Beam quality and expected energy resolution from the TRIUMF cyclotron

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ABSTRACT

H^- ions accelerated in the TRIUMF isochronous cyclotron will require a minimum of 1250 turns to attain a final energy of 500 MeV. Experience with other cyclotrons indicates that with careful control of imperfections, particularly the first harmonic magnetic field component, the internal beam emittance should be no more than 4π mm mrad at 50 MeV or $1{\cdot}2\pi$ mm mrad at 500 MeV. Multiple scattering at the stripping foil will enlarge this a further 20–70% depending on the foil thickness. The total energy spread in the raw extracted beam at 500 MeV will be ±600 keV.

When higher resolution is required, slits will be placed at low energy to limit the radial emittance, enabling $6\,\mu$A to be extracted with ±150 keV resolution. With the planned addition of third harmonic to the rf cavity separated turn acceleration will be possible out to maximum energy with the magnet and rf control technology available today; $30\,\mu$A should be obtainable with ±25 keV total energy spread and a ±7° phase spread. A third scheme, using slits at the final radius, can give two high current low resolution beams and a simultaneous low current beam with ±60 keV total spread.

1. BEAM QUALITY

The effective beam quality of the external beam will depend upon the initial emittance at the H^- ion source and the treatment it is accorded during acceleration. Non-linear and coupling effects occurring at injection, or during acceleration and extraction, can distort and fragment the emittance, as Banford[1] has illustrated for a gridded electrostatic mirror. The spiral inflector proposed for TRIUMF should improve on this. Moreover, orbit calculations[2] show that non-linear

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effects over the first few turns are negligible, presumably because of the relatively small beam size and high injection energy (300 keV).

Radial oscillations which displace the beam bunch from the equilibrium orbit (E.O.) are produced both by the phase dependence of radial centring[2] and by first harmonic magnetic field errors near the centre. A cyclotron's sensitivity to the latter varies roughly as $B_C^{-3/2}$ where B_C is the central magnetic field,[3] and is thus a factor ten worse for TRIUMF than in most cyclotrons. Special care will be taken to shim out such errors and a set of 12 × 6 harmonic coil pairs is included for final smoothing. The machine design avoids serious resonances and with only a limited range of operating conditions needed it should be possible to maintain beam quality during acceleration. Multiple scattering at the stripping foil is expected to increase the emittance by a further 20-70%, depending on the foil thickness; the effect of higher order terms during extraction is <10%.[4]

Except for multiple scattering none of these effects contravenes Liouville's theorem by increasing the actual phase space occupied by ions; what they do is to distort and twist it, increasing the effective volume occupied, and in such a way that 'All the King's horses and all the King's men couldn't put Humpty-Dumpty together again'. Thus, for example, in the Cyclotron Corporation compact cyclotron the emittance of the internal H^- beam is four times larger than that from their Ehlers hot filament arc H^- source (2 mA within 65π mm mrad at 12 keV). In view of the larger size of TRIUMF at least as good an internal beam emittance should be attainable, i.e. 4π mm mrad at 50 MeV, or $1{\cdot}2\pi$ mm mrad at 500 MeV.

This assignment results in the radial parameters shown in Table 1.

Table 1. RADIAL CYCLOTRON PARAMETERS

Energy	30 MeV	50 MeV	200 MeV	500 MeV
Radius (in)	101	129	232	311
$\frac{dE}{dR}\left(\frac{\text{MeV}}{\text{in}}\right)$	0·61	0·79	2·30	6·25
$\Delta\rho$ (in)	0·65	0·51	0·17	0·064
ϵ_R (mm mrad)	5π	4π	2π	$1{\cdot}2\pi$
ϕ_{Rm} (mrad)	1·5	1·1	0·6	0·5
A_{Rm} (in)	0·14	0·14	0·12	0·10
ΔE (MeV) $= A_{Rm}\frac{dE}{dR}$	0·087	0·11	0·28	0·60

Here the maximum amplitudes A_{Rm} of radial oscillation and ϕ_{Rm} of radial divergence from the equilibrium orbit (E.O.) are related to the emittance ϵ_R and radial tune ν_R at mean radius $R = \beta R_c$ by

$$\epsilon_R = \pi A_{Rm}\,\phi_{Rm} = \pi A_{Rm}^2\,\nu_R/R = \text{const}/\beta\gamma$$

β and γ being the usual relativistic factors. $dE/dR = \beta\gamma^3 m_0 c^2/R_c$ is the rate of change of E.O. energy with radius and $\Delta\rho$ is the turn separation for 0° phase ions, which gain 400 keV per turn.

2. ENERGY RESOLUTION OF THE RAW BEAM

At energies above ~150 MeV, Table 1 shows that the turns overlap radially ($2A_{Rm} > \Delta\rho$) so that for a stripper of width s at radius R_s we might expect the stripped ions to have an energy spread $\delta E = \delta\rho\ (dE/dR) = (2A_{Rm} + s)\ (dE/dR)$. However, this argument does not take detailed account of the radial oscillations, which can be used to narrow δE. The oscillations of the ions about the E.O. may be viewed in terms of the rotation either of their representative points in phase space (ν_R times per turn) or of their centres of curvature in real space ($\nu_R - 1$ times per turn) (Fig. 1), which at any instant occupy a disc of radius A_{Rm} at the cyclotron centre (we can ignore orbit scalloping).

Then if an ion hits a stripper at radius R_s while its centre of curvature is at (1), its radius of curvature must be $\rho = R_s - A_R \sin\theta$. For ions near 400-500 MeV $\nu_R \simeq 1{\cdot}5$, so that one turn later (or earlier) its centre of curvature is at (2),

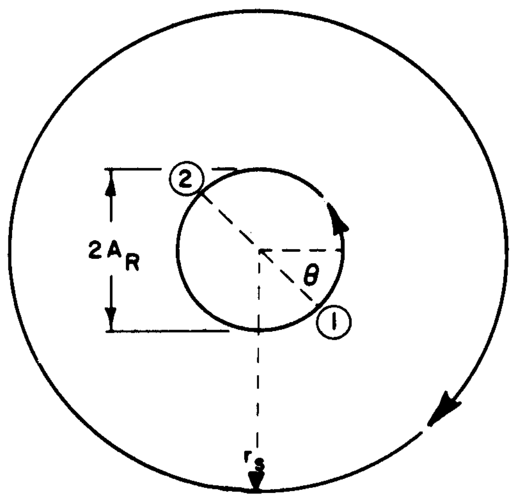

Fig. 1. Radial oscillations of amplitude A_R *and phase* θ *about the equilibrium orbit (large circle) cause the centres of ion motion to precess around the small circle at frequency* $1 - \nu_R$

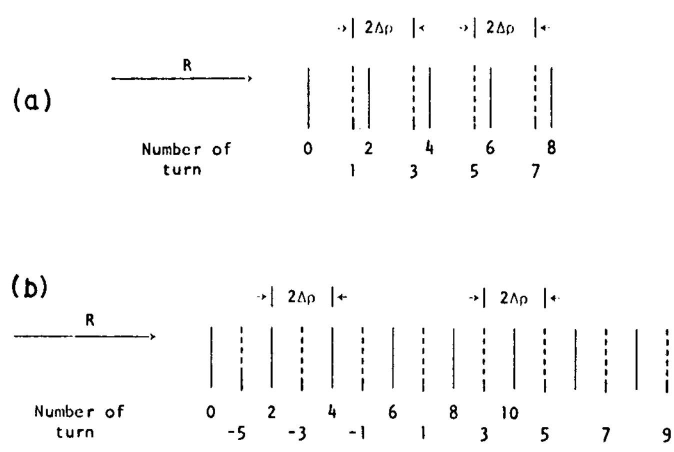

Fig. 2. Ion radius R *at a particular azimuth on successive turns, when* ν_R *= 1·5 and (a)* $A_R \sin\theta \quad \Delta\rho$*, (b)* $A_R \sin\theta = 6\Delta\rho$

180° away, where $\rho = R_s + A_R \sin\theta$. Superimposing on this picture the increase in curvature by $\Delta\rho$ each turn, we see that an ion gains radius in alternate jumps of $\Delta R = \Delta\rho \pm |2A_R \sin\theta|$ at a given azimuth. The result when $|A_R \sin\theta| = 6\Delta\rho$ is shown in Fig. 2(b).

When ions with all phases and amplitudes $A_R \leqslant A_{Rm}$ of oscillations are considered, their distribution in R and ρ is as shown in Fig. 3 (though A_{Rm} is exaggerated for illustration); the continuous distribution in ρ is a consequence of the spread in rf phases.

Fig. 3 also shows the effects of strippers of widths $s < 2\Delta\rho$ (a) and $s \geqslant 2\Delta\rho$ (b). The narrower stripper strips only part of the beam but takes ions from

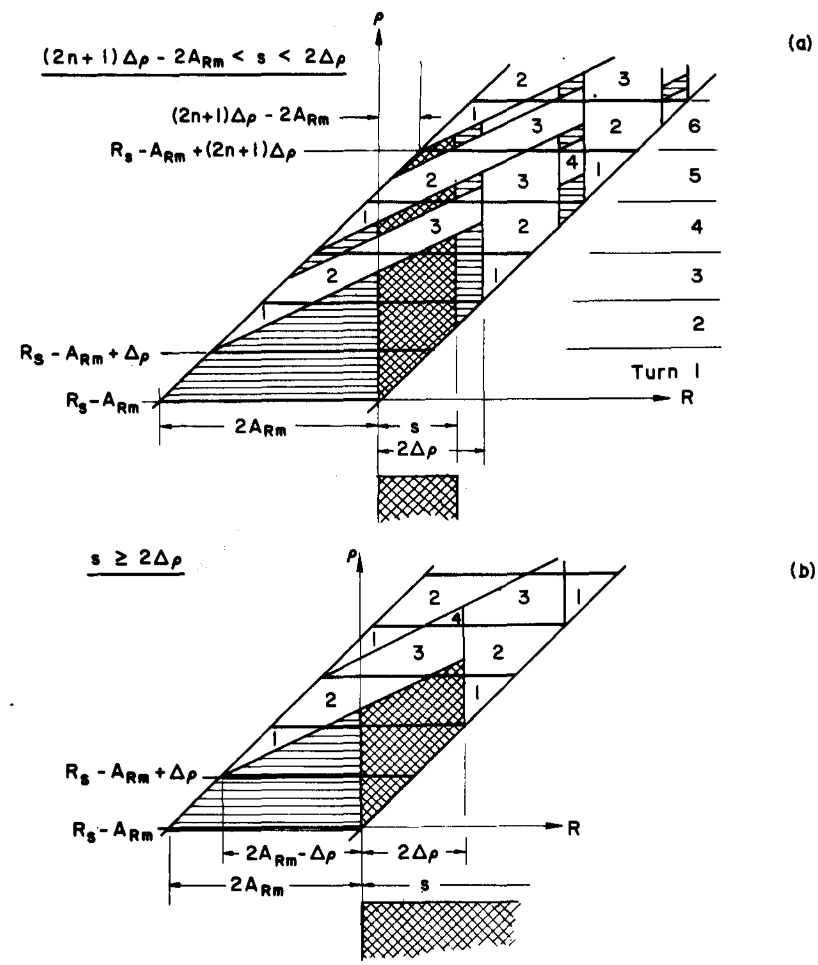

Fig. 3. Ion distribution in radius R *and curvature* ρ *(i.e. energy) at the stripper azimuth during successive turns, when* $\nu_R = 1{\cdot}5$ *and the beam is wider than the turn separation* ($2A_{Rm} > \Delta\rho$)*; in (a) the stripper width* $s < 2\Delta\rho$*, (b)* $s \geqslant 2\Delta\rho$*. The hatched areas are unstripped, the cross-hatched areas are being stripped, and the blank areas have been stripped on the turn indicated. The diagram may be regarded as a median cut through an elliptical tube in (*ρ*,* R*,* ρ_R*) space, the tube being rotated* $\nu_R = 1{\cdot}5$ *times about the* ρ *axis on each turn*

$(2A_{Rm} + s)/\Delta\rho$ turns, with a corresponding spread in ρ. Paradoxically, the wider stripper, stripping the whole beam, produces the smaller spread in curvature $\delta\rho = A_{Rm} + (1\cdot5)\Delta\rho$. Thus at 500 MeV, where $2A_{Rm} \simeq 3\Delta\rho$, we obtain a total energy spread ±600 keV by stripping the whole beam, a 25% improvement through using the effects of $\nu_R \simeq 1\cdot5$. In practice we will not have $\nu_R = 1\cdot5$ exactly and there will be a small tail of poorer resolution. At 200 MeV ν_R is no help, so $\delta\rho \simeq 2A_{Rm} + \Delta\rho$ and $\delta E = \pm 480$ keV.

We now go on to consider three methods of reducing the energy spread of the internal beam.

3. IMPROVED ENERGY RESOLUTION WITH LOW ENERGY DEFINING SLITS

One step to improve the energy resolution of the extracted beam is to restrict the amplitude of radial oscillations so that at extraction $2A_{Rm} < \Delta\rho$. At least three slits are needed; if they are of width w and are placed θ apart in azimuth, the oscillation amplitude is restricted to $A^{w}_{Rmax} = (w/2)\ \mathrm{cosec}^2(\nu_R\theta/2)$. At low energies where $\nu_R \simeq 1, \theta = 90^\circ$ is a convenient choice, though others are possible. For an E.O. of mean radius δ greater than the central slit radius R_w a rectangular cut of emittance $\epsilon_w = w(w - 2|\delta|)(\nu_R/R_w)$ is passed with $A^{w}_{Rm} = \sqrt{(\tfrac{1}{2}w^2 + 2\delta^2)}$. No ions are passed unless $-\tfrac{1}{2}w \leqslant \delta \leqslant +\tfrac{1}{2}w$ so that $w/\sqrt{2} \leqslant A^{w}_{Rm} \leqslant w$; after acceleration to the extraction radius where $\gamma_s \simeq 1\cdot5$ we have to a good approximation $A^{s}_{Rm} = A^{w}_{Rm}\ \gamma_w\gamma_s^{-1} \simeq \tfrac{1}{2}w$. The limits on δ also imply a phase selection by the slits. Optimum intensity is transmitted through the phase band centred at 0° if $\delta(0^\circ) = +w/6$. Then $\langle\epsilon_w\rangle = (2/3)w^2(\nu_R/R_w)$ and the phase limits are $\alpha_m = \pm 2\sqrt{(2w/3R_w)} = \pm 2\cdot0^\circ$ for $R_w = 101$ in, $w = 0\cdot048$ in.

With the reduction in beam width to $2A^{s}_{Rm} < \Delta\rho_s$ a narrow stripper (width s) produces a proton beam with a curvature spread of $\delta\rho = s + 2A^{s}_{Rm} = s + w$ (see Figs 2a and 4a). To keep the total energy spread better than 400 keV, however, $\delta\rho < \Delta\rho$, and this places a limit $\Delta\rho_s - w$ on the stripper width s, and hence on the extractable beam current. If, however, the selected central phase band $\pm\alpha_m$ is narrow enough (<2·3°) that the corresponding energy bands for each turn are separated at extraction, i.e.

$$\Delta R_m \equiv R(\alpha_m) - R(0^\circ) = \frac{2R_s\ \gamma_s(\gamma_s + 1)}{3R_w\ \gamma_w(\gamma_w + 1)} w < \Delta\rho_s$$

then we can again take advantage of the approximately 3/2 rotation of the emittance per turn in radial phase space near 500 MeV. Figure 4b shows how the unpopulated curvature band enables the stripper width to be increased by $2(\Delta\rho_w - \Delta R_m)$ without the curvature spread $\delta\rho$ exceeding ΔR_m. Maximum intensity is to be expected for $\Delta R_m = 107\ w/R_s$ (in) $\simeq 0\cdot75\ \Delta\rho_s$, i.e. for an energy spread of ±150 keV. For the same energy spread, somewhat paradoxically, the available intensity increases with slit radius R_w, for although the phase acceptance $\pm\alpha_m$ is reduced, this is more than compensated by the larger slits and hence larger selected emittances for which the energy bands remain separate. The slits must of course be outside the central region where most of the factors which tend to worsen the radial emittance, such as electric forces, dee voltage asymmetries, and first harmonic magnetic field components, do their work. A radius somewhere between 70 in (15 MeV) and 101 in (30 MeV) would perhaps

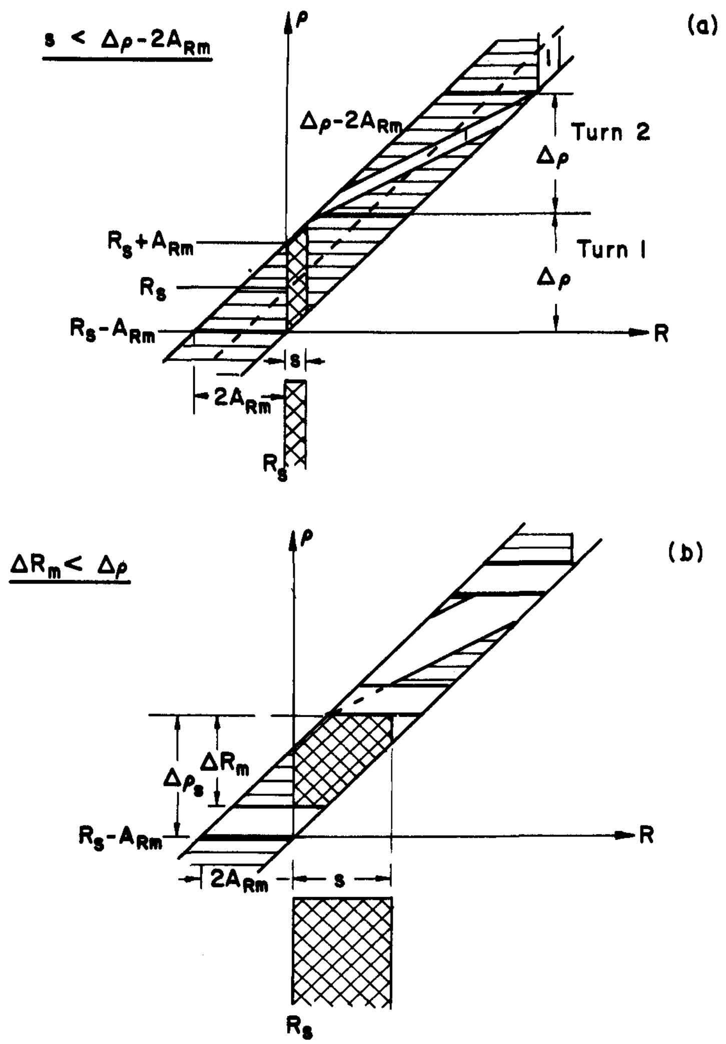

Fig. 4. Ion distribution in R *and* ρ *at the stripper azimuth when the beam width* $2A_{Rm} < \Delta\rho$. *In (b) phase selection has restricted the range of* ρ *to* $\Delta R_m < \Delta\rho$ *on each turn*

Table 2. TOLERANCES REQUIRED FOR SEPARATED TURN ACCELERATION

Operating mode	*Normal*	*Sep. turns*	*Sep. turns*
Third harmonic ϵ	0	0	−11·33%
Trim coils	35	54	54
$\delta(d\bar{B}/dR)$ (G/ft)	±2	±0·25	±1
$\Delta B/\bar{B}$ (pk. to pk.)	10^{-5}	$0{\cdot}3 \times 10^{-6}$	$0{\cdot}25 \times 10^{-5}$
$\Delta\omega/\omega$ (pk. to pk.)	$0{\cdot}25 \times 10^{-5}$	$0{\cdot}15 \times 10^{-6}$	10^{-6}
Phase limit	±60°	±1·1°	± 10·1°
Phase acceptance	±40°	±0·5°	±6·7°
$\Delta V_d/V_d$ (pk. to pk.)	$0{\cdot}4 \times 10^{-3}$	10^{-4}	$0{\cdot}5 \times 10^{-4}$
Total energy spread (keV)	±600	±50	±25

be an optimum. Higher energies would lead to radiation problems from the discarded beam. For R_w = 101 in the required slit width w for ±150 keV energy spread is 0·048 in; the stripper width s = 0·044 in and the estimated intensity is 6 μA. This calculation assumes that the narrow side phase bands at $\alpha = \pm 9{\cdot}2^\circ\sqrt{n}$ (n = 1, 2, 3, . . .) are later rejected; they would broaden the energy spread while contributing less intensity altogether than the central band.

4. SEPARATED TURN ACCELERATION

To achieve still higher energy resolution the phase spread $\pm\alpha_m$ may be still further reduced and the turns completely physically separated permitting complete extraction of individual turns. To achieve energy resolution $\Delta E/E$ the phase limits $\pm\alpha_0$ and dee voltage fluctuations ΔV_d, assumed uncorrelated, should satisfy

$$\left(\frac{\Delta V_d}{V_d}\right)^2 + \left(\frac{\alpha_0^2}{2}\right)^2 < \left(\frac{\Delta E}{E}\right)^2$$

For $\Delta E = \pm 50$ keV the tolerances required on isochronism and on the magnetic field and rf fluctuations (see Table 2) are very strict, even for 1975, if the rf voltage has a purely sinelike waveform, as the authors have pointed out previously.[3] The introduction of a certain percentage of third harmonic[5] (about $-1/9$)

$$V/V_d = V' = \cos\alpha - \epsilon\cos 3\alpha$$

gives a flat top to the rf voltage wave and results in a great relaxation of these technical problems. The simple rf electrode structure in TRIUMF has in fact permitted successful model tests[6] in which the third harmonic has been introduced electrically from a separate transmitter, thus obviating the need for additional electrodes.

For $\epsilon > 1/9$ the rf voltage rises to a peak $V'_M(\alpha_M)$ and then drops to $V'(\alpha_3) = V'(0) = 1 - \epsilon$. The equivalent α_0 is defined by $V'_M \cos\alpha_0 = 1 - \epsilon$.

For $\epsilon \simeq 1/9$ these quantities are related by

$$\frac{1}{2}\alpha_0^2 = \frac{3}{8}\alpha_M^4 = \frac{3}{32}\alpha_3^4 = \left(\frac{3}{2}\right)^7 \left(\epsilon - \frac{1}{9}\right)^2$$

so that there is a large improvement factor α_3/α_0. Because of power limitations V_d rather than V_m will be fixed for TRIUMF; and hence the phase wander out to R_f can be expressed

$$\Delta s'(\alpha) = 2\pi\nu \left[\frac{m_oc^2}{4qV_d}\int_0^{R_f} \frac{\delta B}{B_c}\frac{R\mathrm{d}R}{R_c^2} + N_o\sqrt{\left(\frac{\delta B_c}{\gamma_f B_c}\right)^2 + \left(\frac{\delta\omega}{\omega}\right)^2}\,\right]$$

where $s'(\alpha) = \int V'(\alpha)\,\mathrm{d}\alpha = \sin\alpha - \frac{\epsilon}{3}\sin 3\alpha$

and $\nu = 5, N_0 = 1250$, and δB is the isochronous field error. The treatment of fluctuations in magnetic field δB_c and in radio frequency $\delta\omega$, assumed uncorrelated, follows Joho.[7]

These equations lead to the parameters in Table 2, from which it is clear that with $\epsilon = 11{\cdot}33\%$ a total energy spread as small as ± 25 keV can be obtained with tolerances well within the present state of the art. Moreover, this can be obtained for a $\pm 6{\cdot}7°$ phase spread ($3{\cdot}7\%$ duty factor) which should permit eventual extraction of a 30 μA beam. The requirements on the third harmonic are that ϵ be controlled to $1/2 \times 10^3$ and its phase relative to the fundamental be held to $\leqslant \pm 0{\cdot}15°$.

5. ENERGY SELECTION ON THE FINAL ORBIT

B. L. White[8] has suggested the possibility of making the energy selection in the final orbit by means of two slits and a narrow stripper, the beams stripped by the slits also being led outside for use as external beams. The slits and stripper would be placed a quarter wavelength of a radial oscillation apart ($\theta = 90°/\nu_R \simeq 60°$) as shown in Fig. 5 (orbit scalloping can be ignored for our purposes). This conveniently matches the sixfold symmetry of the cyclotron. The spread in curvature of orbits transmitted by the slits but hitting the stripper (all of width Δs) is $\delta\rho = \pm\frac{1}{2}\Delta s \cot^2\frac{1}{2}\theta \simeq \pm 1{\cdot}5\,\Delta s$. The energy spread δE cannot be reduced indefinitely by narrowing Δs because the axial variation in magnetic field produces a spread in ρ for a given E.O. energy (and vice-versa) for ions not

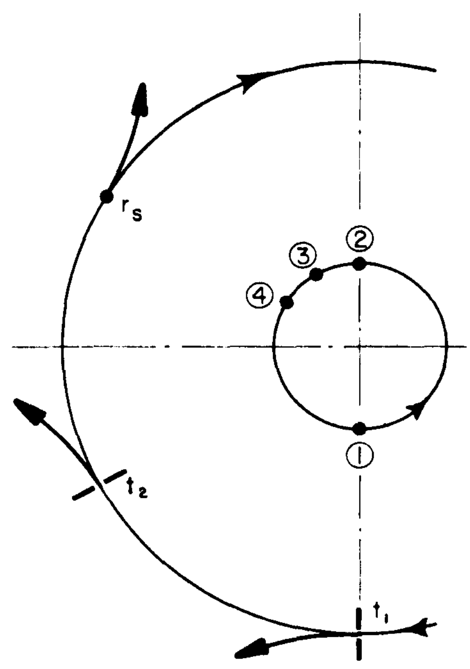

Fig. 5. Arrangement of two slits and a narrow stripper about 60° apart on the final orbit for energy selection; the precessional circle of the ion centres of motion is also shown

confined to the median plane. Thus, a group of 450 MeV ions with representative axial oscillations, tracked by the general orbit code GOBLIN (with magnetic field expansion and equations of motion correct to second order) showed a radial spread of 0·005 in (i.e. $\delta E = \pm 15$ keV). A practical choice for Δs might be 0·005 in also, giving a total $\delta E = \pm 60$ keV. The associated phase acceptance would be $\pm 0{\cdot}5^\circ$ and the estimated intensity 0·05 μA.

To avoid interference with the penultimate turn the slit thickness $t < \Delta\rho - \Delta s = 0{\cdot}059$ in; then if the radial oscillation amplitude is limited to $A_{Rm} = t$, not only is all the beam stripped eventually but ions are prevented from hitting the stripper on their penultimate turn. The low energy slits required for this are relatively wide ($w = 0{\cdot}12$ in) and transmit a 25 μA beam over $\pm 3{\cdot}2^\circ$ phase for extraction on the final two turns. By halving t, w and A_{Rm}, extraction is restricted to the final turn, improving the energy resolution of the high current beams, at the expense of a four times smaller intensity, but without affecting the high resolution beam.

An alternative scheme, enabling the full beam to be extracted simultaneously with a high resolution beam, is suggested by Fig. 3a, where a stripper width $s < 2\Delta\rho$ leaves two diamonds of ions unstripped. If $2A_{Rm} \leqslant \Delta\rho + s \simeq 3\Delta\rho$ (as is always so—Table 3), only one diamond is left and this can then be extracted by a stripper (width $\Delta s > 2\Delta\rho - s$) at radius $R_s + 3\Delta\rho$, two sectors (120°) beyond the main stripper, and without interfering with the primary beam. The curvature spread $\delta\rho = 2\Delta\rho - s$, and for a total energy spread ± 50 keV it appears that about 1% of the primary beam, or 2 μA, could be obtained; for ±100 keV, 4% or 8 μA. The absence of low energy slits and phase selection leads to good intensities and duty factor, but also implies a need for uniform radial centring at all phases if the quoted energy spread is to be attained.

Table 3. SUMMARY OF BEAM CHARACTERISTICS

	Full width energy spread (500 MeV)	*Spread in phase*	*Estimated intensity*	*Duty Factor*
Raw Beam	±600 keV	±30°	200 μA	15%
		±50° (3rd)	200	27
Low Energy Slits				
0·048 in	±150	±2·0°	6	1·1
0·032 in	±100	±1·8°	2	1·0
		±14° (3rd)	16	8
Separated Turn	±50	±0·5°	5	0·3
Acceleration	±25	±6·7° (3rd)	30	3·7
Final Orbit	±60	±0·5°	0·05	0·3
Selection		±5·0° (3rd)	0·5	2·8

(3rd) indicates that the proper mixture of third harmonic of the rf voltage is used.

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REFERENCES

1. Banford, A. P., *RHEL PLA Acc. Phys.* 29 (unpublished report).
2. Craddock, M. K., Louis, R. J., and Reiser, M. Proceedings of this Conference, p. 666.
3. Craddock, M. K., and Richardson, J. Reginald, *I.E.E.E. Trans. Nucl. Sci.* **NS-16**, 3, 415, (1969).
4. Robertson, L. P., Auld, E. G., Otter, A. J., and Mackenzie, G. Proceedings of this Conference, p. 245.
5. Gordon, M. M., *Nucl. Instr. Methods* **58**, 245, (1968).
6. Erdman, K. L., Prochazka, A., Fredriksson, O. K., Grundman, W. A., and Thomas, R. Proceedings of this Conference, p. 105.
7. Joho, W., SIN-TM-11-4 (unpublished report).
8. White, B. L., private communication.