 to 550 MeV. The Columbia conversion scheme also involves a modification of the rf system and improvement of the central region as at CERN; however, it goes a step further than CERN in the change of the magnetic field to a three-sector pole-tip configuration with rising average field. A unique feature of the Columbia approach is the use of so-called 'floating shims' mounted inside the dee structure to increase the flutter and thus the vertical focusing at the centre of the cyclotron.

This paper presents the major results of a study concerning the possible conversion of the 600-MeV synchrocyclotron at the Space Radiation Effects Laboratory (SREL), Newport News, Virginia. The main objective of this study for the SREL machine (which is a copy of the CERN synchrocyclotron) was to investigate the feasibility of achieving internal beam currents in the range of 20 to 50 μA. Three schemes of SC conversion have been considered.

(1) Conversion into an isochronous cyclotron.
(2) Increase of rf voltage and repetition rate, with improvement of the central region but no change of the main magnetic field (CERN solution).
(3) Conversion into a sector-focused synchrocyclotron (similar to Columbia approach).

Of the three alternatives the last scheme appears to be the most promising and was therefore studied in greater detail. In particular we have investigated the feasibility of using a bump to achieve good vertical focusing in the central region rather than shims in the dees. The ion capture process in various field shapes was studied with a computer program which integrates the equation of motion. Of special interest is a field which is isochronised for deuterons (360 MeV) permitting the converted machine to operate in either a fixed frequency mode (for deuterons and alphas) or with the same fields in a frequency-modulated mode for protons.

Isochronisation for protons appears to be impractical due to difficulties in field shaping, while the CERN solution would not give the desired beam intensity.

2. CONVERSION TO AN ISOCHRONOUS CYCLOTRON

The desired beam intensities of 20 to 50 μA could readily be achieved by conversion to an isochronous machine, if this were possible. However, there are several formidable problems which, though not entirely insurmountable, would make such a drastic modification of the SREL synchrocyclotron extremely difficult and, therefore, in our opinion impractical.

(1) Resonances in the radial motion, such as $\nu_r = 3/2$ or $\nu_r = 2$, pose a limit to the energy that can be achieved in an isochronous cyclotron of given pole diameter with number of sectors less than six. By increasing the number of magnet sectors, N, it is possible to shift these resonances to higher energies. In the case of the SREL machine at least six sectors would be

necessary and even then the energy would be limited to ~515 MeV, as Blosser[3] found in an independent study. Given the fixed pole diameter of 5 m it would be very difficult to obtain the necessary flutter at large radii.

(2) To achieve adequate vertical focusing in the centre, three sectors would have to be employed in this region. This introduces the problems of the gap-crossing resonance[4] and of the transition from $N = 3$ to $N = 6$ at some intermediate radius.

(3) The most severe difficulty of such a conversion to an isochronous field is posed by the extremely tight field tolerance requirements. With an assumed energy gain of 250 keV per turn the total number of turns would exceed 2400 which is almost 10 times larger than in existing isochronous cyclotrons. To avoid total loss of beam, field tolerances in this case would have to be better than 1 in 10 000; this is not entirely beyond the state of the art, but clearly an order of magnitude beyond the design and engineering difficulties of existing cyclotrons.

Although the conversion to an isochronous machine appears to be an impractical solution for the SREL synchrocyclotron we have investigated in some detail a four-sector isochronous field configuration. Of particular interest was the question whether the $\nu_r = 4/3$ resonance can be passed without excessive beam distortion in such a field geometry. Since the four-sector field is to be considered for the case of conversion to a non-isochronous, sector-focused SC, we wanted to know whether a field shape could be chosen in which the

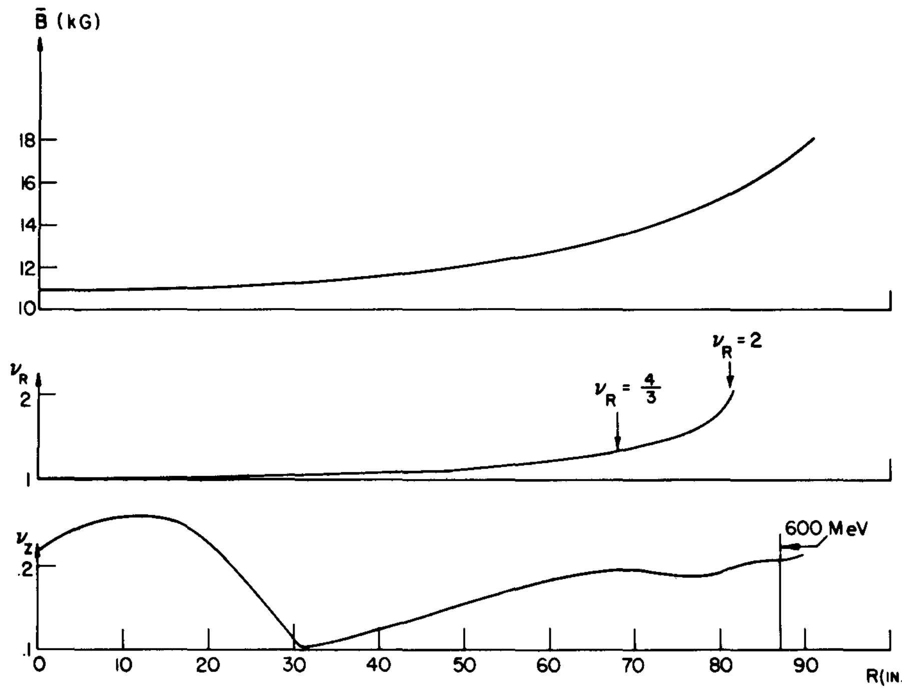

Fig. 1. Computed $\bar{B}(r)$, ν_r, and ν_z in the case of a four-sector isochronous proton field (600 MeV, 90 in radius)

4/3 resonance could either be passed or be used for extraction. With the aid of a special computer program a field shape was simulated with a pure fourth harmonic variation in azimuth of constant relative amplitude (flutter) of 0·35 and with a spiral angle such that ν_z was between 0·1 and 0·2 for all radii. The betatron frequencies ν_r and ν_z for this field were calculated with program CYCLOPS and are plotted, together with the isochronous field (for 600 MeV protons) in Fig. 1. As indicated in the figure the $\nu_r = 4/3$ resonance appears at a radius of 68 in, corresponding to an energy of 239 MeV, while $\nu_r = 2$ occurs at 467 MeV. The average field at extraction radius (90 in) is 17·9 kG in the SREL machine.

Figs 2 and 3 show radial phase plots—computed with the GOBLIN program —of accelerated particles traversing the 4/3 resonance. In both cases the energy gain per turn was assumed to be 170 keV. If the beam has an incoherent betatron amplitude of 0·1 in (Fig. 2) the distortion is acceptable. However, if the amplitude is 0·2 in (Fig. 3) severe phase-space deterioration occurs, and the beam would no longer be useful for further acceleration.

In a sector-focused SC the energy gain would be smaller than 170 keV; one would therefore conclude from these results that traversal of the 4/3 resonance in this case is not possible. (Indeed, we did a computer run for an amplitude of 0·1 in with an energy gain of 30 keV per turn, and found that the phase space was severely distorted.) The field shape should therefore be chosen such that $\nu_r < 4/3$ everywhere except perhaps at extraction where the resonance could be employed to get the beam out. Using $\nu_r = 4/3$ for beam extraction in the case of a four-sector machine is being investigated and the preliminary results are promising.

3. SYNCHROCYCLOTRON WITH GRADIENT FOCUSING

3.1. *Capture efficiency and beam current*

The well-known disadvantage of the frequency-modulated cyclotron as compared to the CW machines is that ions are only captured during a small time interval in which the electric frequency is close to the orbital frequency near the ion source. According to the analytical theory of Bohm and Foldy[5] the frequency interval during which capture takes place is given by:

$$\Delta\omega = 2\omega_o \left(\frac{KeV}{\pi E_o}\right)^{1/2} L(\varphi_o, \varphi_s), \qquad (1)$$

where K depends on the field index, $k = r/B \cdot \mathrm{d}B/\mathrm{d}r$, and $\beta = v/c$ in the centre:

$$K = 1 - \frac{k}{(1+k)\beta^2} \qquad (2)$$

E_o is the particle's rest energy, ω_o the orbital frequency at the centre ($r = 0$), eV is the maximum energy gain per turn; $L(\varphi_o, \varphi_s)$ is a function of starting phase φ_o of the ions (with respect to the rf voltage) and of the phase φ_s of the synchronous particle which was calculated by Bohm and Foldy for $\varphi_o = 0$ and for other values of φ_o by Kullander.[6]

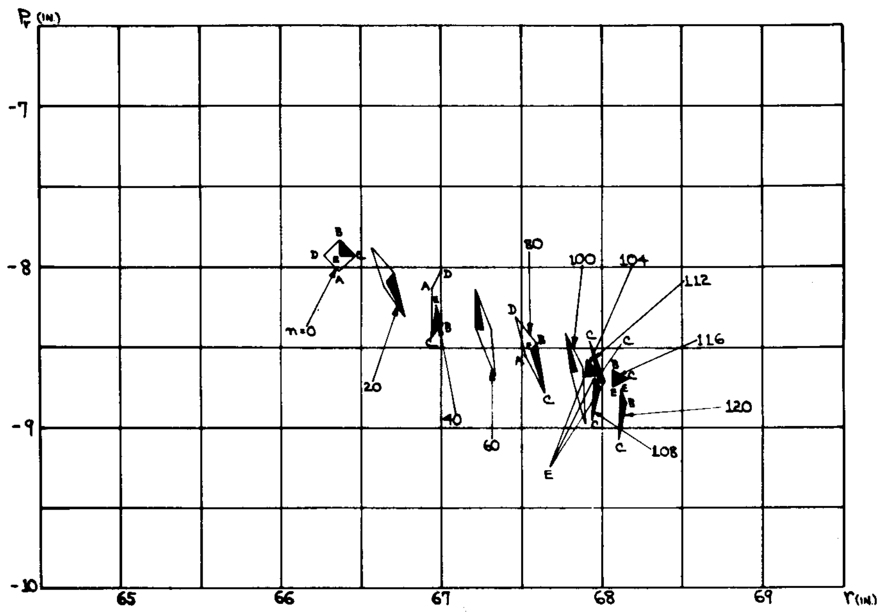

Fig. 2. Phase plots for beam with 0·1 in radial amplitude traversing $\nu_r = 4/3$ resonance

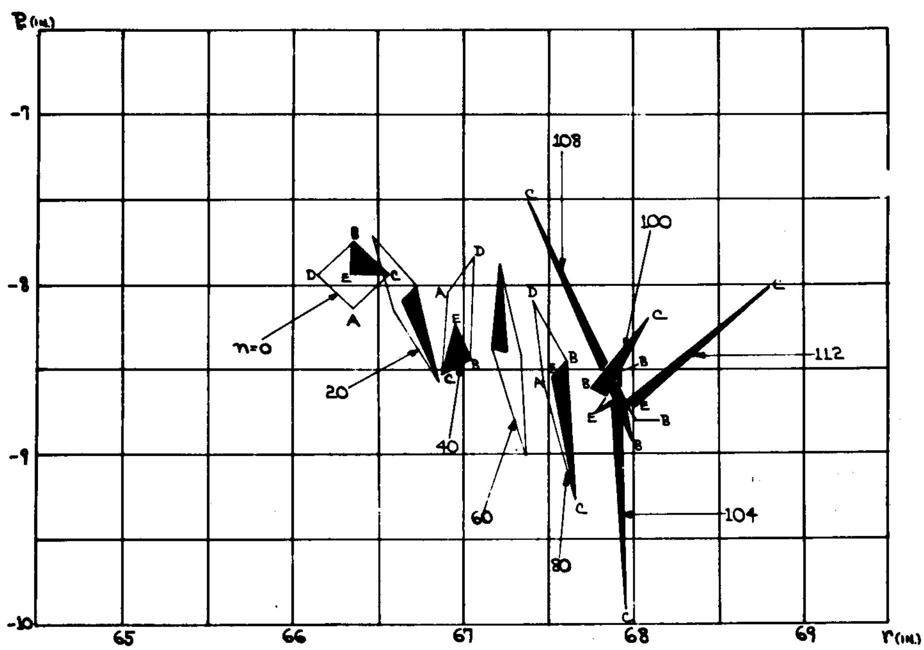

Fig. 3. Phase plots showing effect of $\nu_r = 4/3$ resonance in case of 0·2 in initial amplitude

Assuming a linear time variation of the electric frequency during ion capture, with a slope of $d\omega/dt$, the capture time is given by

$$\Delta t = \frac{\Delta\omega}{\left|\frac{d\omega}{dt}\right|} = \frac{2\omega_o}{\left|\frac{d\omega}{dt}\right|}\left[\frac{KeV}{\pi E_o}\right]^{1/2} L(\varphi_o, \varphi_s). \tag{3}$$

The slope of the electric frequency during capture is determined by the initial rate of change of the orbital frequency of the synchronous particle according to the relation:

$$\frac{d\omega}{dt} = -\frac{\omega_o^2\, KeV \cos\varphi_s}{2\pi E_o} \tag{4}$$

Introducing Eqn (4) into Eqn (3) we find for the capture time:

$$\Delta t = \frac{4}{\omega_o}\sqrt{\frac{\pi E_o}{KeV}}\,\frac{L(\varphi_o, \varphi_s)}{\cos\varphi_s} \tag{5}$$

The capture efficiency ϵ is defined by the ratio of Δt to the rf modulation period, T_m. The latter in turn is proportional to the total acceleration time, T_a, where $\delta = T_m/T_a$ is typically in the range of 1·2–1·5. Thus, with Eqn (5):

$$\epsilon = \frac{\Delta t}{\delta T_a} = \frac{\Delta t}{T_m} = \frac{4}{\omega_o T_m}\sqrt{\frac{\pi E_o}{KeV}}\,\frac{L(\varphi_o, \varphi_s)}{\cos\varphi_s} \tag{6}$$

The total acceleration time of the synchronous particle from the ion source ($E_k = 0$ at $r = o$) to full energy, E_k at radius r_m, is determined by:

$$T_a = \int_{E_o}^{E_o + E_k} \frac{2\pi}{\omega e V \cos\varphi_s}\, dE \approx \frac{E_k}{\bar{f} e V \cos\varphi_s} \tag{7}$$

where $\bar{f}$ denotes an average value of the electric frequency. If φ_s = const one obtains an equivalent expression in terms of magnetic field and radius

$$T_a = \frac{2\pi}{V\cos\varphi_s}\int_o^{r_m} Br(1 + k)dr \tag{8}$$

which in the special case of a uniform field ($B = B_o$, $k = o$) yields:

$$T_a = \frac{\pi B_o r_m^2}{V \cos \varphi_s} \tag{9}$$

and for a field varying linearly in radius (from B_o to B_m):

$$T_a = \frac{\pi B_m r_m^2}{V \cos \varphi_s} \left(1 + \frac{B_m - B_o}{3B_m}\right) \tag{10}$$

Using the approximate expression for T_a of Eqn (7) in Eqn (6) we find for the capture efficiency:

$$\epsilon = \frac{4\bar{f}}{\omega_o \, \delta E_k} \sqrt{\frac{\pi E_o \, eV}{K}} \; L(\varphi_o, \varphi_s) \tag{11}$$

which shows that the efficiency increases with the square root of the energy gain per turn.

The average internal beam current, I, in a synchrocyclotron is proportional to the peak current, I_o, the capture efficiency, ϵ, and the microscopic duty factor, $\Delta\varphi_o/2\pi$:

$$I = I_o \; \epsilon \frac{\Delta\varphi_o}{2\pi} = I_o \frac{\Delta t}{T_m} \frac{\Delta\varphi_o}{2\pi} = I_o \; \Delta t \, f_m \frac{\Delta\varphi_o}{2\pi} \tag{12}$$

As was first pointed out by McKenzie, the peak current in synchrocyclotrons is space-charge limited,[7] rather than by the ion source, and is approximately determined by the relation:

$$I_o = 2\epsilon_o \, \omega h \, V_1 \, \nu_{z\,\mathrm{min}}^2 \tag{13}$$

V_1 is the voltage gain per turn, $2h$ the maximum beam height (less than, or equal to, the internal height of the dee), $\omega \approx \omega_o$ the ion frequency in the centre, and $\nu_{z\,\mathrm{min}}$ the minimum value of the vertical focusing frequency. The minimum value of ν_z is determined by both the vertical magnetic force as well as the electric force in the acceleration gap[8] and occurs usually at a small radius near the ion source.

Eqns (11), (12), and (13) show that the average beam current in a synchrocyclotron is roughly proportional to the 3/2 power of the voltage gain per turn (or the dee voltage) and $\nu_{z\,\mathrm{min}}^2$:

$$I \propto V^{3/2} \, \nu_{z\,\mathrm{min}}^2 \tag{14}$$

The current also depends on the magnetic field shape in the centre, represented by the factor K, and the choice of the synchronous phase φ_s, and starting phase φ_o, as defined by the function $L(\varphi_o, \varphi_s)$.

3.2. *SC improvement without change of main magnetic field*

From the theory outlined in the previous section the low beam intensity in existing FM cyclotrons is readily understood: both the dee voltage as well as the vertical focusing forces are relatively small in these machines. Consequently, any SC improvement scheme is aimed at either an increase of dee voltage or an increase of both dee voltage and magnetic focusing. A noticeable increase in ν_z is not possible without introducing sector-focusing and therefore changing the main magnetic field. Increasing the dee voltage (and repetition rate) without changing the magnet, on the other hand, is less expensive, although the gain in beam current is not as large as in the case of a sector-focusing field. This solution was chosen by the CERN group. They plan to increase the dee voltage by a factor of 6 (from 5kV at injection to 30 kV) and the repetition rate from presently 50 Hz to ~830 Hz. In addition a substantial amount of effort has been devoted to a better understanding and optimisation of the ion capture process and to an improvement of the ion source and central region geometry. Estimates, based on theory as well as measurements in a central region model,[9] indicate that it should be possible to increase the internal beam intensity (presently ~1 μA) by a factor of 10–20.

4. SECTOR-FOCUSED SYNCHROCYCLOTRON

4.1. *General considerations*

The use of sector focusing improves the vertical betatron frequency and enables the designer to choose a radially rising—rather than falling—average magnetic field with slopes somewhere between the present field and the isochronous field. One can readily see that changing the field shape in the direction of isochronism will increase the capture efficiency which reaches the maximum of 100% in the isochronous field. In practice one would therefore aim at a rising field shape with a large slope approaching the isochronous field as close as is possible. Before discussing the factors which limit the slope and determine the choice of the actual field shape, let us first point out what can be gained by such a change of the field configuration. Given the same increase in dee voltage and repetition rate as in the case of unchanged field, there appear to be four distinct advantages of a rising average field, $\bar{B}(r)$, compared to the present falling field.

(1) The range of frequency modulation decreases which alleviates the problems of getting a high repetition rate.
(2) With the field at extraction being fixed, the central magnetic field is lower, hence (for given dee voltage) the initial orbit radii larger, which means that the ions have a better chance to clear the source.
(3) The use of sector focusing (and a magnetic cone), improves ν_z and thus the space-charge limit.
(4) The reduced frequency range due to lowering of the central magnetic field decreases the initial $d\omega/dt$ as well as ω_o and thus increases the capture efficiency according to Eqns (3) to (6).

In summary, the possible improvements in ion-source design and central geometry, vertical focusing and capture efficiency should all contribute to a further increase of beam current beyond the value that can be achieved without changing the magnetic field.

What then are the factors limiting the choice of the field shape and what are—

apart from the higher costs—possible problems associated with the conversion to a sector-focused synchrocyclotron with rising magnetic field? The considerations concerning the field shape are basically the same as those applying for the isochronous field (Section 2), i.e. the shape of $\bar{B}(r)$ is mainly determined by: (a) the limit in flutter amplitude at extraction radius due to geometrical restraints, (b) avoidance of dangerous resonances in ν_r, and (c) use of a resonance value of ν_r for extraction. As regarding the azimuthal field variation one would use three or four sectors to obtain good focusing in the centre. Although the $N = 3$ case gives better flutter, we would slightly favour the $N = 4$ field to avoid the gap-crossing resonance.[4] The major problem of a conversion to a sector-focused synchrocyclotron is probably the traversal of the $\nu_r = 1$ resonance in the centre which many particles must pass three times during the initial phase oscillation. This problem needs a careful study to determine the sensitivity to first-harmonic errors in the magnet field. At any rate, the field shape in the centre should provide a large slope, $d\omega_{rf}/dr$, at $\nu_r = 1$ to permit rapid traversal of this resonance with a minimum of beam deterioration.

4.2. *Field shapes investigated for the SREL project*

As shown in Fig. 4 the present magnetic field in the SREL synchrocyclotron (as well as in the CERN machine) falls almost linearly from 18·9 kG at the centre ($r = 0$) to 17·9 kG at extraction radius ($r = 90$ in). Corresponding orbital proton frequencies are 28·6 MHz at $r = 0$ and 16·8 MHz at $r = 90$ in. In the isochronous case the central field would be lowered to a value of 10·8 kG (if the extraction field remains fixed).

To study the capture process under fm conditions for fields in between the present case and the isochronous curve we decided to use the following analytical representation of the $\bar{B}(r)$ shape:

$$\bar{B}_I(r) = B_o + \Delta B\ \left[1 - 3\left(\frac{r}{r_1}\right)^2 + 2\left(\frac{r}{r_1}\right)^3\right] \text{ for } r < r_1 \tag{15}$$

$$\bar{B}_{II}(r) = B_o\ [1 + h(r - r_1)^2] \text{ for } r_1 < r < r_2 \tag{16}$$

$$\bar{B}_{III}(r) = B_2 + \frac{B_m - B_2}{r_m - r_2}\ (r - r_2) \text{ for } r_2 < r < r_m \tag{17}$$

$\bar{B}_I$ represents the bump field in the central region designed to improve vertical focusing. $\bar{B}_{II}$ is a parabolic curve which connects the bump field with a linearly increasing field, $\bar{B}_{III}$, extending from radius r_2 to full radius, r_m. The curves are joined smoothly together (continuous first derivative) at their intersection. B_o is the minimum field value at $r = r_1$, while $\Delta B = \bar{B}(o) - B_o$ denotes the height of the bump.

From the general considerations discussed in the previous section it appeared that in the case of the SREL project $\bar{B}(r)$ curves with central fields between 13 and 15 kG appeared to be promising. We therefore decided to select the three field shapes plotted in Fig. 4, with B_o of 13, 14, and 15 kG respectively, for detailed investigation of the capture efficiency. The transition radius r_1, which defines the radial extent of the bump region, was fixed at 6 in. (Studies with

$r_1 = 9$ in showed that it was not a critical parameter.) The bump height was varied between 0 and 2%.

It should perhaps be noted that the shape of the $\bar{B}(r)$ curve outside the central region has little effect on the capture efficiency. The linear shape was chosen for convenience; in practice the designer has a large degree of freedom in shaping $\bar{B}(r)$ to suit other requirements such as extraction on a resonance value of ν_r,

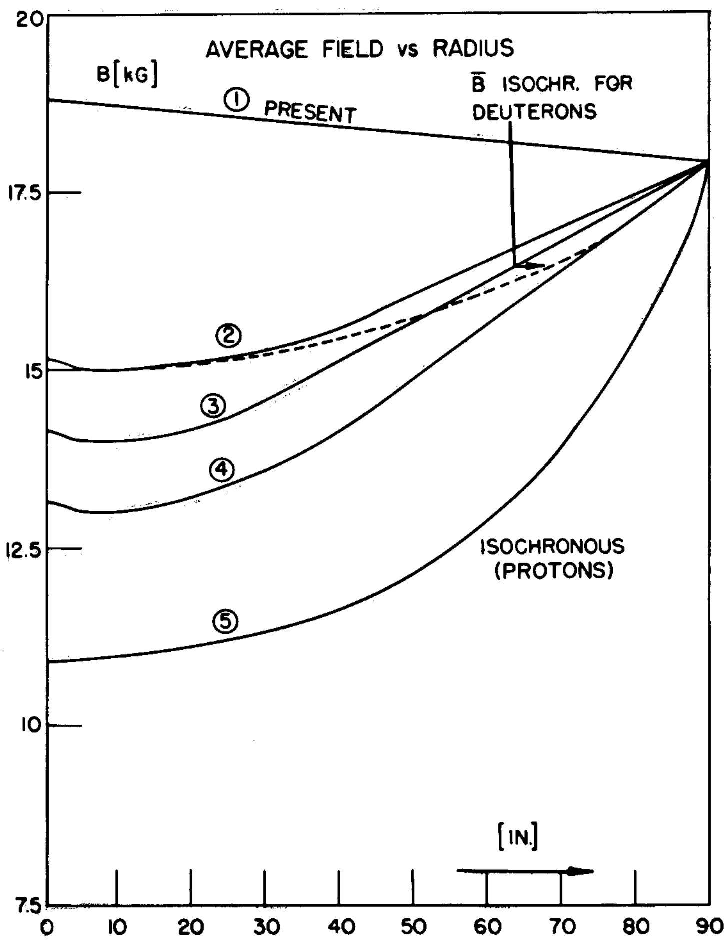

Fig. 4. B(r) *field shapes investigated for conversion of SREL Synchrocyclotron*

minimisation of the frequency modulation,[6] etc. Since the capture time depends only on the average field, the azimuthal field variation could be neglected in the calculations of capture efficiency. The flutter will, however, be considered in the estimate of $\nu_{z\,min}$.

4.3. *Method of calculating capture time*

Ions injected from the source into the synchrocyclotron at a given time t_o of the rf modulation cycle and with phase φ_o with respect to peak voltage ($\varphi_o = 0$)

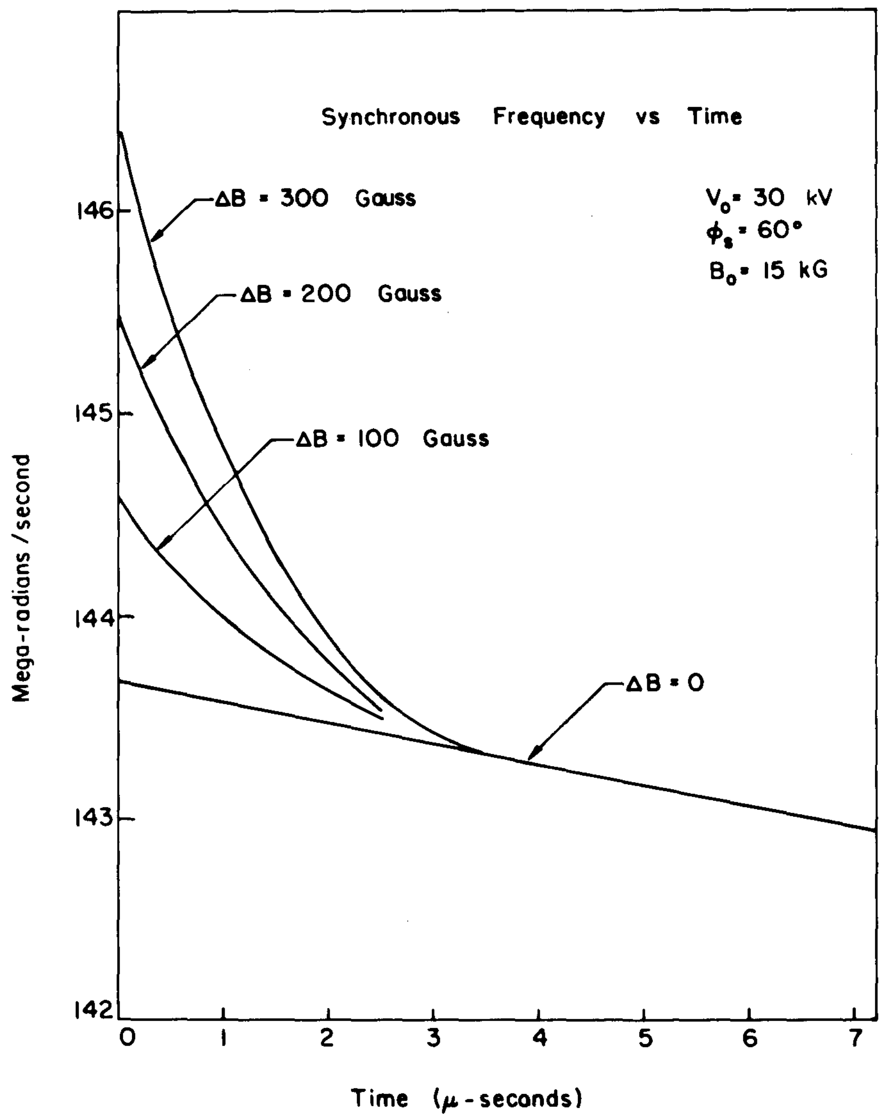

Fig. 5. Initial change of ω(t) for synchronous particle ($\varphi_S = 60°$, $V_0 = 30$ kV) in 15-kG field with several bump heights

undergo a phase oscillation about the synchronous phase, φ_S, which is mathematically described by the phase equation:

$$\frac{d}{dt}\left[\frac{E_S}{\omega_S{}^2 K}\frac{d\varphi}{dt}\right] = \frac{eV}{2\pi}(\cos\varphi - \cos\varphi_S) \tag{18}$$

This equation expresses the time variation of the phase φ of a non-synchronous particle for given energy E_s and orbital frequency ω_s of the synchronous particle. The oscillation in phase of a captured particle is accompanied by an oscillation in energy and radius about energy E_s and radius r_s of the synchronous particle. During the first phase oscillation a non-synchronous particle will swing out to a maximum radius while being accelerated, return to a minimum distance, r_{min}, during deceleration phases and then increase its radial distance again when the phase returns into the accelerating half-period of the rf voltage. When the minimum radius is less than a critical value, r_c, the particle hits the ion source and is lost. The criterion for capture is then simply that the first radial minimum is larger than r_c, i.e. $r_{min} \geqslant r_c$.

Since Bohm and Foldy's theory is based on simplifying assumptions (such as K = const) we wrote a computer program for numerical calculation of the capture time. Rather than integrating the phase Eqn (18) we based the program on the more elementary equation of motion. As betatron oscillations play only a secondary role in the capture process we designed the computer program to trace the motion of the particles on successive equilibrium orbits, which between acceleration gaps are simply semi-circles with radius determined by the (analytic) field shape. This approach makes the program relatively simple and fast, keeping computer time to a minimum.

The electric frequency, ω_{rf}, can either be put into the program in the form of a table of values at given increments in time, or it can be derived by defining the synchronous phase, φ_s, and setting ω_{rf} equal to the orbital frequency, ω_s, of the synchronous particle as computed by the program. A single 180° dee with two acceleration steps of $eV_o \cos\varphi$ per turn was assumed.

4.4. *Computer results*

A detailed discussion of all the computation is beyond the scope of this paper. We shall only present the computer results of capture time for a dee voltage of V_o = 30 kV and a synchronous phase of φ_s = 60°. The capture process was calculated at each field level for three bump heights ΔB (0, 1%, 2%) starting ions with initial energy of 60 keV at various times t_o of the fm cycle with different phases φ_o with respect to the rf voltage.

The computer results for the 15-kG field level are shown in Figs 5-7. First, in Fig. 5, is plotted the initial orbital frequency of the synchronous particle (φ_s = 60°) vs time during the first few microseconds for different bump heights of 0, 100, 200, and 300 G. Since it would be difficult and in fact undesirable to program the electric frequency, ω_{rf}, to follow the steep curves in the non-zero bumps, we have assumed that ω_{rf} is defined by the practically linear curve in the case of $\Delta B = 0$. This curve was extrapolated to negative times using the slope of $-0{\cdot}103 \times 10^6$ rad/s/μs, or $df/dt = -16{\cdot}4$ GHz/s. If ω_{rf} follows the steeper bump curve in the field case where $\Delta B \neq 0$, computer runs show that the capture time is reduced. This follows also from Eqn (3) which implies that Δt is the larger the smaller the initial slope $d\omega_{rf}/dt$.

Fig. 6 shows radius and phase vs time for the synchronous particle and for a non-synchronous captured particle starting at a phase $\omega_o = 0°$ at $t_o = 0{\cdot}43\ \mu$s after the synchronous ion.

If one evaluates the computer data by plotting starting phase φ_o vs starting time t_o one finds a region within which the particles are trapped into phase-stable orbits whereas particles whose starting points (φ_o, t_o) lie outside this area are not captured. Fig. 7 shows these capture regions in the 15-kG field for bump heights of 0, 150, and 250 G. (As pointed out, the $\omega_{rf}(t)$ program was the same in

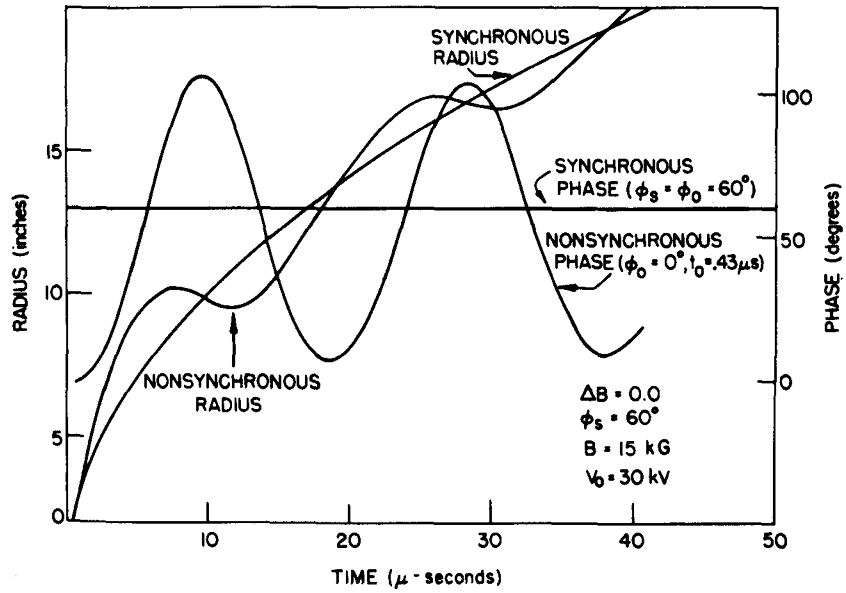

Fig. 6. Radius and phase vs time for the synchronous and a non-synchronous particle

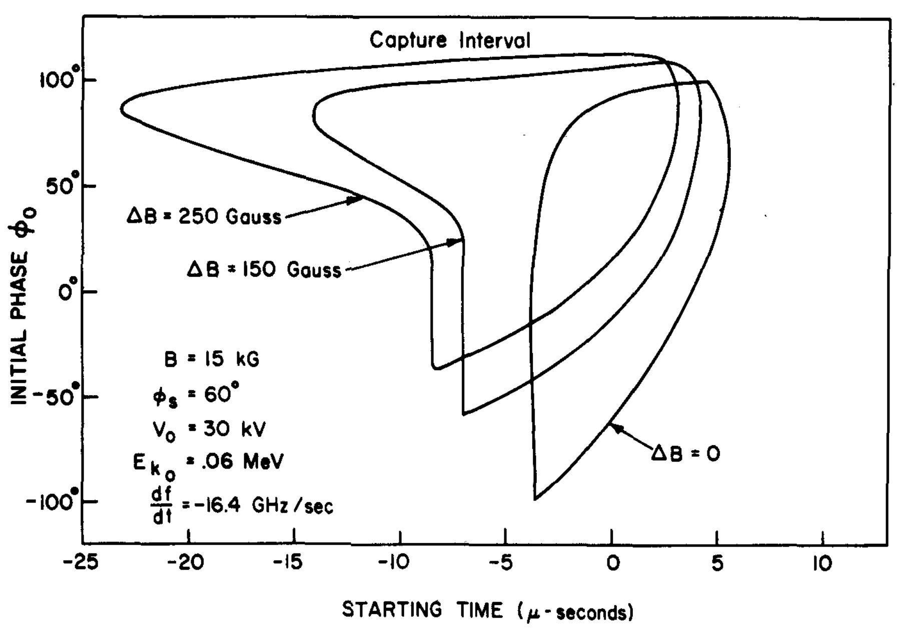

Fig. 7. Capture regions (starting phase φ_0 vs starting time t_0) for three different bumps in the 15-kG field

each case.) As one would expect the capture region moves towards more negative times as the bump is increased since a particle, in order to find a value of ω_{rf} which is in agreement with its orbital frequency, ω, must start earlier in time as ΔB, and hence ω, increases. What is at first sight quite surprising though is the apparent increase of the capture region with increased bump height in contrast to what one would expect. This result can, however, be explained by the following two arguments. According to the theory, ion capture takes place during a given frequency interval $\Delta\omega_c$ corresponding to a given field change, ΔB_c, between $r = 0$ and some radius r in the central region. The way the field changes within this ΔB_c region has little effect on the capture process. Thus, if a bump ΔB is introduced one would expect no significant change of the capture time as long as ΔB and the corresponding $\Delta\omega$ stay within the capture interval, i.e. $\Delta\omega < \Delta\omega_c$. Only if ΔB exceeds this limit would the capture time decrease. The bumps used in our case are less than the capture interval. The reason why the capture region in Fig. 7 increases—rather than remaining roughly constant by previous argument—is due to the fact that $\omega_{rf}(t)$ curve was the same in each case. For a given central field shape there exists a minimum slope, $d\omega_{rf}/dt$, for maximum capture time which is usually less than the slope of the orbital frequency. Obviously our $\omega_{rf}(t)$ curve could have been better optimised by using smaller slopes for the $\Delta B = 0$ and $\Delta B = 150$ G cases.

The main conclusion one can draw from Fig. 7, considering the previous two arguments, is that a bump up to 2% or perhaps more appears to be feasible without reducing the capture efficiency. The second interesting feature of Fig. 7 is that capture time is a maximum for starting phases between 40° and 90°. It is therefore desirable[6] to launch the particles (by off-setting the puller angle and thus increasing the path length between first and second gap) in a phase interval close to 90°. Central orbit studies[10] based on electrolytic-tank data show, however, that in order to clear the source on the first turn the ions have to be in the phase interval between 0° and 60°; in fact the useful beam in our studies has a phase width of 20°-60° at the second gap.

The computer results for the 15-kG field as well as for the other field levels are summarised in Table 1. Listed are the total acceleration time, T_a, for the synchronous particle ($\varphi_s = 60°$), the capture time Δt and capture efficiency, defined by $\Delta t/T_a$, for three particles starting in the phase interval between 20° and 60°. The computations in the other field levels also used the linear slope, $d\omega_{rf}/dt$, of the $\Delta B = 0$ case. For comparison the capture efficiency was also computed with the present field of the SREL synchrocyclotron which was represented by a straight line from $r = 0$ to $r_m = 90$ in. The assumptions concerning $B(r)$ and $\omega_{rf}(t)$ are somewhat different from the actual case. The figures on capture efficiency are therefore not quite representative of the present situation; they do, however, permit a relative comparison which is in good agreement with theoretical expectations. Thus one finds that the capture efficiency, given the same $B(r)$, increases roughly proportional to $\sqrt{V_o}$ in accordance with the theory, Eqn (11), when the dee voltage is raised from presently 10 to 30 kV (CERN solution). (The fact that CERN current estimates for the improved machine give factors of 10–20 is due to the better optimisation[1] of capture conditions especially $\cos\varphi_s$ and $d\omega_{rf}/dt$.)

Lowering the central field to 15 KG brings an increase between 2 and 3 in capture efficiency while only little is being gained with a further decrease to 14 or 13 kG.

The computer results were checked with Bohm and Foldy's theory for the case $\Delta B = 0$, where $K = 1$, and found to be in very good agreement. In all the

Table 1. COMPUTER RESULTS OF CAPTURE TIME

Magnetic field	*Acceleration time T_a (μs)*	*Capture time* Δt (μs)			*Capture efficiency* $\epsilon = \Delta t/T_a$ ($\times 10^{+3}$)		
		20°	40°	60°	20°	40°	60°
15 kG, 30 kV	1·05						
Bump: 0		8·1	8·5	8·4	7·7	8·2	8·0
1%		9·5	11·4	15·1	9·0	10·8	14·3
2%		9·0	12·3	19·3	8·5	11·7	18·3
14 kG, 30 kV	1·07						
Bump: 0		9·1	9·5	9·5	8·5	8·9	8·9
1%		11·0	12·3	16·2	10·3	11·5	15·1
2%		11·5	13·6	21·5	9·7	12·7	20·1
13 kG, 30 kV	1·10						
Bump: 0		10·7	12·0	12·1	9·7	10·9	11·0
1%		13·3	14·9	17·7	12·1	13·5	16·1
2%		10·7	14·3	23·4	9·7	13·0	21·3
Present, 30 kV	0·93						
Bump: 0		4·5	4·8	4·8	4·7	4·9	5·0
Present, 10 kV	2·89						
Bump: 0		7·3	7·7	7·5	2·5	2·6	2·6
15 kG, 40 kV	0·79						
Bump: 0		7·0	7·6	7·2	8·8	9·5	9·1
1%		8·8	9·7	11·9	11·1	12·2	15·0
2%		8·0	11·3	15·1	10·1	14·3	19·1

other cases K is a function of radius and discrepancies are always large no matter what value of K one uses.

4.5. *Vertical focusing and current estimates*

The captured beam intensity can be roughly estimated with Eqns (12) and (13). To get the space-charge limit, I_o, of the beam current we need to know $\nu_{z\,\mathrm{min}}$. A rough figure for $\nu_{z\,\mathrm{min}}$ can be obtained by calculating ν_z due to the electric field using Rose's formula[11] and combining it with the ν_z due to average field and flutter. The field index $\bar{k}$ is obtained from $\bar{B}(r)$, which is given analytically in Eqns (15)-(17), and for the betatron frequency due to the magnetic field we used the formula:

$$\nu_z^2 = -\bar{k} + \frac{f^2}{2} \tag{19}$$

The flutter rise, $f(r)$, in the centre was represented by the following analytical approximation:

$$f(r) = ar^2 + br^3 \tag{20}$$

Assuming values of $a = 2{\cdot}1 \times 10^{-3}$, $b = 2{\cdot}0 \times 10^{-5}$ (for r measured in inches), corresponding to the flutter in the Maryland cyclotron (four sectors), we calculated the combined electric and magnetic ν_z vs r curves plotted in Fig. 8. The minimum ν_z value for the useful phases ($\varphi > 0$) occurs at radius $r = r_1$ (where $\bar{B} = B_0$). In the specific case shown $r_1 = 6$ in, $B_0 = 15$ kG, $\Delta B = 225$ G and $\nu_{z\,\min} = 0{\cdot}05$. By shifting r_1 from 6 to 8 in, which, as was mentioned, has no effect on capture efficiency, the minimum value is raised to $\nu_{z\,\min} = 0{\cdot}1$; an additional increase to 0·15 can easily be obtained by different shaping of the $\bar{B}(r)$ curve. Assuming the values of $\nu_z = 0{\cdot}15$, $h = 0{\cdot}02$ m, $\omega = 1{\cdot}43 \times 10^8 \mathrm{s}^{-1}$, and $V_1 = 2\, V_0 \cos \varphi_s = 3 \times 10^4$ V, one obtains $I_0 = 34$ mA for the space-charge limited current in Eqn (13). The average current is then given by Eqn (12). As was pointed out we have not optimised $\mathrm{d}\omega_{rf}/\mathrm{d}t$ to get the largest capture time Δt. If we take the very conservative value of $\Delta t = 12\,\mu$s (for $B = 15$ kG, 2% bump, $\varphi_0 = 40°$ from Table 1), $T_m = 1{\cdot}2$ ms, $\Delta\varphi_0/2\pi = 0{\cdot}12$, we find that the average internal beam current is approximately $I = 40\,\mu$A. By better optimisation of $\mathrm{d}\omega_{rf}/\mathrm{d}t$, or a small increase in dee voltage, one should be able to get a current of 50 μA.

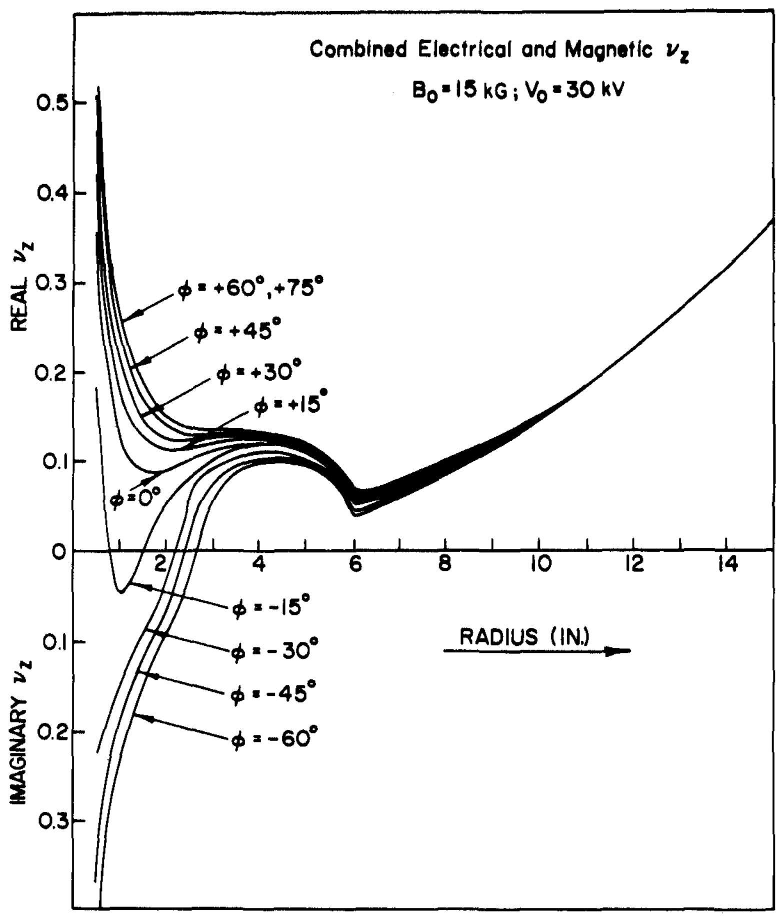

Fig. 8. Combined vertical frequency of electric and magnetic focusing in 15-kG field with 2% bump height ($V_0 = 30$ kV)

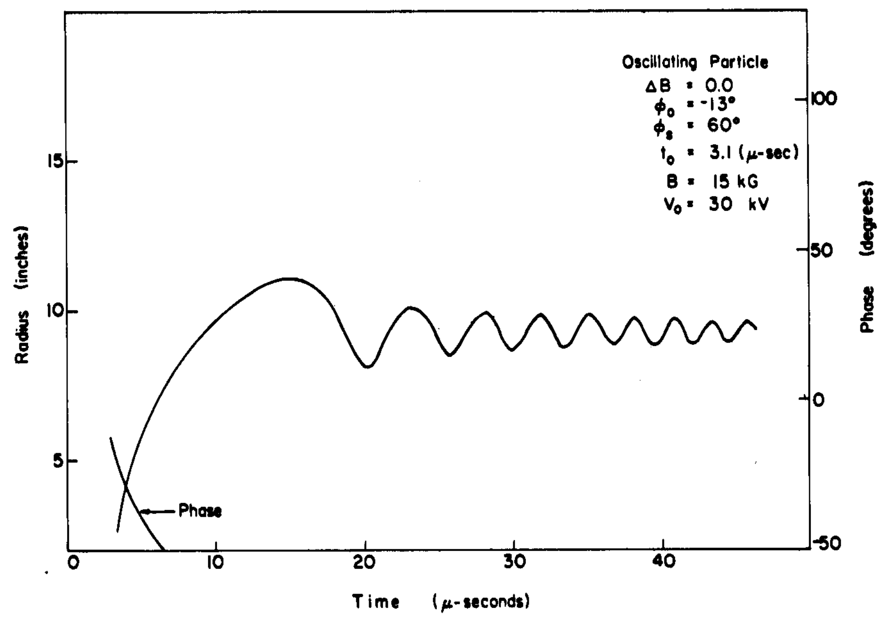

Fig. 9. Radial motion of a trapped, oscillating particle

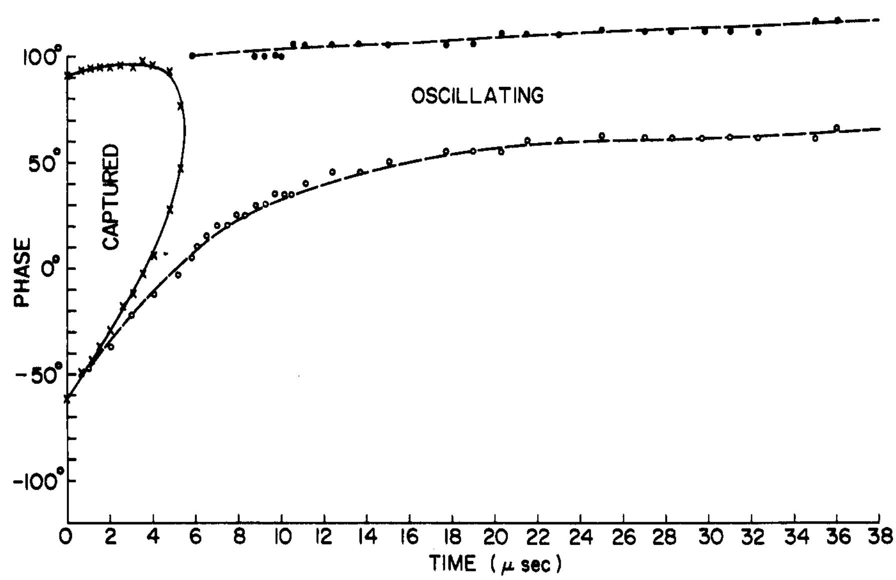

Fig. 10. Region of oscillating particles

4.6. *Oscillating trapped particles*

When we analysed our computer data we made the surprising discovery that there are particles which are neither captured and accelerated to full radius nor do they return to the ion source. The particles are actually accelerated to a distance not too far from the ion source where they oscillate about an equilibrium radius remaining there through the entire modulation cycle of the rf. The radial motion of such an oscillating particle is plotted in Fig. 9.

The phenomenon of trapped, oscillating particles, which to our knowledge has not been discussed in connection with synchrocyclotrons, is quite interesting insofar as it occurs over a relatively large region in the φ_o vs t_o diagram. In fact the region of oscillating particles is much larger than the capture region as is illustrated in Fig. 10. Since the useful beam starts between 0° and 60° it is seen that the portion of beam comprising oscillating particles is much larger than that of captured particles. (The 'oscillating' region extends far beyond the limits of the figure.)

We have discussed this phenomenon with the Columbia as well as the CERN group. The latter has confirmed[12] the existence of these particles by independent calculations with their computer programs.

Although this effect needs further study and explanation one can say the following.

(1) The oscillating particles are ions starting at a time and phase such that they are first accelerated to some radius; subsequently, during the continuous negative phase shift the energy gained in accelerating half-periods of the rf is balanced by the energy lost in decelerating half-periods.
(2) The radial amplitudes about the equilibrium radius are damped.
(3) Preliminary computer results seem to indicate that the oscillating particles are not captured when the rf returns to the initial value and repeats the modulation cycle.
(4) Since a particle's position in space at any given time is a unique function of the starting condition there should be no overlap of oscillating and captured beam; one expects therefore that the oscillating particles should not significantly influence the space-charge limit [Eqn (13) is based on a uniform infinite charge distribution].

5. CONCLUSIONS

The conversion of the SREL synchrocyclotron into an isochronous machine is in our opinion impractical considering the many difficulties and problems associated with the magnetic field design and tolerance requirements. Internal beam currents in the range of 20-50 μA can be achieved by changing the field to a non-isochronous sector-focused configuration which lowers the central field to 15 kG. Use of a 1-2% bump in the centre is feasible without loss in capture efficiency. This provides a substantial increase in ν_z even if the flutter is relatively small as in a four-sector machine with large gap. One problem which needs further investigation is the traversal of the $\nu_r = 1$ resonance in the centre which effects the internal beam quality and hence the extraction efficiency.

The investigated 15-kG field shape is very close to the isochronous field for 360 MeV deuterons and 720 MeV α-particles. By choosing a field shape which is isochronised for deuterons one could operate the converted machine in either a cw mode for deuterons and alphas or, with the same field, in fm mode for protons.

DISCUSSION

Speaker addressed: M. Reiser (Maryland)

Comment by J. R. Richardson (UCLA): It seems to me that these oscillating particles are the same as those we discovered experimentally in the first work on fm cyclotrons, done at Berkeley in 1946. These results were published in the *Physical Review* in 1948.

Comment by J. Rainwater (Columbia): I agree with your results that a bump in B at small radius (to improve ν_z^2 focusing) does not give a net $(1/K)^{1/2}$ reduction in capture time, if it is mainly limited in net effect to the first part of the first cycle of phase oscillation. It then just gives a net phase displacement $\int(\omega_s - \omega)dt$ due to the bump, but the subsequent part of the first and future cycles of phase oscillation are only slightly reduced. I have believed that this is a necessary conclusion for a few years now. But I am not sure whether Dr. Dmitrievsky would agree with this.

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