# NUMERICAL STUDY OF THE RESONANCES IN SUPERCONDUCTING CYCLOTRON C400

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*Abstract*

The paper concerns to the beam dynamic simulations in a new superconducting isochronous cyclotron C400 [1] for carbon and proton beam therapy which is under design by the Belgian company IBA together with a Russian group from JINR (Dubna). The cyclotron C400 is to deliver carbon ions $^{12}C^{6+}$ with energy 400 MeV/amu and protons with energy close to 260 MeV.

All resonances up to 4$^{th}$ order that are crossed at acceleration have been studied by a numerical way using the magnetic field maps obtained with TOSCA model [2].

## INTRODUCTION

During a whole range of acceleration the beam crosses the lines of 18 resonances (see Fig. 1). The list of studied resonances consists of 6 internal resonances ($nQ_r \pm kQ_z=4$, n, k=0, 1, 2, 3, 4, n+k≤4) having the main 4$^{th}$ harmonic of magnetic field as a driving term, resonance $Q_r$-$3Q_z$=0 with an average field as the driving term and 11 external resonances ($nQ_r \pm kQ_z=m$, m=1, 2, 3) that could be excited by the magnetic field perturbations. Impact of all resonances on the ion dynamics has been studied by means of ion tracking through the corresponding regions of acceleration.

To simulate the acceleration an analytical presentation of RF electric field was implemented for two dees of spiral form.

The conditions which provide an intersection of the most important resonances $Q_r$=1, $3Q_r$=4 and $Q_r$-$Q_z$=1 without essential worsening in the quality of beam are determined. The rest resonances impose also the acceptable requirements on the starting amplitudes of the ion oscillations and on the magnetic field imperfections.

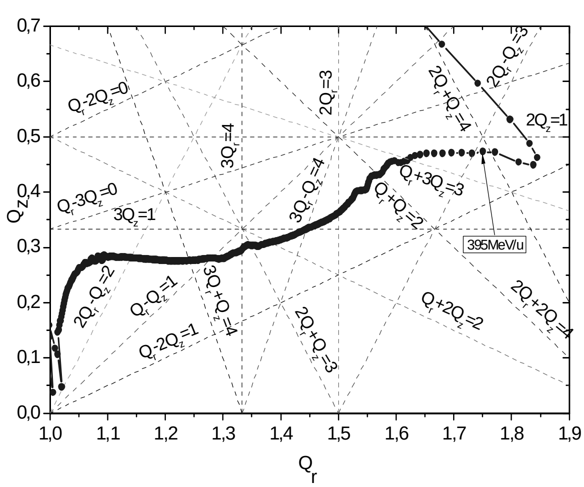


Figure 1: Working point diagram of the cyclotron.

## INTERNAL RESONANCES

Full list of the resonances includes the following internal ones: $4Q_r$=4(2-10), $3Q_r$+$Q_z$=4(131), $3Q_r$=4(154), $3Q_r$-$Q_z$=4(167), $2Q_r$+$2Q_z$=4(177) and $2Q_r$+$Q_z$=4(181). (Radial position of the resonances in centimetres is marked in the brackets). Quadratic or cubic nonlinearities of the main 4$^{th}$ harmonic of the magnetic field could be the driving terms for these resonances. Second derivatives of the 4$^{th}$ harmonic amplitude and of its phase are presented in Fig. 2. These nonlinear terms could be the reason of exciting the resonances of 3$^{rd}$ order.

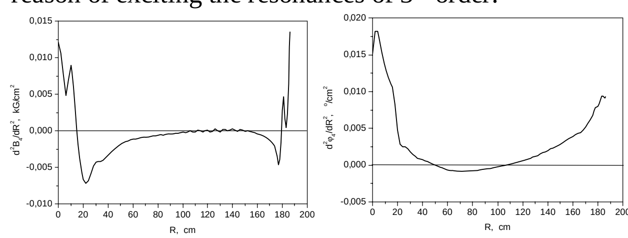


Figure 2: Second derivatives along radius of the 4$^{th}$ harmonic amplitude (left) and its phase (right).

### *Resonance* $4Q_r=4$

The resonance impact on the stability of radial motion was studied in the static and dynamic regimes. The results of first study are shown in Fig. 3. Maximum steady amplitude of the radial oscillations smoothly grows from 0.2 cm to 1.2 cm inside a radial range 2-10 cm.

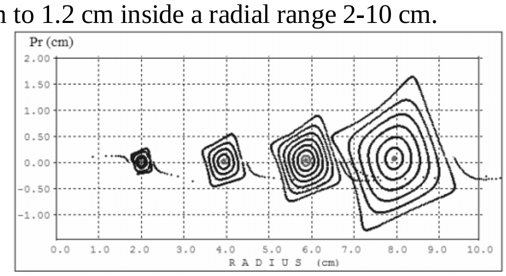


Figure 3: Ion static motion on the radial phase plane

For calculations of the resonance dynamic intersection two bunches of 1000 ions with maximum starting amplitude of radial oscillations 0.25 cm and 0.5 cm at radius 4 cm were specified. Amplitude of RF voltage in the center of cyclotron was taken equal to 40 and 80 kV.

The results of beam acceleration in the radial range 4-12 cm confirmed a fact that none of the ions lost their radial stability. This good stability of beam is provided by the sufficiently large energy gain per turn. We only observed small distortion of the beam radial phase plane if the dee voltage is 40 kV. On the basis of carried out calculations we can say that the resonance $4Q_r$=4 does not present any danger if the dee voltage is 80 kV in the center of cyclotron.

### *Resonance* $3Q_r=4$

Again, the resonance impact on the stability of radial motion was studied in the static and dynamic regimes. One sees in Fig. 4 that the limiting amplitude of the radial oscillations of ions in the static regime is of about 0.3 cm.

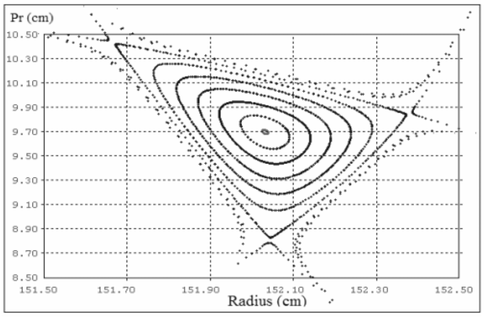


Figure 4: Static motion of 9 ions on radial phase plane. Range of starting radial amplitudes is 0.0 – 0.4 cm with step 0.05 cm.

Then an acceleration of 500 ions with different radial amplitudes was simulated for RF voltage 90 and 180 kV at resonance region. Impact of the resonance on the radial motion was also corrected by means of the average magnetic field perturbation of special form [3] which ensures an increase of the derivative $dQ_r/dr$ at the resonance crossing. Based on the results of these computations the resulting radial amplitude after resonance crossing was plotted (see Fig. 5) as a function of the initial one.

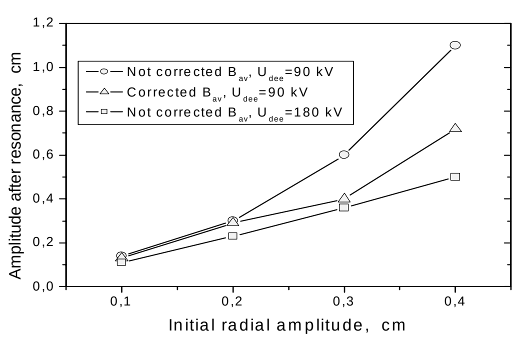


Figure 5: Resulting radial amplitude after the resonance crossing versus initial radial amplitude.

One observes that the special field perturbation noticeably decreases the amplitudes of radial oscillations. However, the increase in the radial amplitudes can be most effectively limited by the dee voltage enlargement.

### *Resonances* $3Q_r$-$Q_z=4$, $2Q_r+2Q_z=4$, $2Q_r+Q_z=4$ *and* $Q_r$-$3Q_z=0$

These internal coupling resonances are situated in final acceleration region 167-181 cm. Three bunches containing 500 ions were initially generated at radius ~142 cm and then accelerated through the resonance region up to an entrance of deflector. These bunches had different value of the initial betatron amplitudes: ($A_r$=0.1, 0.2, 0.4 cm), ($A_z$=0.3 cm). Correction [3] of the resonance $3Q_r=4$ has been applied during these calculations. Figure 6 shows typical view of the radial and axial amplitudes dependence on radius in the considered area.

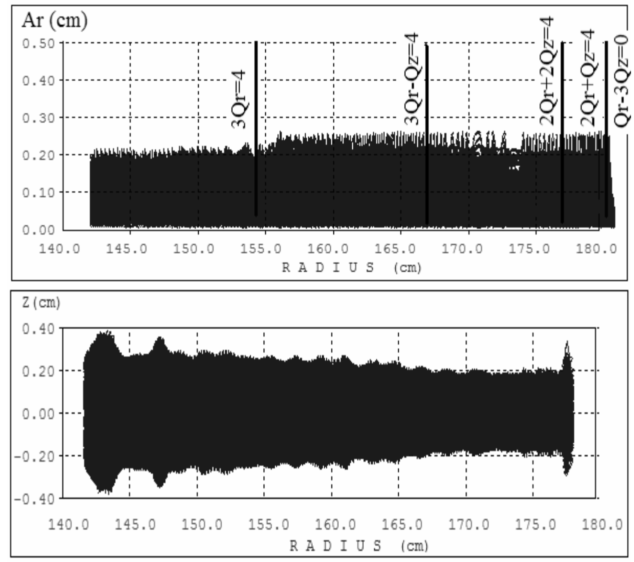


Figure 6: Amplitudes of radial oscillations versus orbit average radius (above), projection of the beam on plane (R-Z) four times in each turn (below).

It is seen that some permissible increase in the axial amplitudes is detected just in front of the deflector. It is difficult to distinguish which resonance $2Q_r+Q_z=4$ or $Q_r$-$3Q_z=0$ is a reason of this since they occur at the same radius 181 cm.

## EXTERNAL RESONANCES

From the 11 external resonances the most important ones were found the following: $Q_r=1$ (2-10), $Q_r$-$Q_z=1$ (145), $Q_r+2Q_z=2$ (162) and $2Q_r=3$ (172).

### *Resonance* $Q_r=1$

The driving term of this resonance is 1st harmonic of the magnetic field imperfections. To study the resonance, the isochronous magnetic field and starting bunch of emittance $\varepsilon_r=150\ \pi$ mm•mrad have been used. It was interested to see how the 1st harmonic, existing only in the very center of cyclotron, influences on the beam parameters at the end of acceleration. Amplitude of the 1st harmonic used in simulations had the parabolic dependence along radial range 0-20 cm. Maximal value of the amplitude was on radius 10 cm and it was varied inside limits 0-20 G. Results of simulations are illustrated in Fig. 7.

Negative action of the 1st harmonic is mainly appeared when the beam crosses the resonance $3Q_r=4$ on radius 154 cm. If the harmonic is absent then this resonance leads to increase in the radial amplitudes from 0.3 cm to 0.4 cm. But even 1st harmonic of amplitude 5 G causes the increase in the resulting amplitudes up to 0.6 cm.

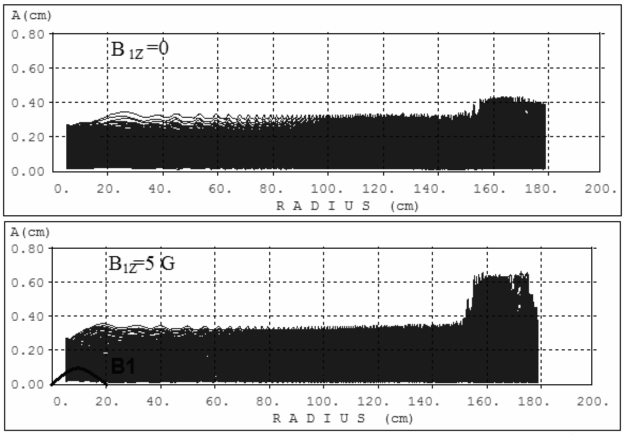

Figure 7: Distribution of radial amplitudes without 1st harmonic (above) and with its amplitude 5 G (below).

## *Resonance $Q_r$-$Q_z$=1*

This resonance driven by the 1st harmonic of the radial magnetic field perturbation ($B_{1r}$), which could be caused, for example, by a main coil tilt. This resonance leads to increase in the axial amplitudes. A number of computations were done varying form of $B_{1r}(r)$. A typical view of the beam axial size near the resonance for imperfection with maximal $B_{1r}$=10 G is shown in Fig. 8.

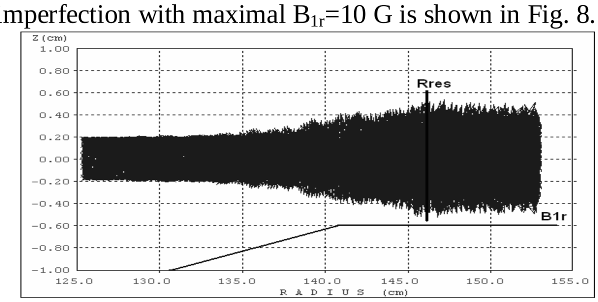

Figure 8: Projection of the beam on the radial-axial plane marked four times in each of 400 turns.

The larger the beam radial amplitudes, the larger the $B_{1r}$ impact on the axial amplitudes. For the proposed radial oscillation 3 mm the increase in the beam axial size by not more than 50% after the resonance crossing is provided if the maximum of $B_{1r}$ is not larger than 5-7 G.

## *Resonance $Q_r$+2$Q_z$=2*

Two types of the 2nd harmonic representing the axial and radial magnetic field imperfections were used in simulations.

First, an influence of the axial harmonic was studied. These computations have clear shown the action of this harmonic with amplitude of some tenths Gausses on the resonance excitation of the axial amplitudes. Figure 9 illustrates this fact for the ion with initial amplitudes $A_r$=4 mm and $A_z$=3 mm when $B_{2z}$=80 G.

Using the computations it was concluded that a tolerance to the amplitude of $B_{2z}$ harmonic on radius 162 cm is of about 20 G. In this case an increase in the amplitude of axial oscillations does not exceed 0.5 mm for the beam with radial amplitudes up to 4 mm.

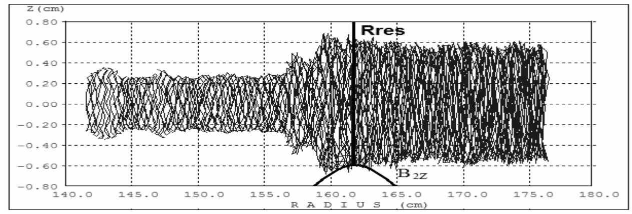

Figure 9: Ion position on plane (R-Z) 4 times on each turn for a perturbation harmonic $B_{2z}$=80 G.

Calculations with radial harmonic $B_{2r}$ have shown that this type of perturbation leads also to increase in the axial oscillations, but its influence is rather weak relatively the axial harmonic impact.

## *Resonance 2$Q_r$=3*

The 3rd harmonic of axial component of the magnetic field was used in simulations as the driving term of resonance.

Figure 10 shows how the amplitudes of radial oscillations vary in final accelerating region if maximum value of 3rd harmonic at resonance radius is 20 G. Bunch containing 500 ions had a range of the initial transverse amplitudes ($A_r$, $A_z$)=(0.4, 0.3) cm.

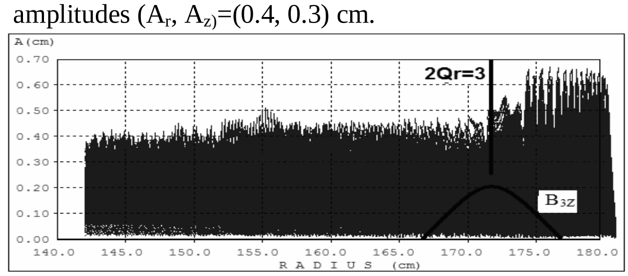

Figure 10: Amplitudes of radial oscillations versus orbit average radius.

Amplitude of the 3rd harmonic must be not greater than 10 G inside the radial range 170-180 cm in order to provide weak influence of this resonance.

## CONCLUSIONS

From 10 resonances briefly described above only 3 of them are looked as really important ones: $Q_r$=1, $Q_r$-$Q_z$=1 and 3$Q_r$=4.

Computations showed that the rest not mentioned here resonances are not dangerous, since they either tolerate the permissible betatron amplitudes as larger than 5-7 mm or tolerance to the harmonic of imperfection at resonance crossing is larger than 10 G.

## REFERENCES

[1] G.Karamysheva et al., “IBA C400 Cyclotron Project for Hadrontherapy”, this conference.

[2] Y.Yongen et al., “Computer Modelling of Magnetic System for C400 Superconducting Cyclotron”, EPAC’06, p. 2589.

[3] Y.Yongen et al., “Simulation of Ions Acceleration and Extraction in Cyclotron C400”, EPAC’06, p. 2113.