

Design and Performance of a 12-Mev Isochronous Cyclotron

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An isochronous cyclotron, accelerating protons to 12 Mev, has been in use in our laboratory for almost one year now⁽¹⁾. Our main object in building this machine has been to find out whether it would be possible with a low dee voltage to get a beam through the complicated azimuthally varying field necessary to obtain isochronism as well as sufficient focusing, and to study the various factors influencing the beam current.

From the start we decided not to bother about a region within a radius of about 10 cm at the center of the machine, since conditions are very confused there. The particles are moving between the dees for the greater part of time, the electrical field is distorted by the ion source and puller, the magnetic field focusing is negligible in the center, the electric field may be defocusing, the particles may have all phases with respect to the rf during the first few turns, etc. Since the number of revolutions in this region is small, about 10% of the total number in our machine, the phase shift will not be very large even if the angular frequency of the particles is not correct. It was assumed that a considerable difference between the actual angular frequency of the particles and the circular frequency of the oscillator would be permissible in the central region, and we made the field uniform to a radius of about 7.5 cm. Later on the field in the center has been adjusted for maximum beam current.

Furthermore, we tried to keep the calculations as simple as possible and to design the field so as to render measurements simple and quick to perform. We also wanted to avoid model measurements which consume much time, to avoid pole-face windings and to use a simple stepped pole profile easy to machine.

Basic considerations

A cyclotron will be isochronous if the average flux density $\langle B \rangle$ along an equilibrium orbit is

$$\langle B \rangle = \frac{B_0}{(1-\beta^2)^{1/2}}, \tag{1}$$

where B_0 is determined by the circular frequency $\omega_0 = (eB_0/m_0)$ of the oscillator (e is the charge, m_0 the rest mass of a proton). If Eq. (1) is satisfied the average angular velocity $\langle \omega \rangle$ of the protons on the equilibrium orbits will be equal to ω_0 . Putting $R = (1/2\pi) \oint ds$, where the integral is the length of an equilibrium orbit, we obtain

$$\beta = \frac{v}{c} = \frac{\langle \omega \rangle R}{c} = \frac{R}{\lambda} \text{ with } \lambda = \frac{c}{\omega_0} \text{ and, hence, } p = e \langle B \rangle R.$$

Eq. (1) can thus be written

$$\langle B \rangle = \frac{B_0}{(1-(\frac{R}{\lambda})^2)^{1/2}} \tag{2}$$

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(1) F. A. Heyn and Khoe Kong Tat, R.S.I. 29 (1958) 622.

The focusing properties of this field are determined by the tunes ν_x and ν_z where $\nu_x^2 = \alpha$ and $\nu_z^2 = 1 - \alpha$.

The momentum compaction α is given by

$$\alpha = \frac{d \ln p}{d \ln R} = 1 + \frac{d \ln \langle B \rangle}{d \ln R} = 1 + \frac{\left(\frac{R}{\lambda}\right)^2}{1 - \left(\frac{R}{\lambda}\right)^2} \quad (3)$$

and, therefore, the defocusing of the field, given by (2), is proportional to

$$\frac{\left(\frac{R}{\lambda}\right)^2}{1 - \left(\frac{R}{\lambda}\right)^2}.$$

Focusing can be obtained by azimuthal variation of the field. Let the field have N identical radial sectors and let $\theta = (N/R) \int ds$. Then the field in the median plane can be expressed by

$$B = \langle B \rangle (1 + \mu_o(R, \theta)) \quad (4)$$

with $\langle \mu_o \rangle = 0$. The vertical tune is now determined by

$$\nu_z^2 = 1 - \alpha + \langle \mu_o^2 \rangle + \dots$$

and the field will be focusing provided $\nu_z^2 > 0$. Therefore, the amplitude of μ_o is taken proportional to the square root of

$$\frac{\left(\frac{R}{\lambda}\right)^2}{1 - \left(\frac{R}{\lambda}\right)^2}, \text{ and we write}$$

$$\mu_o = C f(\theta) \frac{\frac{R}{\lambda}}{\left(1 - \left(\frac{R}{\lambda}\right)^2\right)^{1/2}}, \quad (5)$$

where C is a constant and $f(\theta)$ a certain function of θ . Thus the field in the median plane required to obtain isochronism can be written:

$$B(R, \theta) = B_o \left[1 + \frac{1}{2} \left(\frac{R}{\lambda}\right)^2 + \dots + C f(\theta) \left\{ \frac{R}{\lambda} + \left(\frac{R}{\lambda}\right)^3 + \dots \right\} \right]. \quad (6)$$

For practical use the field must be expressed in polar coordinates and Eq. (6) takes the form

$$F(r, \varphi) = B_o \left[1 + a \left(\frac{r}{\lambda}\right)^2 + \dots + g(\varphi) \left\{ A \frac{r}{\lambda} + B \left(\frac{r}{\lambda}\right)^3 + \dots \right\} \right], \quad (7)$$

where a, b, \dots and A, B, \dots are to be determined.

To simplify calculations, measurements, and machining of the poles, $g(\varphi)$ rather than $f(\theta)$ must be chosen carefully and as simple as possible. From preliminary

estimations it was known that

$$g(\varphi) = (-1)^k a_k(r) \cos 2(k+1) N \varphi, \quad (8)$$

with

$$\alpha_k = \frac{\sinh N \frac{z_m}{r}}{\sinh(2k+1)N \frac{z_m}{r}}, \quad \alpha_0 = 1, \quad (9)$$

where $2z_m$ is the largest distance between the poles at radius r (see below) that can be obtained with a simple pole profile. All terms in $g(\varphi)$ with $k > 0$ are small, i.e., the azimuthal variation of the field is almost sinusoidal. This is a disadvantage with respect to focusing, but it renders calculations and measurements easier. In order to determine a, b, \dots the coordinates r, φ in Eq. (7) are transformed to R, θ and Eq. (7) is compared with (6). Transformation formulas are obtained from the equilibrium orbit equation

$$p = e B_0 = e \langle B \rangle R.$$

For a machine of the size of the one under consideration these formulas become rather simple. Terms containing third or higher powers of R/λ can be neglected and one needs only the coefficient a . One obtains

$$a = \frac{1}{2} - A^2 \sum_k \frac{a_k^2}{(2k+1)^2 N^2 - 1} \quad (10)$$

For $K = 0$ this reduces to

$$a = \frac{1}{2} - \frac{A^2}{N^2 - 1}$$

From the same comparison μ_0 is obtained. Transformation formulas

$$r = R \left(1 + \frac{R}{\lambda} \frac{1}{N^2 - 1} \cos N \varphi \right) \text{ and } \theta = N \varphi \quad (11)$$

are accurate enough for this purpose; with them one gets

$$\mu_0 = \left(1 - \left(\frac{R}{\lambda} \right)^2 \right)^{1/2} \left[A \frac{R}{\lambda} + \dots \right] \cos N \varphi + \dots = \bar{A} \cos N \varphi + \dots \quad (12)$$

Terms containing $\cos 3N\varphi, \cos 5N\varphi,$ are so small that they can be neglected.

With this expression for μ_0 and using a smooth approximation the tunes are calculated. This yields:

$$v_x^2 = \frac{1}{1 - \left(\frac{R}{\lambda} \right)^2} + \frac{\bar{A}^2}{2N^2} \left[\frac{1}{\left(1 - \left(\frac{R}{\lambda} \right)^2 \right)^2} + \frac{1}{1 - \left(\frac{R}{\lambda} \right)^2} \left(2 + 2 \frac{\bar{R}\bar{A}'}{\bar{A}} \right) \right] + \dots \quad (13)$$

$$v_z^2 = - \frac{\left(\frac{R}{\lambda} \right)^2}{1 - \left(\frac{R}{\lambda} \right)^2} + \frac{\bar{A}^2}{2} + \frac{\bar{A}^2}{2N^2} \left[\frac{1}{\left(1 - \left(\frac{R}{\lambda} \right)^2 \right)^2} - \left(2 - 2 \frac{\bar{R}\bar{A}'}{\bar{A}} \right) \frac{1}{1 - \left(\frac{R}{\lambda} \right)^2} + 2 \left(1 + \frac{\bar{R}^2 \bar{A}'^2}{\bar{A}^2} \right) + \frac{5}{8} \frac{\bar{A}^2}{2} \right] + \dots \quad (14)$$

where $\bar{A}' = (\partial\bar{A}/\partial r)$. This is in agreement with the results obtained by L. C. Tang⁽²⁾. Since v_z^2 must be greater than zero for focusing, the minimum value of A can be determined from Eq. (14) for all values of R/λ and N . In our machine $N = 4$ and the limiting value of \bar{A} is found to be approximately $1.25 R/\lambda$ for all R/λ . To ascertain sufficient focusing, A was chosen $1.5 R/\lambda$ so that $A \approx 1.5$ and $a \approx 0.35$ and, finally, the required field in the median plane becomes

$$B = B_0 \left[1 + 0.35 \left(\frac{r}{\lambda}\right)^2 + 1.5 \left(\frac{r}{\lambda}\right) \sum_k (-1)^k \alpha_k \cos(2k+1)N\varphi \right] \quad (15)$$

Poleface design

For a field in the median plane given by

$$B = B_0 \left[1 + a \left(\frac{r}{\lambda}\right)^2 + \dots + \left\{ A \left(\frac{r}{\lambda}\right) + B \left(\frac{r}{\lambda}\right)^3 + \dots \right\} \sum_k (-1)^k \alpha_k \cos(2k+1)N\varphi \right], \quad (16)$$

the profile of the poles was calculated⁽³⁾. Let $V = V_1 + V_2$ be the scalar potential of the field which satisfies Laplace's equation. V_1 is determined so as to satisfy

$$\frac{\partial^2 V_1}{\partial r^2} + \frac{1}{r} \frac{\partial V_1}{\partial r} + \frac{\partial^2 V_1}{\partial z^2} = 0 \quad (17)$$

Using the Hankel integral transformation one gets

$$V_1(r_1, z) = -B_0 z \left[1 + a \left(\frac{r}{\lambda}\right)^2 + b \left(\frac{r}{\lambda}\right)^4 + \dots - \frac{1}{3!} \left\{ 2^2 a \left(\frac{r}{\lambda}\right)^2 + 4^2 b \left(\frac{r}{\lambda}\right)^4 + \dots \right\} \left(\frac{z}{r}\right)^2 + \frac{1}{5!} \left\{ 2^2 4^2 b \left(\frac{r}{\lambda}\right)^4 + \dots \right\} \left(\frac{z}{r}\right)^4 + \dots \right] \quad (18)$$

Putting $V_2(r, \varphi, z) = U(r, z) F(\varphi)$ and substituting this into Laplace's equation this equation can be divided into two parts for U and F , respectively. For F one readily obtains

$$F = A_1 \cos(2k+1)N\varphi$$

and for U we get the equation

$$\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial U}{\partial r} \right) + \frac{\partial^2 U}{\partial z^2} - (2k+1)^2 N^2 \frac{U}{r^2} = 0 \quad (19)$$

Now we write

$$U = U_1 + U_3 + U_5 + \dots = A \left(\frac{r}{\lambda}\right) G_1\left(\frac{z}{r}\right) + B \left(\frac{r}{\lambda}\right)^3 G_3\left(\frac{z}{r}\right) + C \left(\frac{r}{\lambda}\right)^5 G_5\left(\frac{z}{r}\right) + \dots$$

(2) L. C. Tang, R.S.I. 27 (1956) 1051-1058

(3) Khoe Kong Tat, Thesis, Delft.

Substituting this into Eq. (19) a tedious calculation yields.

$$V_2(r, \varphi, z) = -B_0 d \sum_k \left[K \frac{r}{\lambda} \left(\frac{r^2 + z^2}{r^2 + z_m^2} \right)^{1/2} \frac{1 + \frac{1}{2} p_k^{-2} \left\{ 1 + \left(\frac{z}{r} \right)^2 \right\}^{-1} \sinh S \left(\frac{z}{r}, p_k \right)}{1 + \frac{1}{2} p_k^{-2} \left\{ 1 + \left(\frac{z_m}{r} \right)^2 \right\}^{-1} \sinh S \left(\frac{z_m}{r}, p_k \right)} \right. \\ \left. + L \left(\frac{r}{\lambda} \right)^3 \frac{\sinh \lambda_{k3} \frac{z}{r}}{\sinh \lambda_{k3} \frac{z_m}{r}} + M \left(\frac{r}{\lambda} \right)^5 \frac{\sinh \lambda_{k5} \frac{z}{r}}{\sinh \lambda_{k5} \frac{z_m}{r}} + \dots \right] (-1)^k \frac{\cos p_k \varphi}{2k+1}; \quad (20)$$

$$p_k = (2k+1)N; \quad S = p_k \left\{ \ln \left(\frac{z}{r} + \left(1 + \left(\frac{z}{r} \right)^2 \right)^{1/2} \right) - \frac{\frac{z}{r}}{p_k^2 \left(1 + \frac{z}{r} \right)^2} \right\};$$

$$\lambda_{ki}^2 = (2k+1)^2 N^2 - i^2; \quad i = 3, 5, \dots; \quad K, L, M, \dots \text{ are constants.}$$

Here $z_m = z_m(r)$ is the pole profile in radial direction in a region where the flux density is minimum, i.e., z_m is a boundary condition which has yet to be fixed. V_2 as given by Eq. (20) satisfies Laplace's equation only for constant values of (z/r) , i.e., in small ringshaped regions where (z_m/r) can be considered approximately constant. From $B_2 = -(\partial V_2 / \partial z)$ one obtains

$$B_2(r, \varphi, 0) = B_0 \sum_k \left[K \frac{r}{\lambda} \alpha_k + L \left(\frac{r}{\lambda} \right)^3 \alpha_{k3} + M \left(\frac{r}{\lambda} \right)^5 \alpha_{k5} + \dots \right] (-1)^k \cos p_k \varphi. \quad (21)$$

The α 's are functions of (z_m/r) and N and are approximately given by

$$\alpha_{ki} = \alpha_{oi} \frac{\sinh N \frac{z_m}{r}}{\sinh (2k+1) N \frac{z_m}{r}} = \alpha_{oi} \alpha_k. \quad (22)$$

Furthermore, the $\alpha_{o3}, \alpha_{o5}, \dots$ which are of interest for large values of (r/λ) only, are practically equal to one for large (r/λ) . Therefore, we get

$$B_2(r, \varphi, 0) = B_0 \sum_k \left[K \frac{r}{\lambda} \alpha_{o1} + L \left(\frac{r}{\lambda} \right)^3 + M \left(\frac{r}{\lambda} \right)^5 + \dots \right] (-1)^k \alpha_k \cos (2k+1) N \varphi. \quad (23)$$

Hence L, M, \dots must be chosen equal to B, C, \dots and $K \alpha_{o1} = A$. However, α_{o1} depends on (z_m/r) for small values of (r/λ) and, therefore, K will depend on (z_m/r) and V_2 will no longer satisfy Laplace's equation exactly. Since $K = (A/\alpha_{o1})$ becomes infinite for $r = 0$, K is chosen constant in the region $(r/z_m < 2)$. For large values of r , K is approximately equal to A .

From $V = V_1 + V_2$ the profile of the poles is obtained putting $z = z_m$ and putting $z = d$ for $r = 0$. This yields an equation in z_m for given values of (z_m/r) which can be solved by approximation methods. Thus z_m is obtained as a function of r .

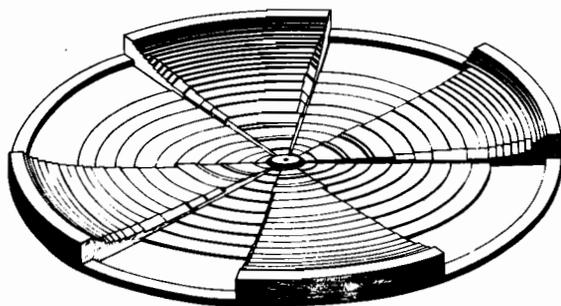


Fig. 21. Shims on pole tips of the Delft cyclotron

The poles of the magnet were made flat and shims, as shown in Figure 21, were screwed onto them. The shims were made out of two steel discs, one for the "valleys" and one for the "hills". They were machined on a lathe, given the calculated shape with an accuracy of 0.1 mm, then both were cut into eight sectors, four of which were screwed onto the upper pole and four onto the lower pole. The edges of the "hill" sectors were partially cut away at an angle of 45° .

The flux density was measured with a small coil connected to a recording millivoltmeter. The coil moved at constant speed in radial or azimuthal direction. The relative flux density was determined with an accuracy of about $5 \times 10^{-4} \text{ Wb/m}^2$. At the larger radii the shims had to be modified a little because of saturation effects. For smaller radii they generate almost exactly the required field in the median plane as given by Eq. (15). The flux density in the "valleys" decreases, on the "hills" it increases with $(1/B) (\partial B / \partial r)$ equal to about 0.6 m^{-1} . In the center the field is almost uniform in a region with a radius of 7.5 cm. The coefficients α_1 and α_2 in Eq. (15) increase with radius, as shown in Figure 22, and a small second harmonic is present. Therefore, at larger radii the azimuthal field variation is not sinusoidal but the tops of the sinewave are a little flat. At 1.5 cm above and below the median plane the field has almost a square wave form. Due to the presence of a second harmonic the flux density at the boundaries of hills and valleys is not quite zero. The determination of $\langle B \rangle (R)$ from the minimum values of B in the valleys and the maximum values of B in the hills was, therefore, impossible. A harmonic analysis of the azimuthal field variation was made to determine $\langle B \rangle$. The result is given in Figure 23. In Table 1 a few data of the magnet are given.

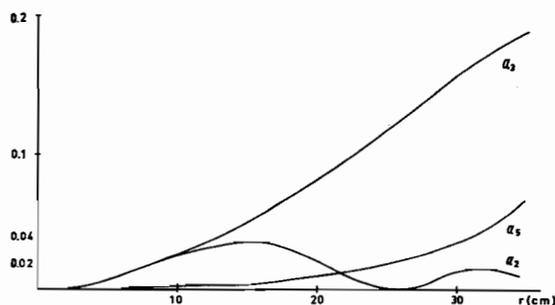


Fig. 22. Fourier coefficients of the field harmonics

Table 1. Magnet Data.

Weight of steel	25 ton
Weight of coils (aluminum)	1.6 ton
Cooling	water
Ampere turns	180,000
Core diameter	85 cm
Pole tip diameter	85 cm
Gap height in center	12 cm
Gap height at edge of poles	7 - 14 cm
Field in center	appr. 1.4 Wb/m^2
Field at edge	$1.7 - 1.75 \text{ Wb/m}^2$
Field flutter amplitude at edge	appr. 0.33 Wb/m^2
Maximum beam radius	36 cm
v_x at $r=35 \text{ cm}$	1.015
v_z at $r=35 \text{ cm}$	0.087

Performance

With a region (radius 7.5 cm) of uniform flux density in the center of the machine only a very small proportion of the particles reached the outer orbits. At a slightly smaller radius a high current was observed, but the energy of the particles was small.

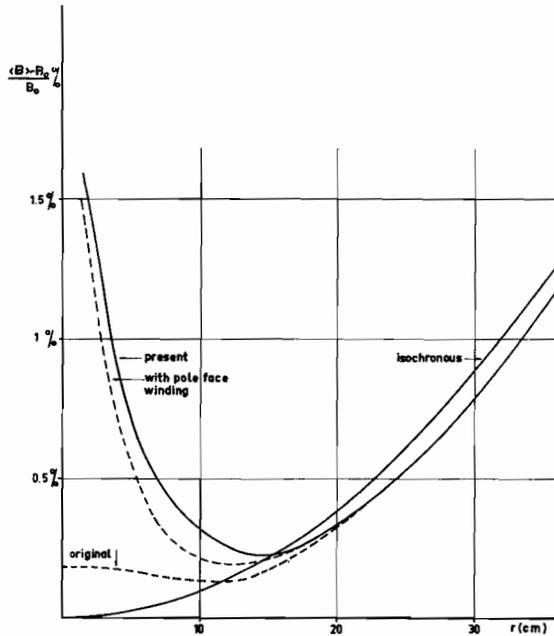


Fig. 23. Variation of average field with radius

This energy as estimated from the heat and radiation produced was much less than one would expect at this radius. Therefore, the location of the orbit was roughly determined with an auxiliary probe and it was found that the greater part of the particles moved on excentric orbits, the excentricity being 10 - 12 cm in the direction of the accelerating gap. Only a small proportion of the particles moved on the correct centric orbits and were accelerated to 12 Mev. Reversing the direction of the field shifted the direction of the excentricity 180°. The radio frequency for maximum current was slightly different (40 k Hz) for the centric and the excentric beam.

Various possible explanations of this excentricity were tested experimentally, but none was found entirely satisfactory. A slight excentricity of a few cm could of course be expected since the ion source was situated exactly in the center of the machine.

Since $v_x = 1$ in a homogeneous field, resonance might be responsible for the excentricity, although the subharmonics in the azimuthal field variation were smaller than 0.1%. Therefore, steel disks of various sizes were fixed on the poles in the center of the magnet. The decrease of the field, thus produced, made v_x smaller than one and v_z larger than zero and caused strong axial magnetic focusing near the center. It was observed that the number of ions in the excentric beam dropped until, finally, all particles moved on centric orbits when the field in the center was made to decrease more steeply by adding more steel.

Of course this enhanced the phase shift but, as already pointed out, the number of revolutions in the central region is so small that even a considerable difference $\Delta\langle\omega\rangle$ between the average angular velocity $\langle\omega\rangle$ of the ions and the r-f circular frequency ω_0 causes only a small phase shift. The total phase shift becomes even smaller since ω_0 is so adjusted as to give maximum current output. The result of this adjustment can be seen from Figure 24 where $(\langle B \rangle - B_0)/B_0$ is plotted as a function of r^2 , curve (1). For an ideally isochronous machine of the size of the present machine this function is practically a straight line through the origin given by

$$\frac{\langle B \rangle - B_0}{B_0} = a \left(\frac{\langle \omega \rangle}{c} \right)^2 r^2 \quad . \quad (24)$$

According to Eq. (7) it is required that $\omega_0 = \langle \omega \rangle$. If ω_0 is made slightly larger, the isochronous curve corresponding to the new circular frequency $\omega_0 + \Delta\omega_0$ is given by

$$\frac{\langle B \rangle}{B_0 + \Delta B_0} - 1 = a \left(\frac{\langle \omega \rangle + \Delta\langle \omega \rangle}{c} \right)^2 r^2 \quad .$$

Since variations of ω_0 of the order of 0.1% are considered here this can be written

$$\frac{\langle B \rangle - B_0}{B_0} \approx \frac{\Delta \langle \omega \rangle}{\langle \omega \rangle} + a \left(\frac{\langle \omega \rangle}{c} \right)^2 r^2, \quad (25)$$

i.e., the new isochronous curve, (2) in figure 24, is practically parallel to the original one, but it is displaced over a distance

$$\frac{\Delta \langle \omega \rangle}{\langle \omega \rangle} = \frac{\Delta B_0}{B_0}.$$

Therefore, the vertical distance between the two curves (1) and (2) gives $(\Delta \langle \omega \rangle / \langle \omega \rangle)$ for all values of r^2 . The actual relation between $(\langle B \rangle - B_0 / B_0)$ and r^2 is given by curve (3) which is partially parallel to (1) and again the vertical distance between (3) and (2) gives $(\Delta \langle \omega \rangle / \langle \omega \rangle)$. Now $\Delta \langle \omega \rangle = d\phi/dt$ or $d\phi/dn = (2\pi / \langle \omega \rangle) \Delta \langle \omega \rangle$ where ϕ is the phase angle of the particles with respect to the rf and n is the number of turns. The kinetic energy of the particles is $T = (1/2)m(\langle \omega \rangle r)^2$ and $T = 2eV \cos \phi$ is the increase of T per revolution (V is the dee voltage). Therefore,

$$\frac{2\pi}{\langle \omega \rangle} \Delta \langle \omega \rangle = \frac{d\phi}{dT} \frac{dT}{dn} = \frac{d\phi}{dT} 2eV \cos \phi, \text{ or}$$

$$(\sin \phi)_r - (\sin \phi)_0 = \frac{\pi m \langle \omega \rangle^2}{2eV} \int_0^r \frac{\Delta \langle \omega \rangle}{\langle \omega \rangle} dr^2. \quad (26)$$

In order that the phase shift be zero the integral must be zero, i.e., the two shaded surfaces in Figure 24 must be equal. However, it is also required that the phase shift is kept as small as possible and in particular that the maximum phase shift is smaller than $\pi/2$. The maximum phase shift is reached at the intersection P of curves (3) and (2) and

$$(\sin \phi)_{\max} = (\sin \phi)_0 + \frac{\pi T_p}{eV r_p^2} A \quad (27)$$

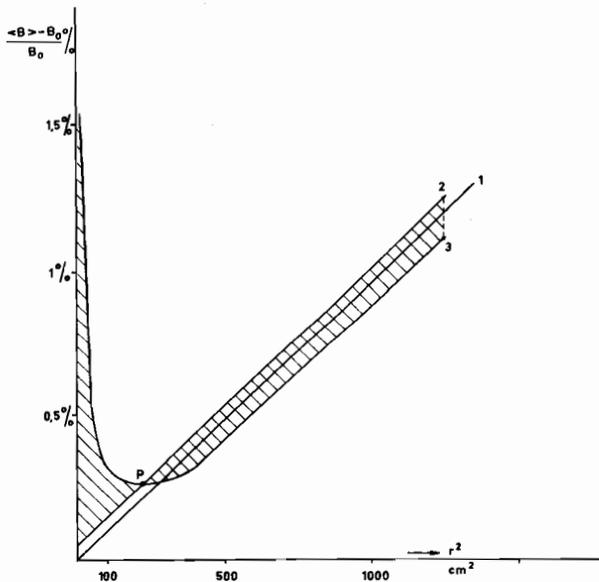


Fig. 24. Variation of average field with r^2 .

where A is the surface of the shaded areas to the right or the left of P . This sets a limit to the field shape near the origin for given dee voltage, or it sets a lower limit to the dee voltage for given field shape.

A circular steel disk with a diameter of 6 cm and a height of 0.35 cm from the center to a radius of 1.25 cm, then decreasing linearly to a height of 0.1 cm at a radius of 3 cm, made the excentric beam disappear entirely, whereas the current of the centric beam increased considerably. It was observed, however, that a large part of the beam was lost in the neighbourhood of $r = 15$ cm. Experiments with probes with

graphite fingers and with a dee covered by a grid showed that the axial focusing was insufficient at this point. In order to decrease defocusing a coil was fixed to the upper and lower pole (inner diameter 14 cm, outer diameter 17 cm). With 80 amp-turns for the lower and 60 amp-turns for the upper coil a maximum in the beam current was obtained.

By varying the current in the main coils slightly it is possible to make the average flux density at large 4-values deviate slightly from the value required for isochronism. This is due to saturation effects. As soon as this is done, the beam disappears entirely. This is in accordance with expectation, since in this region the number of revolutions is so large that a small value of $\Delta\langle\omega\rangle/\langle\omega\rangle$ already gives a large phase shift.

The ion source is of the hooded-arc type with a slot, 2 cm high, and a puller at a distance of 0.5 cm. The source is located at the geometric center of the machine. No efforts have so far been made to find the optimum location or to improve the performance of the source, but the orbits of the ions leaving the source were calculated from electrolytic tank measurements of the electric field in the neighbourhood of the source. These calculations show that ions leaving the source with phase angles between $-\pi/6$ and $+\pi/3$ can be accelerated.

Only one dee and a grounded dummy dee are used at a distance of 4.5 cm from each other. The voltage between dee and dummy dee (ground) is about 21 kv for normal operation. This is much less than required for a conventional proton cyclotron of corresponding size. The radio frequency is roughly 21 MHz. Very little r-f power is needed and a grounded-grid self-excited oscillator is used with one water-cooled tube. Since the dee voltage is low the dee stem can pass through the vacuum tank wall through a quartz insulator. Thus, the r-f line, coupling loops, trimmer condensers, etc., are placed outside the tank at atmospheric pressure.

Table 2. Operating Conditions.

Source	Hooded arc
Filament current	75 amp
Arc voltage	90 volts
Arc current	0.18 amp
Dee voltage	21 kv
Frequency	21.05 MHz
Max energy of protons	12 Mev
Beam current at max energy	0.3 ma
Number of revolutions	>300

Under the circumstances described a beam current of 0.3 ma is obtained on a graphite target at 11 Mev. At 12 Mev the current is essentially the same but a small number of particles strikes the edge of the dee and produce an amount of radiation far above tolerance. This is due to an excentricity of the beam of about 1 cm caused by the location of the source in the center of the machine. From $r = 16$ cm to $r = 34$ cm the attenuation of the beam current is 50%, but this has little significance since it depends largely on the initial beam height. At intervals much higher currents are obtained and it is expected that ion source adjustments and improvements will render it possible to obtain very high currents. The height of the beam at maximum radius is about 1 cm. Increasing the dee voltage from 20 to 28 kv the current increases 50%. Of course this is partly due to improved ion extraction from the source. Below 20 kv

the current drops rapidly, but the voltage could not be lowered enough to observe a threshold. The frequency adjustment is rather critical. The frequency must be kept within a band of a width of 5 kHz. The magnetic field and the frequency stabilizers keep B and ω_0 constant within 2×10^{-4} , but this is not quite good enough. Work on beam extraction is in progress. An electrostatic deflector and a magnetic channel are being used. Since only one dee is used half the vacuum tank is available for the extraction apparatus which is, therefore, very easy to construct.

Discussion

CHAIRMAN JUDD: These remarks by Professor Heyn, practical achievements of the things we are gathered here to learn about, are pure gold! I am sure we will learn a great deal from him during the meeting about his experiences pertaining to all of the topics to be discussed during the meeting.

LIVINGOOD: Have you tried to get the beam out at all?

HEYN: We have not yet tried, but everything is ready. The electrostatic deflector has its entrance at the top of a "hill." There is a difference of 1.2 cm between the equilibrium orbit radius in a strong and a weak field sector which will make extraction easier. The deflector is followed by a magnetic channel.

RICHARDSON: You mentioned that in going from a dee voltage of 21 kv to 28 kv the beam current doubled; what is the threshold?

HEYN: The current drops rapidly below 20 kv, but we could not go low enough with our oscillator to observe a threshold.

SNELL: I would like to ask about the size of the beam spot, and what is the power input to the oscillator?

HEYN: The size of the beam spot at 11 to 12 Mev has not been measured exactly, but it is about 1 cm. The input of the oscillator is about 12 kw; of this, the beam takes 3.5 kw.

YAVIN: What happens if you move the ion source?

HEYN: Since you move the ion source with respect to the extractor, you don't know what happens, or what causes any difference that may appear. We tried a different type of source which we were able to move it in two directions, but without significant results.

SCHMIDT: Is the ion source in the center?

HEYN: Yes, exact center.

KING: What is the amount of eccentricity?

HEYN: The center of the eccentric beam is 10 cm from the center of the machine.

SCHMIDT: We have to place the ion source one inch off center in our cyclotron.

COHEN: The ion source being off center in the standard cyclotron is a well understood effect; the distance is proportional to the voltage. Dr. Heyn's voltage being probably a factor of 4 lower would correspond to only 1/4 in. which would not come near to explain the 4 inches.

HEYN: Yes, we calculate the orbit in the center, of course, and that would explain only 1 centimeter.

CHAIRMAN JUDD: Dr. Symon has a brief comment, before we proceed with a discussion of the analytical work.

SYMON: I have some remarks about the smooth approximation which will perhaps serve as a transition from the first four talks to those to follow on analytical methods. In the first place, since the smooth approximation has fallen into some disrepute, this is perhaps a good time to point out that it originally was due to Sigurgierssen.⁽¹⁾

In the second place, I too would like to express my surprise that this kind of approximation is good enough to serve as a basis for design. It was originally intended as a rough way of making back-of-the-envelope calculations as to what the general properties of the different suggested kinds of accelerators might be. Our experience in comparing it with more accurate calculations is that if the numbers of both the radial and axial oscillations per revolution are less than one-quarter the number of sectors, then the approximation gives both the axial and radial oscillation frequencies within about 10% in the sense that there are two terms, an average focusing term and a flutter focusing term, which are either added or subtracted, and each of those terms is accurate within about 10%.

In the case of the cyclotron, unfortunately, the first condition is not met with reference to radial oscillation frequency unless the number of sectors is large, so one would not expect much accuracy anyway. Moreover, the axial frequency is given by a small difference between two large terms; the approximation would normally be accurate within something like 10% of either one of those terms. This is the reason one would not expect a very good approximation. I would think this is true of all approximations of this kind, except for some which may be discussed next in which one takes explicitly into account the resonance with the sector structure which is approached at higher oscillation frequencies.

⁽¹⁾T. Sigurgierssen, unpublished CERN reports, CERN-T/TS-1, December, 1952; CERN-T/TS-3, May, 1953.