Extraction from medium and high energy cyclotrons

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ABSTRACT

A review of some basic extraction systems is given with emphasis on the properties of non-resonant and resonant extraction. Results are given for extraction efficiency and energy spread of the extracted beam as a function of beam quality and radial focusing frequency. It is shown, that for higher energies, large ring cyclotrons can have extraction efficiencies over 90%. Some formulas for the stability of vertical motion are summarised.

1. INTRODUCTION

Circular accelerators have in the past typically achieved about 60% extraction efficiency. In the new era of high intensity beams extraction efficiency for cyclotrons has to be pushed towards 100% in order to remain competitive with future superconducting Linacs. The meson factories at TRIUMF (Canada)[1] and SIN (Switzerland)[2] will have a beam power of over 50 kW. Already a loss of 10% of the beam at extraction can pose serious problems concerning radiation and heat dissipation. This beam loss will probably determine the intensity limit of these machines.

2. BASIC EXTRACTION SYSTEMS FOR CYCLOTRONS

2.1. *Extraction by stripping*[3]

If ions with bound electrons are accelerated, extraction can be achieved simply by putting a foil into the beam. Electrons are stripped, changing the radius of curvature of the ions instantaneously.

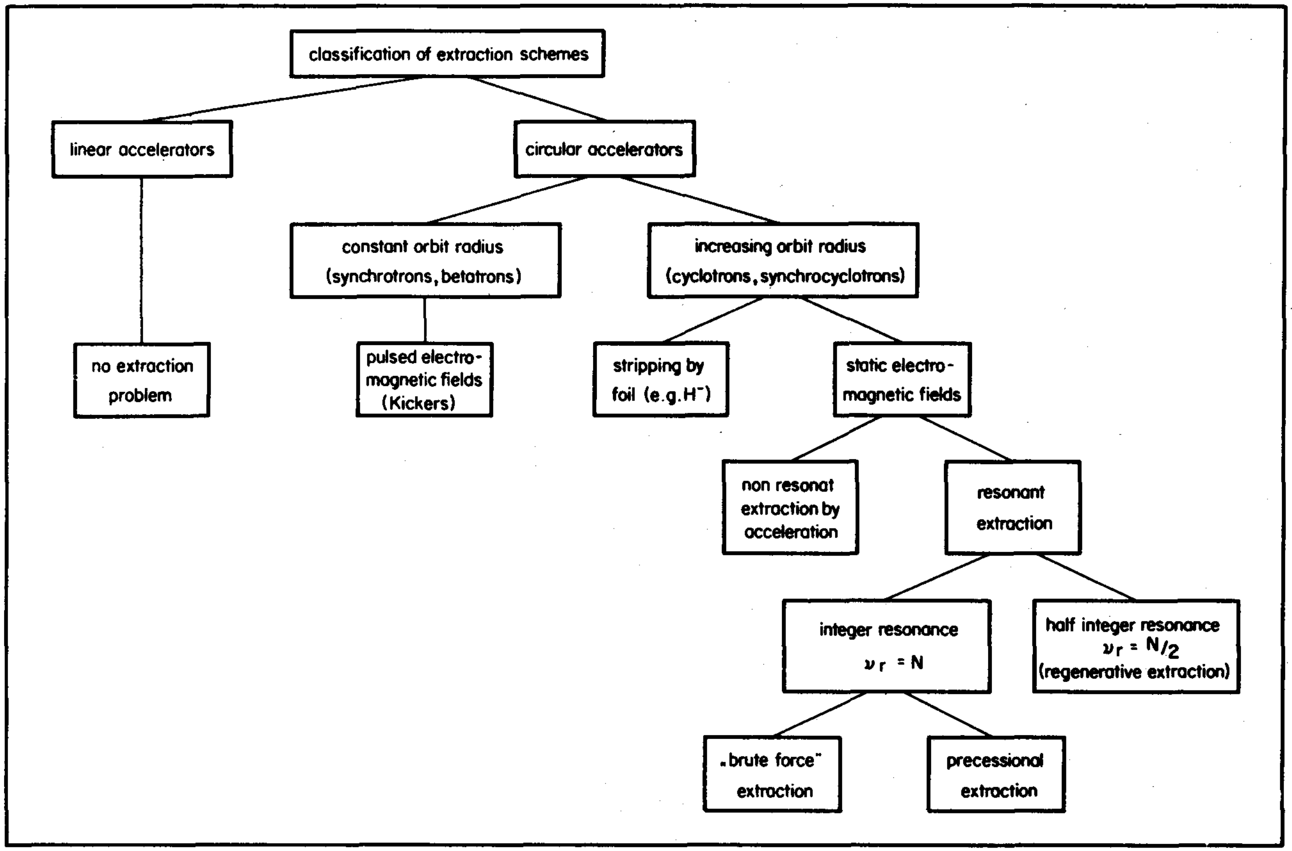

Fig. 1. Survey of extraction parameters of some cyclotrons. NR = *non-resonant extraction,* BF = *brute force resonant extraction,* PREC = *precessional extraction,* STR = *stripping,* RG = *regenerative extraction,* x_o, z_o = *incoherent beam amplitudes at extraction,* e_{max} = *optimal extraction efficiency,* $\delta\phi$ = *total phase width,* ES = *electrostatic septum,* FES = *radially focusing electrostatic septum,* MC = *magnetic channel,* FMC = *radially focusing magnetic channel,* SC = *screening channel,* CC = *coaxial channel,* PR = *peeler+ regenerator,* TVB = *time varying bump,* δ = *septum thickness,* D = *channel width. *The energy spread can be improved with rf-flattopping*

Advantages: 1. Easy to control. 2. Rapid crossing of fringe field. 3. 100% extraction efficiency. 4. Easy energy variation. 5. Possibility of simultaneous beams.

Disadvantages: 1. Lorenz dissociation of electrons produces beam loss. 2. Residual gas stripping requires good vacuum. 3. The stripping process can be used only once in a multi-stage machine.

2.2. *Extraction by acceleration*

In a circular machine with fixed magnetic field the particles spiral outwards due to their energy increase. This makes it possible to peel the beam off with static electromagnetic channels (or a stripper for H^-) at the extraction radius. The key number for successful extraction is, therefore, the radial gain per turn dR/dn. A radial increase of the orbit can be achieved not only by acceleration, but also by magnetic bumps:

$$\frac{dR}{dn} = \frac{dR}{dn}(\text{accel.}) + \frac{dR}{dn}(\text{magn.}) \tag{2.1}$$

Let us concentrate first on the contribution from acceleration only:

$$\frac{dR}{dn}(\text{accel.}) = R \times \frac{E_G}{E} \times \frac{\gamma}{\gamma + 1} \times \frac{1}{\nu_r^2} \tag{2.2}$$

R = average radius of orbit, E_G = energy gain per turn,

E = kinetic energy, $\gamma = 1 + E/E_o$,

ν_r = radial focusing frequency.

Formula (2.2) tells you in a remarkably simple way, how to get a high extraction rate:

(1) build cyclotrons with a large average radius,
(2) make the energy gain per turn as high as possible,
(3) accelerate the beam into the fringe field, where ν_r drops.

The last requirement calls for a large energy gain too, since the phase slip in the fringe field has to be kept small. The desired high energy gain is achieved with a high voltage on the accelerating gap, many gaps or a mixture of both. Some typical examples for medium energy cyclotrons are (all numbers for protons):

ORIC (Oak Ridge): $E_G = 2 \times 80$ keV = 160 keV/turn

Jülich (AEG): $E_G = 6 \times 30$ keV = 180 keV/turn

Michigan State University (MSU): $E_G = 4 \times 60$ keV = 240 keV/turn.

The above values for E_G are still too small to guarantee a good extraction at higher energies, as can be seen from (2.2). One new idea is the *ring cyclotron,*

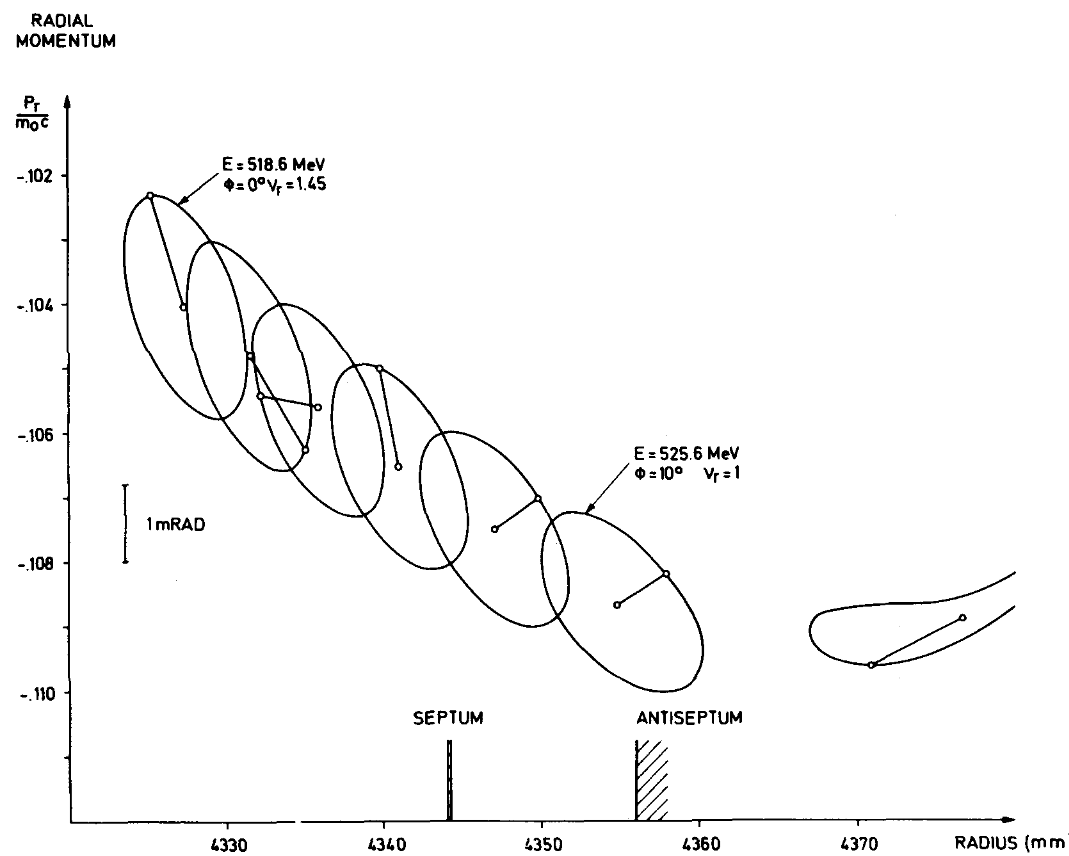

Fig. 2. Radial phase space at septum entrance for SIN ring cyclotron (old design of 525 MeV*). Assumptions were: conservative beam quality of 50 mm mrad at injection (68* MeV*). 400 kV peak voltage on a cavity (1·4* MeV *energy gain at extraction, 10° phase slip). Plot shows rapid change of ν_r distorted beam in fringe field and optimum septum position*

proposed in 1963 by H. A. Willax for the Swiss project of a meson factory.[4] All three factors [(1)-(3)] for a high radial gain per turn can be exploited with such a machine. The use of separate sectormagnets and cheap straight sections allows the construction of a machine with a large average radius. Several high voltage cavities can be inserted into the free sections. The importance of the factor ν_r in (2.2) is illustrated in the following example: for the SIN 580 MeV cyclotron extraction in the isochronous region ($\nu_r = 1{\cdot}6$) would give a radial gain of 4 mm, whereas extraction in the fringe field ($\nu_r = 1{\cdot}1$) gives 8 mm! This improvement is only possible with a high energy gain per turn and a narrow gap.

The free sections in the ring leave room for extraction elements, which have very small interference with the main magnetic field. The magnet gap can be made very small, since nothing has to be squeezed into it. The price which has to be paid for these advantages is an additional accelerator as injector.

2.3. *Resonant extraction*

In most cyclotrons the radial gain per turn from acceleration is not large enough for a good extraction efficiency and some enhancement with magnetic perturbations is necessary. The idea is always to move the beam off centre towards the extraction element. We distinguish two different classes: integer and half integer resonances.

The integer resonance $\nu_r = N$

(a) BRUTE FORCE

With a magnetic bump in the axial field component of the form

$$\Delta B(r, \theta) = b_N \cos N(\theta - \theta_N)$$

the beam is driven off centre, if ν_r is close to the integer value N (for a bump of a general azimuthal shape the influence of Fourier components other than b_N can be neglected). The maximum additional radial gain per turn is given by:

$$\frac{dR}{dn}(\text{brute force}) = \pi \times R \times \frac{b_N}{N \times B_0} \qquad (2.3)$$

B_0 is the average field at radius R. For a given final energy the expression $R \times B_0(R)$ is a constant, hence the radial gain grows with R^2, favouring machines with a large average radius. A nice example of this type of resonance is the first CERN-experiment for the $(g-2)$ factor of the muon.[5] A slight gradient was superposed on a homogeneous magnetic field. The $\nu_r = 1$ resonance made the orbits drift at right angles to the gradient (no equilibrium orbit!). For typical medium energy cyclotrons the additional radial gain from (2.3) is around 0·2 mm for a first harmonic bump of 1 G. To get the desired turn separation one has to produce stronger bumps (brute force). This approach is used for the compact cyclotrons of AEG[6] and Philips.[7] In most cases there is fortunately another method available, which requires only small bumps.

(b) PRECESSIONAL EXTRACTION[8, 9]

In the extraction region of medium energy cyclotrons ν_r drops gently through the resonance value $\nu_r = 1$. This crossing produces a coherent amplitude x_c[10]:

$$x_c = \pi \times R \times \frac{b_1}{B_0} \times n_{\text{eff}}, \qquad n_{\text{eff}} = \frac{1}{\sqrt{\left|\frac{d\nu_r}{dn}\right|}} \tag{2.4}$$

$d\nu_r/dn$ is the rate of change of ν_r at the crossing point, n_{eff} is the effective duration of the resonance (typically around ten revolutions). After the resonance the beam precesses with the frequency $(\nu_r - 1)$ around its equilibrium position. The maximum turn separation from this precession is given by:

$$\frac{dR}{dn}(\text{precession}) = 2 \times x_c \times \sin \pi (1 - \nu_r) \tag{2.5}$$

Because of phase slip problems only cyclotrons with a large energy gain per turn are able to accelerate into the fringe field, where ν_r is substantially below 1.
A beautiful example of this precessional extraction mechanism is given for the MSU cyclotron.[8]

Near $\nu_r = 1$, dR/dn (accel.) = 2 mm and dR/dn (brute force) = 0·3 mm. The resonance duration is 14 revolutions and builds up a 4 mm coherent amplitude. At extraction near $\nu_r = 0{\cdot}8$ (turn 220), dR/dn (accel.) = 3 mm and dR/dn (precession) = $1{\cdot}2 \times x_c$ = 5 mm, giving a total separation of 8 mm. Fig. 3 indicates that turns would be separated even if they had originally overlapped. An energy difference between two particles with the same (r, p_r) values is transformed into a spatial separation after the resonance. The integer resonance is energy selective and not radius selective.[11, 12]

The half integer resonance $\nu_r = N/2$ (regenerative extraction)

Consider a gradient bump of the form

$$\Delta B(r, \theta) = g_N (r - r_N) \cos N(\theta - \theta_N) \tag{2.6}$$

This periodic perturbation changes the focusing properties. A formula for the *frequency shift* $\nu_r \to \tilde{\nu}_r$ was derived by Vogt-Nilsen.[13]

$$\cos\tilde{\sigma} = \cos\sigma - \frac{\pi R^2 \times g_N^2}{2N \times \nu_r \times B_0^2 (N^2 - 4\nu_r^2)} \sin\sigma \tag{2.7}$$

$$\sigma = \frac{2\pi}{N}\nu_r, \qquad \tilde{\sigma} = \frac{2\pi}{N}\tilde{\nu}_r$$

The frequency shift is thus biggest for $\nu_r \sim N/2$. If ν_r is within the so-called *stop band* $\nu_r = N/2 + \Delta\nu_r$, the new frequency $\tilde{\nu}_r$ is complex and the beam becomes unstable, a desirable feature for extraction! Expanding (2.7) around $\nu_r = N/2$ gives the useful formula:

$$\left(\tilde{\nu}_r - \frac{N}{2}\right)^2 = \left(\nu_r - \frac{N}{2}\right)^2 - \Delta\nu_r^2 \tag{2.8}$$

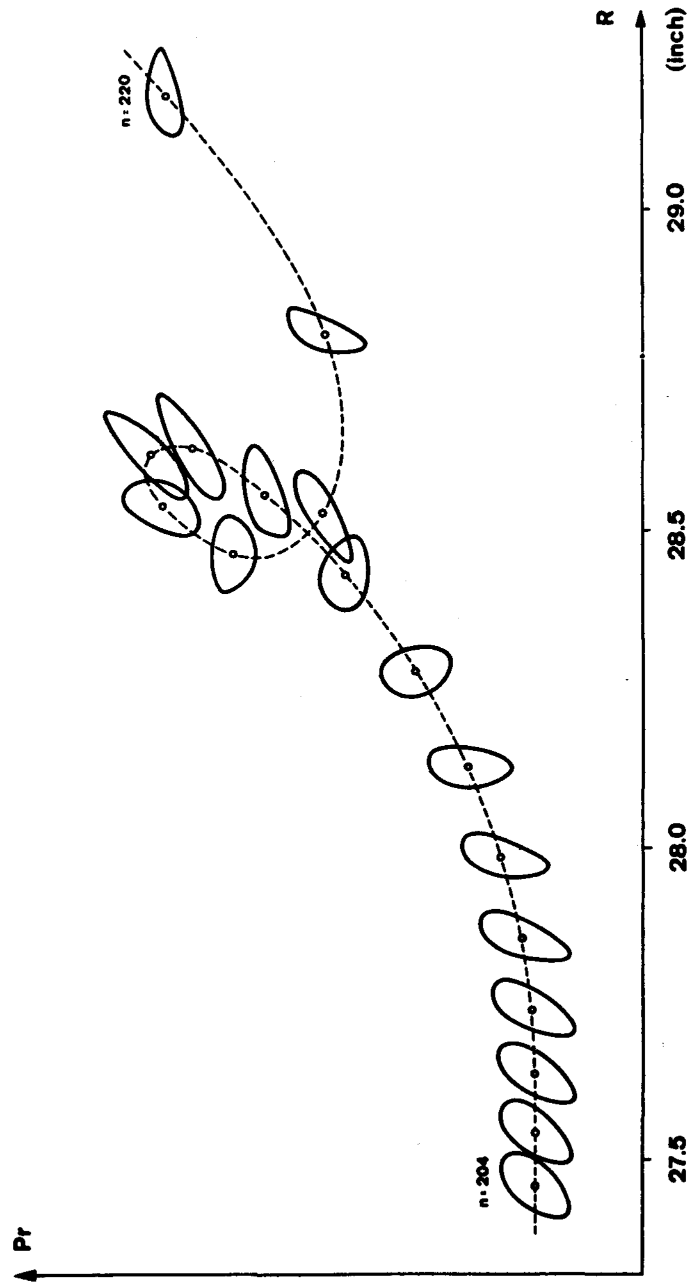

Fig. 3. Radial phase space turn pattern for MSU cyclotron. Turns 204-220 are shown. Resonance $\nu_r = 1$ occurs at turn 202. Post-resonance acceleration gives radial separation through precessional motion of beam

$$\Delta \nu_r = \frac{R \times g_N}{2N B_o} = \text{stop-band half-width} \qquad (2.9)$$

Eqn (2.8) is illustrated graphically in Fig. 4.

The situation is completely analogous to the forbidden energy zones in solids, which are produced by the periodic potential of the crystal lattice. ν_r and $Re\,(\tilde{\nu}_r)$ correspond to the energy and wave number squared of the electrons.

For a particle with positive rotation around the cyclotron centre the betatron oscillation is described by:

$$\begin{aligned} x(\theta) &= c_1 \cos Re\,(\tilde{\nu}_r) \times (\theta - \theta_N + 45^\circ)\, e^{\mu\theta} \\ &+ c_2 \cos Re\,(\tilde{\nu}_r) \times (\theta - \theta_N - 45^\circ)\, e^{\mu\theta} \qquad (2.10) \\ \mu &= Jm\,(\tilde{\nu}_r \qquad \mu_{max} = \Delta\nu_r. \end{aligned}$$

The maximum amplitude increase per turn is given by:

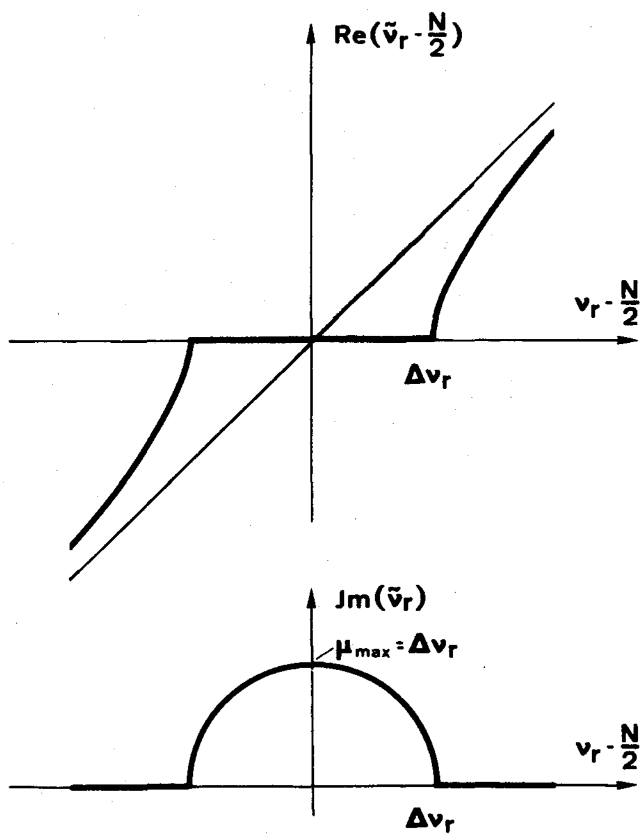

Fig. 4. Stop band ν_r = N/2. A gradient bump $\Delta B\,(r, \theta) = g_N\,(r - r_N) \cos N\,(\theta - \theta_N)$ shifts the betatron frequency ν_r to the new value $\tilde{\nu}_r$, which is complex inside the stop band. The imaginary part $Im(\tilde{\nu}_r)$ is responsible for an exponential growth of the radial amplitude

$$\frac{dx}{dn} = 2\pi \times x \times \mu_{max} = \pi \times R \times \frac{g_N \times x}{N \times B_0} \quad (2.11)$$

This formula matches exactly with (2.3) for the integer resonance. $b_N = g_N \times x$ is the bump seen by a particle of amplitude x. Eqns (2.10), (2.11) together with Fig. 4 shows three facts:

(1) Inside the stop band the amplitude initially grows or decreases exponentially, according to the starting position, but finally ends up growing.
(2) Even if ν_r is slightly different from $N/2$, the resonance locks the phase of the oscillation to a fixed value (production of a common node).
(3) Particles near the central particle hardly move.

Since for medium energy cyclotrons ν_r is around 1, the half integer resonance $\nu_r = 2/2$ can be used for extraction. The cos 2 θ-dependence of the field perturbation is normally achieved with a negative gradient bump (the peeler) and a positive gradient bump (the regenerator) about 90° displaced. This scheme is called 'regenerative beam extraction'.[14] Sometimes the falling fringe field is used as the peeler. Since the half integer resonance favours big betatron amplitudes [see Eqn (2.11)], regenerative extraction is especially attractive for synchro-cyclotrons,[15,16] where the radial gain from acceleration is orders of magnitude lower than the incoherent amplitude. For isochronous cyclotrons with their instrinsically higher extraction efficiency the regenerative system never got much popularity. At Berkeley it was found through experience that precessional extraction is easier to handle and gives slightly better results than the regenerative method.[17]

Table 1. CHARACTERISTICS OF INTEGER AND HALF INTEGER RESONANCE

$\nu_r = 1$	$\nu_r = 2/2$
Flat bump, shape not critical	Gradient bump, shape critical
No shift of betatron frequency	Shift of betatron frequency towards half integer value
Coherent off-centre drift of beam	Off-centre drift dependent on radial amplitude
Energy selective	Radius selective
Constant radial gain per turn	Exponential growth of radial gain (asymptotically)
Post resonance acceleration to $\nu_r \sim 0{\cdot}8$ gives additional turn separation through precession	Remedy against low energy gain per turn and bad beam quality (synchrocyclotrons)

Resonance extraction at 'high' energies

All existing cyclotrons are extracting the beam around $\nu_r = 1$. The situation changes drastically, however, for kinetic energies of the order of the rest mass. ν_r ($\sim \gamma = 1 + E/E_0$) can rise so high above unity, that even a sharp field turnover would not bring ν_r back to 1 at extraction, as illustrated for the SIN cyclotron in Fig. 5. One can think of using the $\nu_r = 3/2$ resonance[18] or the $\nu_r = 4/2$ resonance.[19] A qualitative behaviour is shown in Fig. 5. One feature of these systems could be the extraction of beams at different azimuths. The azimuthal dependence of the resonant oscillation is $\cos N/2\,(\theta - \theta_N)$—see Eqn (2.10)—which means three possible azimuths for $\nu_r = 3/2$ and two for $\nu_r = 4/2$. The $\nu_r = 4/2$ resonance was examined in some detail for the SIN ring cyclotron.[20] The idea is to shift ν_r into the stop band $\nu_r = 4/2$ and then use the integer resonance $\nu_r = 2/1$ to build up a coherent amplitude. The orbits do not drift off centre as with $\nu_r = 1$, but become elliptical. It was indeed possible to change ν_r from 1·6 up to 2, but the necessary fourth harmonic gradient perturbation of about 70 G/cm [see Eqn (2.9)] is rather difficult to achieve and causes large non-linearities. This method is, therefore, more attractive if ν_r is already close to $\nu_r = 2$ (energies above 700 MeV).

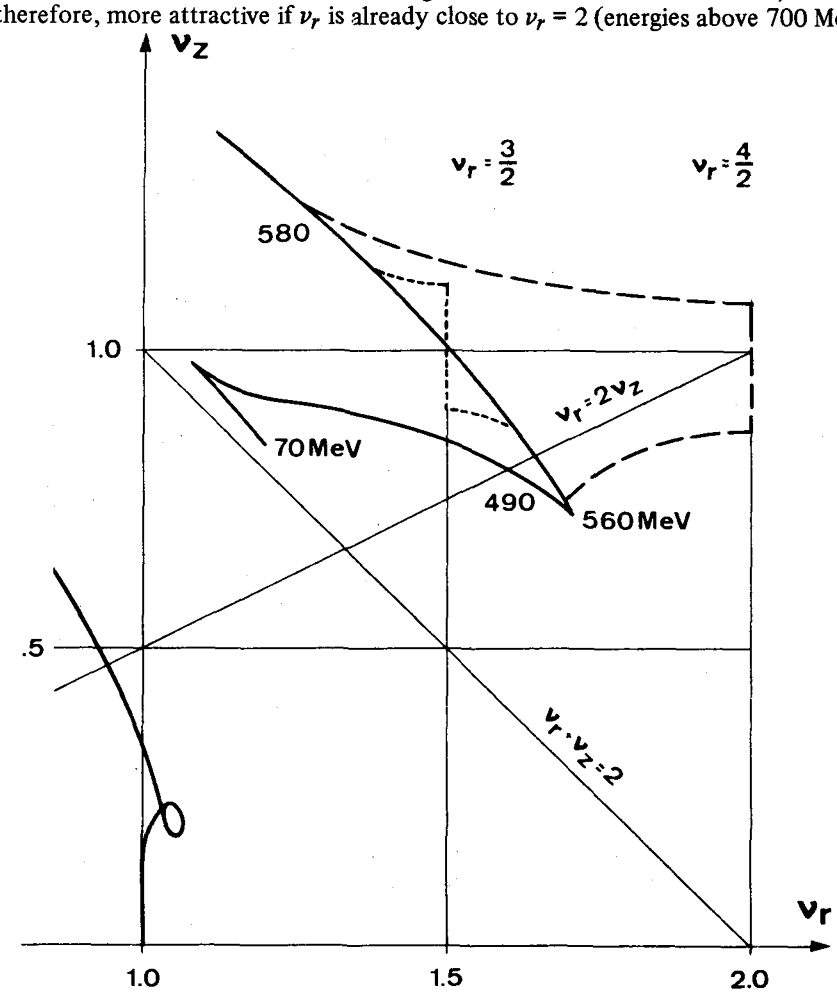

Fig. 5. ν_r, ν_z resonance diagram. The upper solid curve shows the working part for the SIN ring cyclotron. Injection occurs at 72 MeV *and extraction above 580* MeV. *The picture shows the change in the working path, if the radial resonance $\nu_r = 3/2$* (- - - -) or *$\nu_r = 4/2$* (– – –) *would be used. The lower solid curve shows a typical curve for a medium energy cyclotron (MSU, 50* MeV *protons)*

Non-linear resonances

Besides the integer and half integer resonances there are higher order resonances. The third order resonance $\nu_r = N/3$ is of some interest, since it occurs naturally in three-sector machines ($N = 3$). It has turned out, however, that the strong amplitude dependence of this resonance can introduce large beam distortions.[11]

For synchrotrons the third integer resonance was successfully used for the slow extraction mode.[21] The stability limits of the resonance shrink with time and force the beam to spill out slowly.

Variable energy

If a resonance is used for extraction, a fixed turn pattern should be chosen for all energies. The fringe field has to be carefully shaped with trim coils, to keep the ν_r-value at extraction constant. For higher energy cyclotrons energy variability requires drastic changes in the magnetic field shape. Preliminary studies have shown, however, that for ring cyclotrons extraction at earlier radii looks possible. For the SIN-Cyclotron an external beam of 400 MeV seems feasible, but would require a second set of extraction elements.

3. ENERGY DISTRIBUTION AT EXTRACTION

3.1. *The monoenergetic beam*

A beam is considered monoenergetic if the energy spread δE for a fixed turn number n is much smaller than the energy gain E_G. An example is shown in Fig. 6.

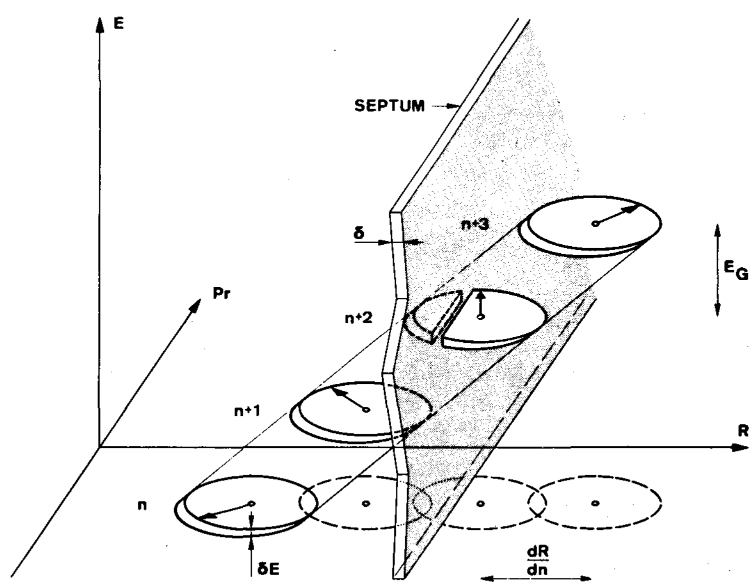

Fig. 6. Energy distribution of beam at septum entrance. Movement of particles in three-dimensional phase space as a function of turn number n. *In the 'monoenergetic case all particles are inside a thin energy slice* δE. *In the continuous energy case the distribution is uniform. The plot displays the energy gain per turn* E_G, *the radial gain* dR/dn, *the projected beam ellipses and the rotation of particles around the centre of the beam* (ν_r)

We can think of thin 'salami slices' flying towards the septum with a rotation given by ν_r.

A monoenergetic beam can be achieved in two cases:

(a) NARROW PHASE WIDTH

The sinusoidal voltage $E_G = E_G(0) \times \cos\phi$ introduces an energy spread δE depending on the phase width $\delta\phi$ of the beam. If we ask, e.g. for $\delta E < 1/4\, E_G$, we get the requirement

$$\delta\phi < \sqrt{(2/n)} \tag{3.1}$$

Even for a low turn number like $n = 200$ (MSU) the accepted phase range is only 6°.

(b) FLATTOP VOLTAGE

A rectangular waveform for the rf-voltage would be ideal to get a monoenergetic beam with a broad phase. An approximation consists of using the second or third harmonic in addition to the main harmonic.[22]

In ring cyclotrons a 'flattop cavity' operating on the higher harmonic is easily inserted into a free sector. A third harmonic of only 11·7% of the main voltage gives an energy spread of 5×10^{-4} for a phase width of 30°! The crucial point is the stabilisation of the phase ripple α between main cavities and flattop cavities:[23]

$$\alpha < \frac{1}{\delta\phi}\,\frac{\delta E}{E} \tag{3.2}$$

which gives $\alpha < 0{\cdot}05°$ for the above example.

Both methods (a) and (b) for monoenergetic beams require a good stability of voltage, magnetic field and frequency.[8]

3.2. *The continuous energy beam*

A broad phase width or large fluctuation of the rf- and magnet-parameters smear out completely the turn structure at extraction. Instead of 'salami slices' we have now a 'continuous salami' moving towards the septum. Even for an extremely good beam quality extraction efficiency will always be less than 100%.

4. EXTRACTION ELEMENTS

A standard set of extraction elements is:

(a) Electrostatic channel with a tungsten or copper-septum ~ 0·1 to 0·5 mm thick, water cooled, V-slit at entrance. Problems are: sparking with operation close to $V \times E$-limit, deformation of septum by heat. Specialities are: the wire septum (AEG), the Berkeley septum, which can stand 6 kW power loss.[17]

(b) Magnetic channel, if possible radially focusing. Interesting achievements in this field are: the air-core channel of MSU[24] (compensated fringe field, radial gradient of 240 G/cm), the compensated weakening channel of AEG,[25] and the coaxial channel of Oak Ridge.[26]

Electric channels are still useful at high energies for two reasons:

(1) The separation between internal and extracted orbit is proportional to the machine radius. Therefore, a large radius compensates for the smaller deflection angle at higher energies. At SIN we get 5 cm separation with a moderate field of 40 kV/cm.

(2) The energy loss dE/dx drops drastically for higher energies, reducing substantially the heating problem of the septum.

The field-free regions of the SIN Cyclotron allow a nice solution for the focusing of the extracted beam. A 3 X 3 cm 'Panofsky-quadrupole' will be built[27] with a gradient of 300 G/cm. Two additional current sheets give a small dipole field for angular corrections. The focusing channel helps to reduce the dispersion of the extracted beam. The position of the SIN extraction elements are shown in the accelerator layout as presented in the status report by H. A. Willax.[28]

5. EXTRACTION EFFICIENCY

For an estimation of the extraction efficiency we make the following assumptions:

(1) The first extraction element is an electrostatic channel with a large aperture (no losses on antiseptum).

(2) Negligible losses on following elements.

The parameters influencing the extraction efficiency are: the radial gain dR/dn, the effective septum thickness δ_{eff}, ν_r and the beam amplitude x_o. A lower limit for the loss is given by:

$$\text{loss} \lesssim \delta_{eff}/(dR/dn), \quad \delta_{eff} = \delta_o + \delta_g + \delta_s \tag{5.1}$$

δ_o = thickness of septum entrance.

δ_g = additional thickness from fabricational errors (typically ~ 0·3 mm for curved septums).

δ_s = 'shadow thickness', takes into account particles which hit the septum walls. A typical shadow thickness is given by:

$$\delta_s \sim \frac{L\phi^2}{2\alpha_e} \tag{5.2}$$

L = channel length.

α_e = angular deflection of particle orbit.

ϕ = $|x'|_{max}$ = maximum divergence of beam.

δ_s is only of importance for a small angular deflection and long channels (for the SIN Cyclotron $\delta_s \sim 0{\cdot}15$ mm).

To see the influence of beam amplitude x_o and ν_r on extraction efficiency, a Monte Carlo programme was written at our laboratory by G. Tripard.[29] The assumptions are: continuous energy distribution near extraction (gives a pessimistic value for the particle loss), uniform phase space density inside the 'beam ellipse', and no coherent beam amplitude. The programme calculates particle loss at the septum, beam quality, and the energy spread ΔE (*FWHM*) of the extracted beam. The results of a generalised case are shown in Figs 7 and 8.

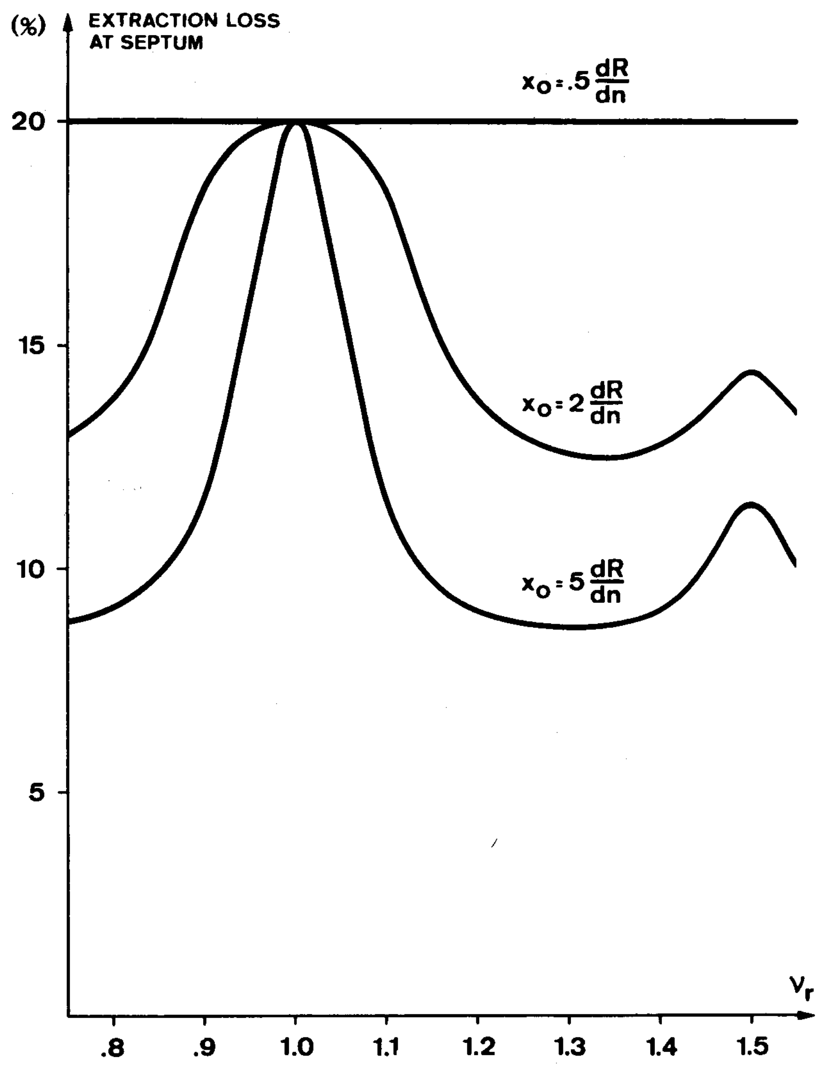

Fig. 7. Monte Carlo calculation of particle loss at septum. Assumptions are: septum thickness δ_{eff}, no losses at side walls of septum nor at the antiseptum, fixed radial gain per turn dR/dn = 5 × δ_{eff}, uniform energy distribution of particles. The plot shows losses as a function of ν_r for different incoherent amplitudes x_o

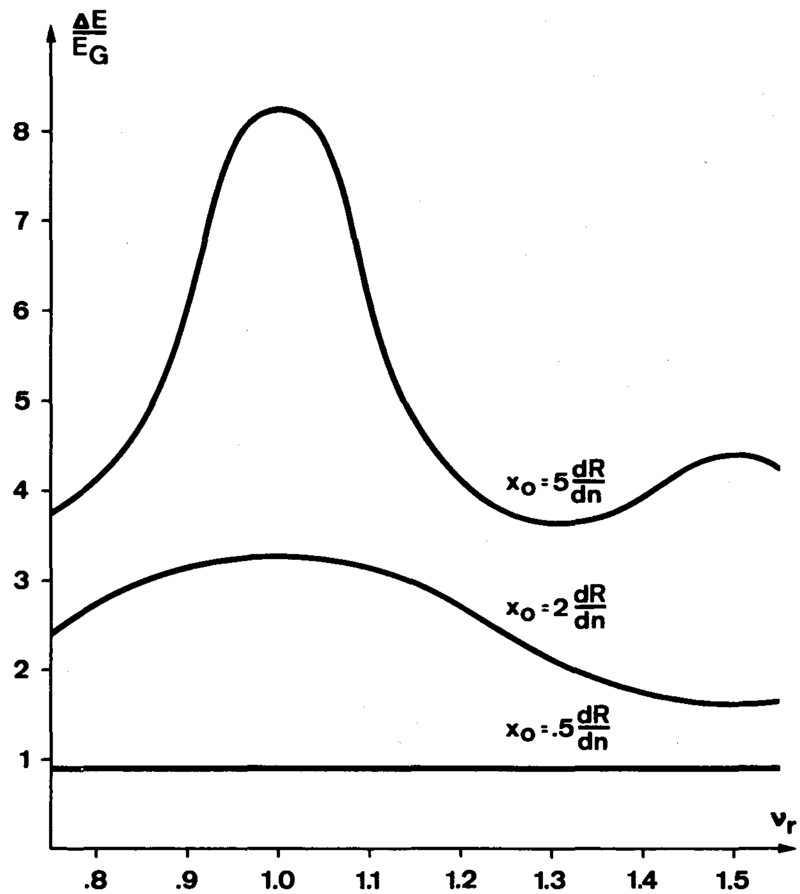

Fig. 8. Monte Carlo calculation of energy spread (FWHM) *of extracted beam. The assumptions are the same as for Fig. 7. The plot shows the energy spread* ΔE *divided by the energy gain per turn. The advantage of a good beam quality is obvious*

We can clearly distinguish two domains:

(a) good beam quality: $x_0 < dR/dn$.

Extraction efficiency and energy spread are almost independent of ν_r

loss $\sim$ $\delta_{eff}/(dR/dn)$, $\Delta E \sim 0{\cdot}9\, E_G$ (5.3)

(b) bad beam quality: $x_0 > dR/dn$

Extraction loss and energy spread show large fluctuations as a function of ν_r with a sharp maximum at $\nu_r = 1$. The energy spread is much worse than for good beam quality and practically independent of the energy gain.

$$0{\cdot}7\, x_0\, dE/dR < \Delta E < 1{\cdot}7\, x_0\, dE/dR \quad (5.4)$$

It seems paradoxical that the extraction efficiency can be higher for a bad quality beam! But a precession caused by $\nu_r \neq 1$ can really swing many particles with big amplitudes past the septum (synchrocyclotron!). In practice, however, most of these particles get lost on the anti-septum due to the limited aperture of the channel. Fig. 2 shows a radial phase plot of the SIN Cyclotron for the early design energy of 525 MeV and a conservative peak voltage of 400 kV per cavity. The picture shows that the radial gain from acceleration alone is enough to guarantee a good extraction rate. The Monte Carlo calculations determined an energy spread of 0·3% and a particle loss of 5% for a 0·2 mm septum. With the addition of a flattop cavity and an improved beam quality the extraction efficiency can be pushed towards 100% and the energy spread lowered to 0·1%.

6. STABILITY OF VERTICAL MOTION

Near extraction special attention has to be paid to the vertical motion. Resonances which can cause growth of the vertical amplitude are: $\nu_z = \frac{1}{2}$, $\nu_r = 2\,\nu_z$, and in addition for ring cyclotrons $\nu_z = 2/2$ and $\nu_z = 1$. The characteristics of these resonances are listed in Table 2.

Table 2

Resonance	*Driven by*	*Maximum growth per turn of z-amplitude*	*Stop band half width* $\Delta\nu$
$\nu_r = 2\,\nu_z$	–	$2\pi \times x_0 \times z_0 \times \frac{d\nu_r}{dR}$	$\sqrt{(4x_0^2 + z_0^2)}\ \frac{d\nu_r}{dR}$
$\nu_z = 1/2$	First harmonic of gradient bump	$\pi \times R \times \frac{g_1 z_0}{B_0}$	$\frac{Rg_1}{2B_0}$
	Eccentricity δx_1	$2\pi\ \delta x_1 \times z_0 \times \nu_r \times \frac{d\nu_r}{dR}$	$\delta x_1 \times \nu_r \times \frac{d\nu_r}{dR}$
$\nu_z = 2/2$	Second harmonic of gradient bump	$\pi \times R \times \frac{g_2 z_0}{2B_0}$	$\frac{Rg_2}{4B_0}$
	Eccentricity δx_2	$\pi \times \delta x_2 \times z_0 \times \nu_r \frac{d\nu_r}{dR}$	$\delta x_2 \times \frac{\nu_r}{2}\frac{d\nu_r}{dR}$
$\nu_z = 1$	First harmonic of horizontal field component	$\pi \times R \times \frac{b_{h_1}}{B_0}$	–

The important fact is that an eccentric beam, represented by

$$x(\theta) = \delta x_1 \cos(\theta - \theta_1) + \delta x_2 \cos 2(\theta - \theta_2) + \ldots \tag{6.1}$$

can destroy vertical stability. In medium energy cyclotrons with precessional extraction the resonances $\nu_r = 1$, $\nu_r = 2\nu_z$, and $\nu_z = \frac{1}{2}$ are all crossed shortly before extraction (see Fig. 5). In order to preserve vertical stability, one has to limit the radial amplitude, induced from the $\nu_r = 1$ resonance, to a few mm.

	Operating Cyclotrons								Future Cyclotrons			
	Berkeley 88 in	*Davis*	*Harwell*	*MSU*	*Grenoble*	*UCLA*	*Milan*	*Jülich*	*Columbia SC*	*Cern SC*	*TRIUMF*	*SIN*
Particle	α	α	p	p	p	H^-	H^-	d	p	p	H^-	p
E [MeV]	130	80	50	50	50	46	44	90	550	600	500	580
E_G [keV]	2 X 130	2 X 120	120	240	140	100	90	180	60	50	400	1700
R [m]	1·0	0·61	0·79	0·75	0·88	0·53	0·7	1·54	2·0	2·25	7·9	4·4
dR/dn [mm]	~2	0·9	2·5	6	0·9	0·6	1·0	1·5	20	–	1·6	8
Extraction	PREC	NR	PREC	PREC	BF	STR	STR	NR	RG	RG	STR	NR
ν_r	0·9	1·05	0·9	0·85	–	–	1·06	1·09	1·01	0·95	1·5	1·1
ν_z	0·6	0·2	0·6	0·6	–	–	0·37	0·40	0·4	0·30	0·3	1·3
x_o [mm]	2·5	2	2	0·5	–	5	1	2	7	< 10	2·5	4
z_o [mm]	3	6	4	5	–	7	3·5	4	12	10	5·5	4
E_x [mm mrad]	50	–	40	0·7	80	45	25	20	–	–	6	10
E_z [mm mrad]	70	–	40	5	30	30	50	25	–	–	8	10
I_{max} [μA]	140	20	40	20	1·5	5	2	10	20	15	200	100
ϵ_{max} [%]	60	50	80	100	70	100	100	70	75	75	99	>90
$\Delta E/E$ (*FWHM*) [%]	0·3	0·3	0·5	0·06	<1	2	0·7	–	0·6	0·2	0·2*	0·3*
$\delta\phi$	20°	20°	20°	1·5°	–	–	10°–50°	15°–30°	–	100°	60°	20°
$\Delta V/V$ (p.t.p.) [%]	0·1	0·5	0·1	0·06	1	–	0·1	0·1	–	–	0·04	< 0·1
$\Delta B/B$ (p.t.p.)	10^{-4}	10^{-4}	10^{-4}	$\gtrsim 10^{-6}$	5×10^{-5}	–	–	2×10^{-5}	–	10^{-4}	10^{-5}	10^{-5}
Extraction Elements	ES ES	ES CC	ES FES	ES FMS	ES MC	–	–	ES SC FM	PR TVB FM	PR TVB MC	–	ES FM MC
E [kV/cm]	180	20	80	150	60	–	–	90	–	–	–	40
δ [mm]	0·25	1·5	0·5	0·12	0·2	–	–	0·2	–	–	–	0·1
D [mm]	5	10	4–12	7	4–8	–	–	3–4	-	–	–	12

The coupling resonance $\nu_r = 2\nu_z$ depends strongly on the beam quality. In cyclotrons this resonance can be crossed easily,[9] while synchrocyclotrons with poor beam quality lose some particles there.

The SIN ring cyclotron was originally designed for 500 MeV only, because one feared to cross this coupling resonance. Extensive orbit calculations showed, however, that due to the large energy gain per turn and the expected good beam from the injector, a crossing is not dangerous. We anticipate an axial growth of ~ 1 mm.

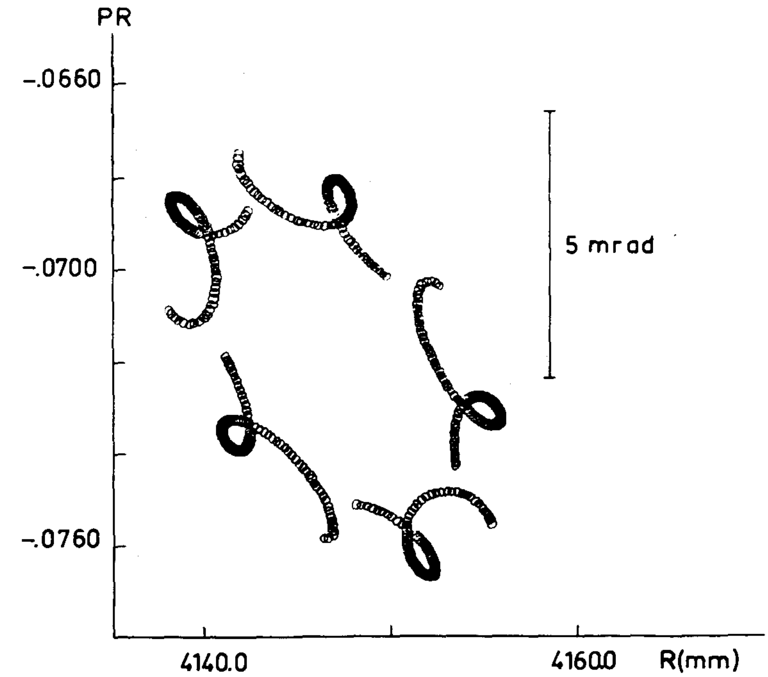

Fig. 9. Coupling resonance $\nu_r = 2\,\nu_z$. The motion of a particle with initial amplitudes of $x_0 = 9$ mm, $z_0 = 10$ mm *is plotted in phase space every 45° in the 8-sector SIN Cyclotron.* $\nu_r \sim 1{\cdot}60$, $\nu_z \sim 0{\cdot}79$, *no acceleration. The curls in the radial diagram indicate the non-linear influence of the vertical amplitude on the radial motion (ν_r – shift)*

A nice mechanical analogue of the coupling resonance is a simple spring pendulum. A mass m is suspended by a spring of length L and spring constant c. The resonance condition is:

$$\omega_{\text{vertical}} = 2\omega_{\text{horizontal}} \tag{6.2}$$

$$\sqrt{\frac{c}{m}} = 2\sqrt{\frac{g}{L}}$$

Let the pendulum swing vertically, after a while it will oscillate horizontally!

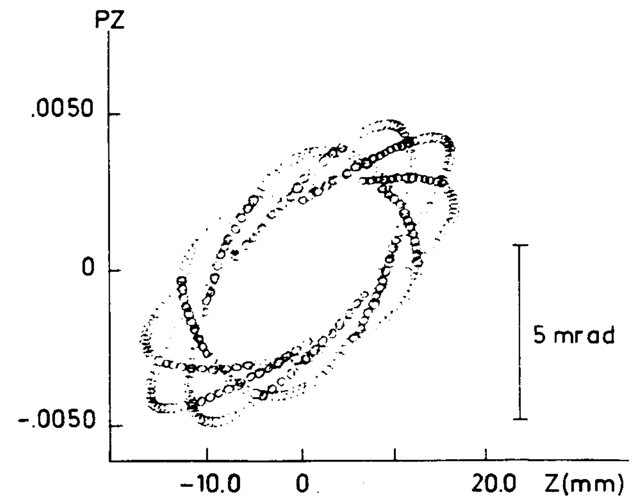

Fig. 10. New orbit patterns for relativistic cyclotrons. The 'eccentric' and the 'cloverleaf cyclotron' have straight sections to incorporate quadrupoles and cavities. The eccentric orbits enhance the radial turn separation at the extraction azimuth

7. OUTLOOK TO THE FUTURE

For high energy cyclotrons resonance extraction is a bit problematic. Why not try, therefore, to make the orbits eccentric from the very beginning? Since a radial separation of orbits is needed only at the injection and extraction point, one could build straight sections with space for quadrupoles, which help to reduce the vertical focusing problem. Fig. 10 shows two very preliminary examples. Since the circumference of the orbits is proportional to the particle velocity, these orbit patterns are only feasible if initial and final velocities are relativistic. Who will build the first cyclotron for K-meson production?

DISCUSSION

Speaker addressed: W. Joho (Zurich).

Question by H. Blosser (M.S.U.): In your calculations of extraction efficiency vs radial amplitude, it appeared that you did not include any allowance for an increase in the effective septum thickness as the radial amplitude increases. I feel that this would, in fact, cause the efficiency for large radial amplitudes to be much less favourable.

Answer: Figs 7 and 8 should indeed be carefully interpreted: not only is the effective septum thickness dependent on beam quality, the radial gain, too, is a function of ν_r. So these figures are meant to compare *different* cyclotrons with given extraction parameters. If you want to know extraction efficiency and energy spread as a function of ν_r for a given cyclotron, then these curves have to be rescaled, and are not symmetrical with respect to $\nu_r = 1$ any more. Low ν_r values below 1 enhance both extraction efficiency and energy spread.

Comment by G. Hendry (Cyclotron Corporation): I would like to comment on Blosser's question. If the object is to extract large amounts of power then the width of the leading edge of the septum is most important. Beam that misses the leading edge but does not get through the channel due to excessive angular divergence is generally spread out over considerable area which can, therefore, be more easily cooled.

Answer: You are right. I would like to mention, however, that at high energies the particles are not stopped in the septum, but rather get Coulomb-scattered out again and are lost somewhere else. Therefore, you are more worried about nuclear radiation than cooling of the septum.

Question by H. L. Hagedoorn (University of Eindhoven): I want to make two remarks. Firstly, it is not necessarily true that all integral resonances are energy sensitive and the half integral resonances are not. In fact it depends on the parameter which drives the resonance. If this parameter is energy dependent then the resonance will be energy sensitive. For example, in precessional extraction the changing ν_r serves to cause this effect, and in regenerative extraction a changing second harmonic first derivative (which means a second derivative). Secondly, if ν_r differs much from unity then the intensity distribution of the beam as a function of radius shows maxima and minima. A septum placed at a minimum results in a better extraction efficiency than the one which is acquired when the distribution function is homogeneous. (Of course, here also one will never get a 100% efficiency but one can approach it.)

Answer: If you take a flat first harmonic bump, then the integer resonance is energy selective only. If you take a radially changing second harmonic bump which drives the half integer resonance, then you have some energy selection too, because, as you point out, all peeler-regenerator arrangements will have non-linear shapes and besides will always have some first harmonic component.

Question by P. Macq (University of Louvain): When you speak of beam quality do you include the factor π in it?

Answer: Yes.

Question by E. Regenstreif (University of Rennes): Is the three-dimensional phase space representation you have shown a projection of the real six-dimensional phase space or has it been drawn under the assumption of complete decoupling?

Answer: Complete decoupling from the three other phase space dimensions (ϕ, z, pz) was assumed.

REFERENCES

1. Vogt, E. W. and Richardson, J. Reginald, *I.E.E.E.* **NS-13, 4,** 262, (1966).
2. Willax, H. A. and Blaser, J. P., *I.E.E.E.* **NS-13, 4,** 194, (1966).
3. Paul, A. C. and Wright, B. T., *I.E.E.E.* **NS-13, 4,** 74, (1966).
4. Willax, H. A., International conference on sector-focused cyclotrons and Meson factories, CERN **63-19,** 386, (1963).
5. Charpak, G., *et al., Phys. Rev. Letters* **6,** 128, (1961).
6. Mahrt, R. and Böttger, O., *Nucl. Instr. Meth.* **71,** 45, (1969).
7. van Nieuwland, J. M., private communication.
8. Gordon, M. M., *I.E.E.E.* **NS-13, 4,** 48, (1966).
9. Hagedoorn, H. L. and Kramer, P., *I.E.E.E.* **NS-13, 14,** 64, (1966).
10. Welton, T.,'Proposal for a southern region accelerator', Oak Ridge, (1957).
11. Blosser, H. G., Gordon, M. M. and Arnette, T. I., *Nucl. Instr. Meth.* **18, 19,** 488, (1962).
12. Blosser, H. G., *Nucleonics* **24,** 54, (1966).
13. Vogt-Nilsen, N., MURA–118, (1956).
14. Kim, H., *I.E.E.E.* **NS-13, 4,** 58, (1966).
15. Tuck and Teng, *Phys. Rev.* 305, (1951).
16. Le Couteur, K. J., *Proc. Phys. Soc.* **B-64,** 1073, (1951).
17. Clark, D. J., private communication.
18. Hopp, D. I. and Richardson, J. Reginald, *Nucl. Instr. Meth.* **44,** 277, (1966).
19. Dmitrievsky, V. P., *et al., I.E.E.E.* **NS-13, 4,** 84, (1966).
20. Blaser, J. P. and Willax, H. A., Progress report on the SIN 500 MeV cyclotron, International conference on nuclear structure, Tokyo, (1967).
21. Barton, M. Q., Fifth international conference on high energy accelerators, Frascati, 485, (1955).
22. Gordon, M. M., *Nucl. Instr. Meth.* **58,** 245, (1968).
23. Joho, W., Tolerances for the SIN ring-cyclotron, internal report TM-11-4, (1968).
24. Berg, R. E. and Blosser, H. G., *I.E.E.E.* **NS-12,** 392, (1965).
25. Müller, R. W. and Berkes, B., *Nucl. Instr. Meth.* **66,** 309, (1968).
26. Lord, R. S., *et al.,* CERN **63-19,** 297, (1963).
27. Vécsey, G., SIN, internal report TM-07-02, (1968).
28. Willax, H. A., Status report on SIN, Proceedings of this Conference, p. 58.
29. Tripard, G. and Joho, W., to be published.