GAP-CROSSING RESONANCE IN CYCIAE-100 CYCLOTRON

Hongjuan Yao #,1,3, R. Baartman[2], Y.-N. Rao[2], Tianjue Zhang [1], Yuzheng Lin [3]

[1] China Institute of Atomic Energy, Beijing, 102413, China
[2] TRIUMF, Vancouver, V6T 2A3, Canada
[3] Department of Engineering Physics Tsinghua University, Beijing, 100084 China.

Abstract

In simulations for the central region of the CYCIAE-100 cyclotron, we found that the gap-crossing resonance causes mismatch of particles with different RF phases in the radial phase space: negative phases (early ones entering the Dee gap) suffer less radial stretching and distortion, so they are more stable than the positive phases. But on the other hand, negative phases are electrically defocused. We carried out analytical calculations and compared with the simulation results. We were able to find best centering conditions and optimum Dee angles to minimize the mismatch and consequently maximize the acceptances in the radial, vertical and RF phase spaces.

INTRODUCTION

CYCIAE-100, which has been designed to accelerate H^- ions and extract protons, is a compact cyclotron with four straight sectors. The characteristics of the cyclotron are given in ref [1]. The acceleration will be achieved at four electric gaps per turn up to the maximum energy of 100 MeV with a fourth harmonic two-dee system. The orbit properties of beams in the central region of the CYCIAE-100 cyclotron were studied numerically[2]. During the calculations we found that the beam orbit centre moves by large steps at each gap crossing, and the acceptance calculation results show that positive phases are limited in radial phase space and negative phases are restricted in vertical phase space. These phenomena were interpreted by studying the gap-crossing resonance.

The electric gap-crossing resonance has been known for a long time; the early study was done by Gordon [3] in the three-sector cyclotron with one or two dees. About 20 years later Schulte and Hagedoorn[4] did the detailed analysis of the effect associated with accelerated particle theory in median plane of cyclotrons using Cartesian coordinates. Gordon and Marti [5] refined the theory of ref [4]. In this paper, analytic calculations were carried out using the formulas of Gordon and Marti to explain the simulation results of the CYCIAE-100 cyclotron.

MATCHING WITH DIFFERENT RF PHASES

The central region of the CYCIAE-100 cyclotron has been designed shown in ref [2]. The basic parameters are: injection energy is 40 keV; RF frequency is 44.4 MHz; harmonic number is 4; and dee voltage is 60 kV. The angular width of the dee provided by RF cavity designers is 36°. The central region design used the dee angle of 45°; the reason will be described in the next section. We should point out that the angle between the first gap and the second one is ~37°, this adjusting is used for getting more energy gain for a wide phase range.

At low energy the beam orbit centre moves by large steps at each gap crossing, so it is important to optimize the central region geometry to obtain a well centered beam. Once the central ray is obtained, the transverse acceptances should be optimized.

At a given energy and azimuth, a specific ellipse (called the matched, or eigen-ellipse which is decided by the cyclotron itself) describes the optimum beam shape in (r, Pr) or (z, Pz) phase space. Beams within the eigen-ellipse will have the minimum envelope during the acceleration process. So the beam should be matched to the eigen-ellipse of certain energy after the acceleration in the central region.

The twiss parameters of the eigen-ellipse were obtained from the equilibrium orbit program CYCLOP[7]. Eight equi-distant points around the boundary of an ellipse in radial phase space with the normalized emittance of 0.5 pi-mm-mrad were tracked backwards to the injection point. These calculations were done for negative phase and positive phase with the interval of 20° relative to the reference phase. Fig.1 gives the graph of the phase as a function of energy at the center line of the first dee. Note the phase slipping and also that an initial 40° phase band is bunched by the central region into 23°.

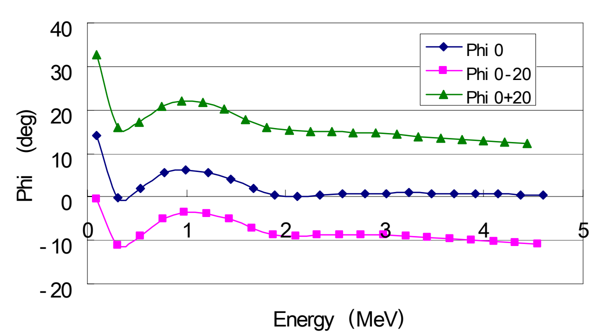


Figure 1: the phase history of particles as a function of energy starting with three initial RF phases, the graph was plotted at the center line of the first dee (θ=0°)

The simulation results are shown in Fig. 2 for three RF phases; the overlapping area of three quasi ellipses is the radial acceptance of the central region. From Fig. 2 we can see that the radial phase space was distorted and

#yhj03@mails.tsinghua.edu.cn

negative phase suffer less radial stretching and distortion. The positive phase limits the radial acceptance.

The vertical acceptance was calculated using the same procedure as for the radial; the results are given in Fig. 3. We found from Fig. 3 that the negative phases become the dominant of the vertical acceptance.

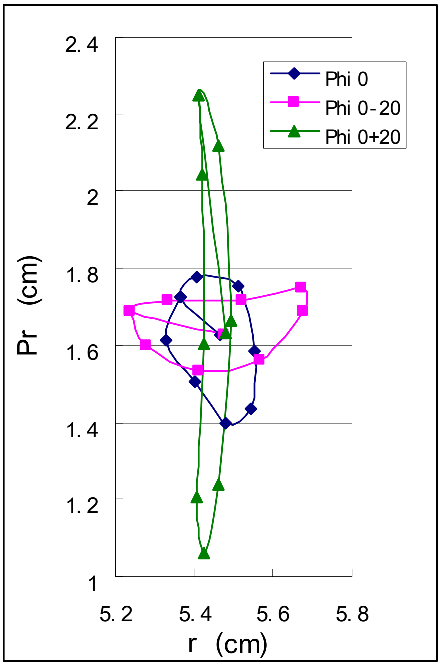


Figure 2: the radial phase results, corresponding to three RF phases, from the eigen-ellipses (~1.89MeV for Phi0) with 0.5 pi-mm-mrad emittance tracking backwards to the injection point (0.04MeV)

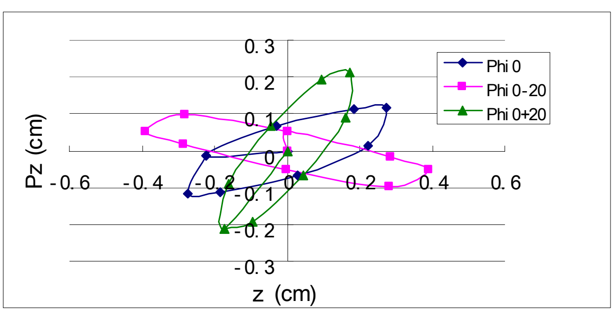


Figure 3: the vertical results, corresponding to three RF phases, from the eigen-ellipses (~1.89MeV for Phi0) with 0.5 pi-mm-mrad emittance tracking backwards to the injection point (0.04MeV)

These observations from the numerical calculations for the central region of the CYCIAE-100 cyclotron can be interpreted by a phase dependent oscillation of radial and vertical tune driven by the gap-crossing perturbation. The analysis is given at the next section.

ANALYTIC CALCULATIONS FOR GAP-CROSSING RESONANCE

Radial Instability

As a result of the gap crossing, the energy increases and the equilibrium orbit shifts outward, this perturbation effect will produce a shift of the radial oscillation frequency υ_r (which is very close to unity in the central region). The detailed analysis is given in ref [5]; the revised radial oscillation frequency arising from the gap-crossing effect is called υ^*_r. Expression (27) in ref [5] can be written as follows.

$$(\upsilon_r^*-1)^2=(\upsilon_r-1)^2-\frac{hqV_0\sin\frac{hD}{2}}{\pi T_c}\sin\phi\cdot(\upsilon_r-1)+\frac{1}{4}\left(\frac{hqV_0\sin\frac{hD}{2}}{\pi T_c}\right)^2\left(\sin^2\phi-\cot^2\frac{hD}{2}\cos^2\phi\right)\sin(\pi-D)\cdot\sin D \tag{1}$$

where h is harmonic number, V_0 is dee voltage; D is the angular width of a dee; T_c is the kinetic energy.

Under certain conditions the value of (υ^*_r -1) may become imaginary and cause an unstable motion in the radial phase plane. This revised radial oscillation frequency depends on the phase and the instability will occur at low energy in the cyclotron. Using the parameters of the CYCIAE-100 cyclotron we did the contour plot shown in Fig. 4 using different values of the dee angle. The energy and phase are x- and y- axis, and 2pi*Imag (υ^*_r-1) are the values of the contour lines. The calculation does not take into account phase slipping, due either to lack of isochronism or phase compression. Nevertheless, from Fig. 4 we can see the following apparent qualitative features:

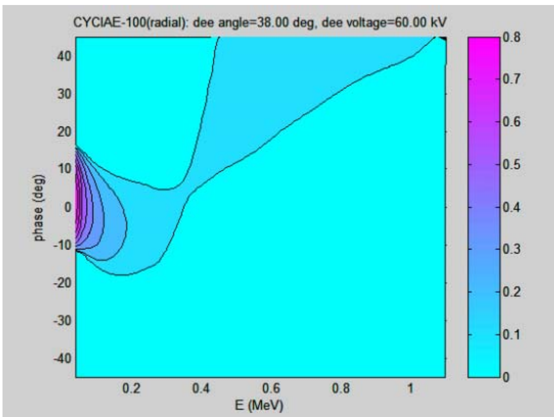
- In radial phase plane, negative phases (early ones entering the dee gap) are more stable than positive phases (late ones entering the dee gap).
- Dee angle of 45° (sin (hD/2) =1) is preferred for central region, but not very strongly; a large range of phases will still have frequency near zero.

Vertical Instability

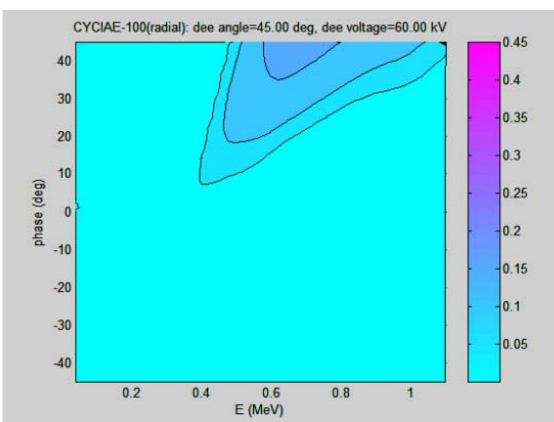
Because of the complementarity of vertical and radial motions, the formula for vertical phase plane is almost the same as for radial. The revised vertical tune υ^*_z can be written as follows.

$$\upsilon_z^{*2}=\upsilon_z^2+\frac{hqV_0\sin\frac{hD}{2}}{\pi T_c}\sin\phi-\left(\frac{hqV_0\sin\frac{hD}{2}}{\pi T_c}\right)^2\left(\sin^2\phi-\cot^2\frac{hD}{2}\cos^2\phi\right)\frac{\pi-D}{2}\cdot\frac{D}{2} \tag{2}$$

The vertical motion will become unstable when υ^*_z is imaginary. The contour plot is shown in Fig. 5. The values of the contour lines are 2pi*Imag (υ^*_z). From Fig. 5 we can see that the scale of the contours is bigger than the radial case, so the vertical instability is much stronger for the wrong phases; and for different dee angle the contour plot is almost the same. Fig. 1 shows that the central region has been carefully optimized to overcome the effects of vertical defocusing. The isochronism is such that the phase of the particles slips toward negative, so that the initial phase band is injected at positive phase and avoids the vertical electrical defocusing. As well, it should be noted that the analytic calculations apply to particles with a constant energy, but in fact the energy changes non-adiabatically on the first turn and this reduces the effects of the vertical defocusing.



(a) the dee angle is 38°



(b) the dee angle is 45°

Figure 4: The contour plot for imaginary part of the radial tune

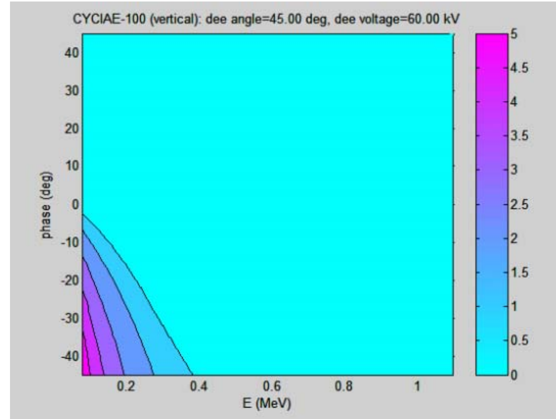


Figure 5: The contour plot for imaginary part of the vertical tune

CONCLUSIONS

From the numerical calculations for the central region of the CYCIAE-100 cyclotron, we observed some features arising from the gap-crossing resonance. The analytic calculations using Gordon and Marti formula were made and contour plots were given for radial and vertical instability. Summarizing the results of numerical calculations and analysis, for two-dee system one can say that negative phases are more stable in radial phase plane and much stronger instability in vertical. The dee angle (denoted by D) in the central region satisfying sin (hD/2) =1 is preferred, which can minimize the mismatch at low energy.

REFERENCES

[1] Tianjue Zhang et al., Proc. 17th Int. Cycl. Conf., Tokyo, 497, (2004).
[2] Hongjuan Yao et al. The study and design of the central region for the CYCIAE-100 cyclotron. this conference, 2007.
[3] M. M. Gordon. Nucl. Instrum. Methods 18-19, 268 (1962).
[4] W. M. Schulte and H. L. Hagedoorn. Nucl. Instrum. Methods 171, 409 (1980).
[5] M. M. Gordon and F. Marti. Particle Accelerators 12, 13 (1982).
[6] B.F. Milton, TRI-DN-99-4, June 1 1999.
[7] M. M. Gordon. Particle Accelerators 16, 39 (1984).