A Simplified Approach

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I know there are some people here with only modest resources who are considering building a spiral clover-leaf cyclotron in the low intermediate energy region (up to 100-Mev protons). It is my purpose in this talk to point out that in my view a simple-minded approach to the design of a cyclotron in this energy region is quite adequate and that people should not be scared out by the complexity and sophistication of the many beautiful analyses that will be presented this morning. I believe these precise calculations are desirable, and I honor the people who do them; I only wish to point out that it is possible for groups with smaller resources to get along with less.

The first figure will more or less define the symbols. I use here simply a step-function concept for the magnetic field. There are a number of conditions one can put on a magnetic field of this type. One has a sector magnetic field with, say, a constant magnetic field on the hill and a constant magnetic field in the valley.

Figure 1 shows the hill field, B_2, and the valley field, B_1. Note the centers of curvature in the valley and on the hill. The angle of turning in the valley is ξ and the angle of turning in the hill is η. Then for the closed orbit indicated by the solid line (the circle is indicated by the dotted line) we would have the condition $\xi + \eta = 2\pi/N$ where N is the number of sectors, and for the angles with respect to the center of the cyclotron we would have $\xi_o + \eta_o = 2\pi/N$. Now for the four-sector case, $\cot \eta/2 = 1 + (B_1/B_2)(\cot \eta_o/2 - 1)$, and for N = 3, N = 6, and so on, there are similar simple relations. The Thomas angle, δ, which is the angle which the orbit makes with the circle, is just $(\eta - \eta_o)/2$. The axial focusing in which one is interested is obtained from the Thomas angle, the angle through the prism. That gives the first term in the usual focusing relation.

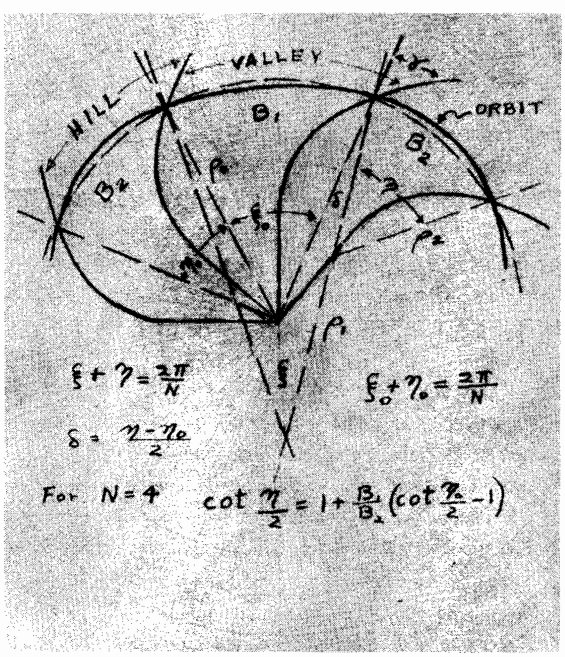

Fig. 1. The solid curve represents part of the ion orbit compared with a circle.

A spiral gamma, here makes two contributions to the focusing. One contribution just depends on the fact that the particle spends a longer time in going through the focusing part of the wedge, than in going through the defocusing part. Then there is also an alternating gradient term.

The isochronous condition, which Judd mentioned, is given by the following simple relations

$$\frac{\eta}{B_2} + \frac{\xi}{B_1} = \frac{2\pi/N}{\bar{B}} \text{ and } \bar{B} = \frac{B_\infty}{\sqrt{1 - \beta^2}},$$

where $\bar{B}$ is the time-averaged magnetic field the particle experiences, which must vary in a relativistic way with respect to the central magnetic field. B_∞ is the central constant magnetic field. The vertical or axial focusing,

ν_z, which I have used is given by

$$\nu_z^{\,2} \simeq - \frac{\beta^2}{1-\beta^2} + F\,[A+(A+D)\ \tan^2\gamma]\ ,$$

$$A = \frac{B_2 - B_1}{B}\left(\frac{\delta}{\pi/N}\right),\ D = \frac{(B_2 - B_1)^2}{B_1 B_2}\left(1 - \frac{\eta}{2\pi/N}\right)\left(\frac{\eta}{2\pi/N}\right)\ .$$

The first term is the usual relativistic defocusing term. The additional terms are usually focusing. The first one is the Thomas focusing term which depends on the magnetic field step function and the Thomas focusing angle. This same term enters into the spiral part of the focusing, as mentioned previously. Finally, there is the alternating gradient term, the magnitude of which depends on the fraction of the orbit in the hill, ($\eta/2\pi/N$,) and the fraction of the orbit in the valley $1 - (\eta/2\pi/N)$. The focusing is maximized when they are approximately equal.

But η is not the angle of the hill with respect to the center of the cyclotron. It is the angle of turning in the hill; actually, one gets more focusing with a sector angle which is smaller for the hill than for the valley.

The parameter F in the above expression is what we call the effective flutter factor. This is unity for true step-function magnetic fields. In practical terms, of course, one does not get a step function but one gets an azimuthal field variation which approaches a step function only at large radii while near the center it is sinusoidal in character. If we consider a plot of the magnetic field against azimuthal angle around the center of the cyclotron, we define a quantity b_1 which is the azimuthal displacement from the angle where the field is equal to the mean field to that where it is 70% up to the hill field B_2 and a quantity b_2 which is the azimuthal displacement from the angle of the mean field to that where the field is 70% down to the valley field B_1. Then, approximately, we have

$$F \simeq 1 - \frac{b_1 + b_2}{\pi/N}\ .$$

In the previous expression for $\nu_z^{\,2}$ the first term is defocusing due to the relativistic expression of the isochronous condition and the second term, with the factor F in front of it, is the azimuthally focusing term. In general these two terms are balanced to give a net positive focusing. If a cyclotron has a design energy of, say, 50 Mev, the first term will be of the order of 0.1, and you want something of the order of $\nu_z^{\,2} = 0.02$ so that there will be some 5 or 10 turns in performing a complete axial oscillation. This means balancing two terms, each of which has a magnitude of about 0.1, to give a net effect of 0.02.

It is only when conditions are of this order of magnitude that analyses such as I have indicated here are sufficiently accurate. When one gets to energies of the order of 400 Mev for the final energy then these balanced terms are much larger and the necessary accuracy requires numerical computation. But, you see, in a situation where an accuracy of only about 10% is needed an analysis of this type is of sufficient accuracy.

Just as an illustration of two of the design concepts which we worked with a little, Figure 2 shows a possible design for 55-Mev final energy. The hill field was assumed constant at 22 kilogauss and the valley field at 16 kilogauss; the resonance condition

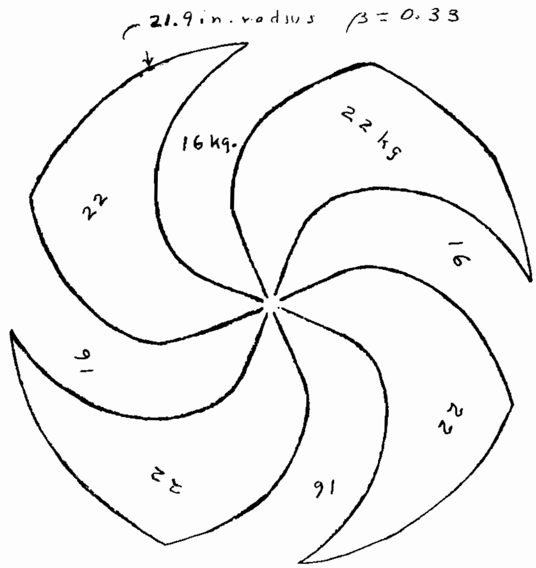

Fig. 2. A magnetic field geometry employing constant hill and valley fields.

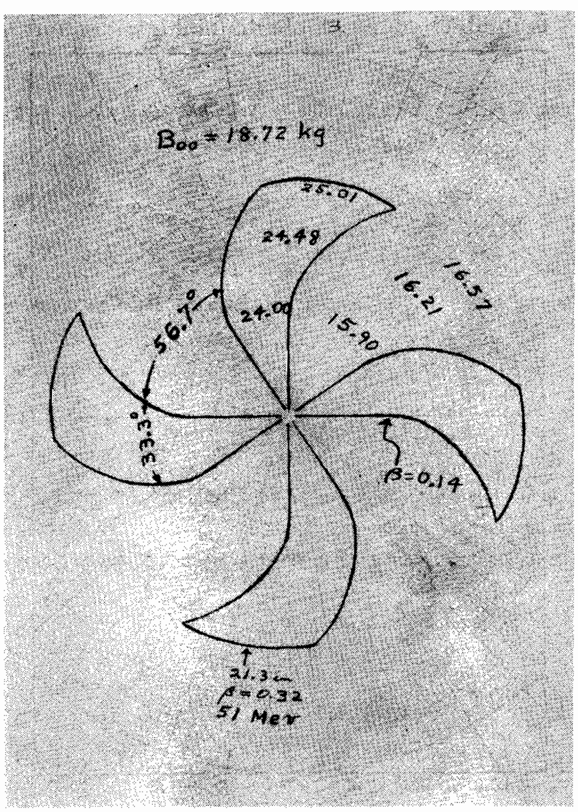

Fig. 3. The actual field configuration being approximated on the UCLA cyclotron.

was satisfied by flaring the hill sectors. There are disadvantages with this sort of design. The angle γ becomes quite large at the large radii. This means that if particles have fairly large radial oscillations the axial focusing will be quite different for different particles. This is disadvantageous, of course, so that in general we would like to keep the angle γ well below 60^o in designs for this energy range.

Figure 3 shows the design we have finally chosen; the central field is 18.7 kilogauss, the hill field goes up to 25 kilogauss, and the angle of the hill is only 33 degrees. In a later session Dr. MacKenzie will discuss putting the dees in the valleys. The two dees are in opposite valleys, so we have a nice system from the r-f point of view. Actually, the contours shown here correspond to an increasing magnetic field and a constant angle for the hill (or valley). What we have ended up with is somewhat of a compromise, but that will be discussed by Dr. Wright in another session.

I should like to point out that this design holds the Thomas field out to approximately half radius. We think this is important because we are satisfied with the performance of the Thomas field at the center and at small radii from our experience with the electron models of the Thomas field. We didn't want to go into another type of design where the spiral would go right into the center.

Next, I would like to discuss briefly the satisfaction of the resonance condition from the center to the outside edge. We propose to take care of that by a number of concentric trimming coils which will tailor the mean field.

In Figure 4 we see that, to the first approximation, the isochronous magnetic field, $\overline{B}$, will be a linear function of the square of the radius. We propose to put in a number of trimming coils which are spaced at equal increments

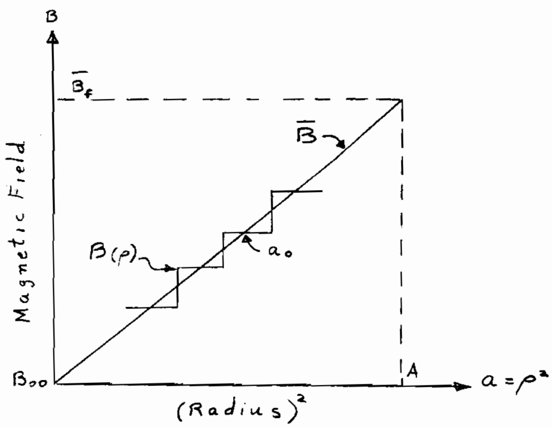

Fig. 4. Approximating the isochronous magnetic field by the use of a number of circular trimming coils.

of r^2 so that we can approximate the magnetic field by a step function. Now this is a very pessimistic way of analyzing the situation since you would, in fact, make some effort to approximate this magnetic field with iron and just do the tailoring by means of the concentric trimming coils. But we can just analyze it as shown and ask ourselves the question as to how many trimming coils, for example, we would need to get a certain threshold dee-voltage for a given final energy.

If we let α be the phase angle between the ion and the dee voltage we have

$$\alpha = 2\pi\bar{f}\,\Sigma\,\frac{f - f_a}{\bar{f}}\,\Delta t,$$

where $\bar{f}$ is the design isochronous frequency, f is the frequency resonant to the actual magnetic field and f_a is the applied frequency. The quantity a is the square of the radius and $a' = a - a_o$; then the rate of change of a' with the number of turns is

$$\frac{da'}{dn} = \frac{2m_oc^2}{\gamma B_{\infty}^2e^2}\,(e\Delta V)\cos\alpha,$$

where $(e\Delta V)$ is the maximum possible energy gain per turn, and γ is the usual relativistic quantity,

$$\gamma = \frac{m_oc^2 + T}{m_oc^2}\ .$$

From the above equation for α we see that under the conditions assumed in Figure 4 we have

$$\frac{d\alpha}{dn} = 2\pi\,\frac{\gamma_f - 1}{\gamma}\,\frac{a'}{A},$$

where γ_f corresponds to the final energy. Let us now assume that the initial phase at the point $a' = 0$ is $\alpha = 0$. This will be accomplished automatically by the tuning-up process with the trimming coils. We now integrate to find the maximum phase angle at the end of the step (α_m);

$$\sin\alpha_m = \frac{\pi}{8\mu^2}\cdot\frac{\gamma_f + 1}{\gamma_f^2}\cdot\frac{T_f}{m_oc^2}\cdot\frac{T_f}{(e\Delta V)},$$

Here μ is the number of trimming coils.

If we let $\alpha_m = \pi/2$ we obtain the threshold energy gain per turn:

$$(e\Delta V)_t = \frac{\pi}{8\mu^2}\cdot\frac{\gamma_f + 1}{\gamma_f^2}\cdot\frac{T_f}{m_oc^2}\cdot T_f\,,$$

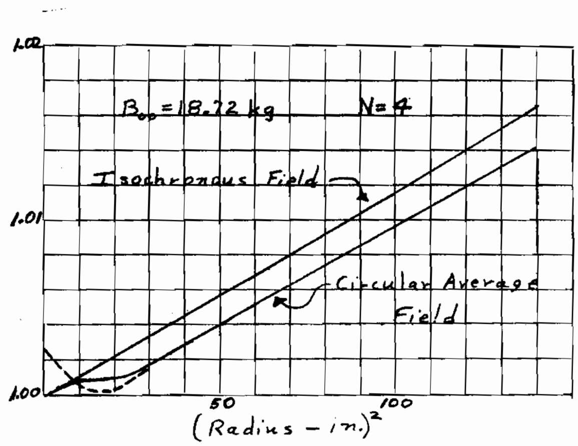

Fig. 5. The isochronous field near the center of the UCLA cyclotron and the required circular average field to produce isochronism.

and we see that it varies inversely as the number of trimming coils, where these coils are spaced evenly as r^2.

For our UCLA design for 50-Mev protons and with 8 trimming coils we expect a threshold energy gain of about 34 kev/turn. This corresponds to a dee-to-ground threshold of 17 kv in our proposed dee geometry.

In practice, we would like to recommend an operating value of roughly twice the threshold dee voltage so that the phase angle of the particle does not differ by more than 30° from that of the dee voltage.

In Figure 5, I will just indicate the situation which we are endeavoring to obtain on our magnet. The isochronous field is shown as a function of r^2. This is approximately a straight line for the field contours we have adopted. The circular average field which would give this desired isochronous field is also shown. The reason for the differences is that the particle spends more time in the hill field than in the valley field; the actual field experienced by the particle is larger than the mean circular field at a given radius.

In actuality we plan to have a magnetic field like that of the dotted curve. Our axial focusing is adequate beyond 4 or 5 inches, but we are concerned with the axial focusing inside that. We plan to have a radially decreasing magnetic field in the central region to be sure of this point. This means that the radial oscillation frequency will go from below to above unity in this region. Our experience with the electron model in the Thomas field indicates that if this is done rapidly enough there is no particular trouble.

This has been a very hit-and-miss discussion, but I want to emphasize that in our view it is possible to make an analysis of a cyclotron of this sort which is rough but adequately accurate for the purpose involved.