HIGH-ENERGY CYCLOTRONS WITHOUT SPIRAL

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Abstract

This paper explores the possibility of reaching high energies in isochronous ring cyclotrons with radial sectors by using negative valley fields to increase the magnetic flutter. A simple model was used to generate field maps for 4-GeV and 13-GeV proton rings, whose orbit properties were then studied using the CYCLOPS equilibrium orbit code. Field maps were also generated for two FFAG designs (both non-scaling, one isochronous and one not) and their orbit properties evaluated with CYCLOPS – the first time that a cyclotron code has been used on FFAGs.

INTRODUCTION

In the past, designs have been presented for isochronous ring cyclotrons to accelerate protons to energies of 10-15 GeV[1,2], as these would allow cw, and therefore very-high-intensity, operation. These would have been three-stage devices, using the 500-MeV TRIUMF cyclotron as injector, followed by a 3.5-GeV booster ring. The fundamental problem in designing a high-energy cyclotron is, of course, how to counter the strong axial defocusing term $-\beta^2\gamma^2$ created by the steeply rising average magnetic field $B_{av} = \gamma B_c$ that is needed to keep the particles (of mass m_0) isochronous at a kinetic energy of $(\gamma - 1)m_0c^2$. For this, these designs relied on edge focusing by spiral magnet sectors to supplement the magnetic flutter $F^2 \equiv \langle(B(\theta)/B_{av} - 1)^2\rangle$:

$$\nu_z^2 \approx -\beta^2\gamma^2 + F^2(1 + 2\tan^2\varepsilon). \quad (1)$$

That axial stability could be achieved in this way was confirmed by tracking orbits through simulated magnetic fields using the equilibrium orbit code CYCLOPS[3]. The feasibility of extracting the beam efficiently by exciting a radial resonance was also confirmed [4] using the general orbit code GOBLIN.

The high spiral angles ε required, however, lead to various practical problems: strong distorting forces on the magnet coils (particularly if these are superconducting), restricted space for rf cavities and injection and extraction equipment, and strong radial kicks during acceleration.

More recently, Rees[5] has proposed an isochronous radial-sector FFAG design (IFFAG) for accelerating muons from 8 to 20 GeV. This employs a novel five-magnet 0doFoDoFod0 lattice cell, termed a "pumplet" from the Welsh word pump (pronounced pimp) for five, where the d magnets (and Fs at low energy) are reverse bending, and the d, F and D magnets each have special field profiles *B(r)*. With long drift spaces between the d magnets, and N = 123 cells, the circumference is 1255 m.

Méot *et al*.[6] have used the ray-tracing code ZGOUBI to follow muons through a simulated field grid and confirm the orbit properties Rees predicts: good isochronism, a radial tune that drops with energy from 0.38N to 0.20N (rather than rising ≈ γ as is usual in isochronous fields), and an axial tune dropping slightly from 0.14N to 0.08N. The physical mechanism by which the lattice achieves axial stability up to γ = 190 is unclear. The reverse fields certainly provide relatively high flutter, but even so, F^2 is only ≈200 at 8 GeV and drops to ≈30 at 20 GeV, far below the ≈6,000 and ≈36,000 respectively, which (1) would require in the absence of spiral.

In view of these intriguing results, and the practical difficulties presented by superconducting spiral magnets, it seemed interesting to explore how far the energies of radial-sector cyclotrons could be raised by inserting reverse-bend magnets to increase the flutter. The next section shows how the parameters for such devices may be determined, and compares the theoretical predictions with the results from a cyclotron ray-tracing code. Finally, we present results for some other FFAG lattices (including IFFAG) obtained with the CYCLOPS code.

RADIAL-SECTOR CYCLOTRONS WITH REVERSE BENDS

As this is an exploratory study, we make the simplest possible assumptions: N radial sectors, hard-edge magnets, no drift spaces, and equal but opposite hill and valley fields:

$$B_h = -B_v = B(r) = \gamma B_0 . \quad (2)$$

Denoting the angular widths of the hills and valleys as $2\pi h/N$ and $2\pi(1 - h)/N$ respectively, and neglecting scalloping effects on the orbit length:

$$B_{av} = 2(h - ½)B . \quad (3)$$

The magnetic flutter is determined entirely by h, and so is the same at all energies :

$$F^2 = ¼(h - ½)^{-2} - 1 . \quad (4)$$

For the axial focusing to remain positive up to some maximum energy γ_m , but no further, (1) tells us that:

$$h - ½ = 1/2\gamma_m. \quad (5)$$

Assuming that the maximum magnetic field available, B_m, is applied at maximum energy γ_m , then the "central field" B_c and "cyclotron radius" R_c are given by:

$$B_c = B_m/\gamma_m^2 \quad (6)$$

$$R_c = (m_0c/e)\gamma_m^2/B_m . \quad (7)$$

From (7) we see that the ring radius required increases as the square of the desired maximum energy, making such designs impractical for very high energies. The recipe for hill and valley field strength is:

$$B(r) = (B_m/\gamma_m)/ \sqrt{1 - (r/R_c)^2}. \quad (8)$$

Note that Symon's circumference factor[7], the ratio of the actual circumference to that obtainable with the same maximum field, but no reverse bends:

$$C = \gamma_m . \quad (9)$$

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We choose the number of sectors $N \approx 3\gamma_m$ so that $\nu_r \approx \gamma$ remains well below the $N/2$ resonance.

As examples we have studied two cases with specifications similar to those of the spiral-sector ring cyclotrons mentioned above: one accelerating protons from 1 to 4 GeV, and the other from 3 to 13 GeV. We assume a maximum magnetic field B_m = 5 T. In the first case, $\gamma_m = 5$ leads to $N = 15$, $h = 0.6$, $F^2 = 24$, $B_c = 0.2$ T, and $R_c = 15.65$ m. CYCLOPS was then run on a simulated magnetic field grid with $B(r)$ calculated from (8). The CYCLOPS results showed that both the flutter and the axial tune (Fig. 1) were lower than predicted, ν_z becoming imaginary above 3.4 GeV, where F^2 had dropped to 10.9. This occurs because the orbit scalloping causes the field seen by a proton to vary, rather than being piecewise constant, as the above equations assume. To remedy this, we have attempted to produce a new field map with the B contours shaped to match the orbit arcs:

$$B(r,\theta) = (B_m/\gamma_m))/ \sqrt{1-(R_{hv}/R_c)^2} \qquad (10)$$

where $R_{hv}(r,\theta)$ is the radius at which the orbit through (r,θ) crosses the hill-valley boundary. A fully satisfactory field map remains to be completed, but some preliminary results for low energies show that the full theoretical flutter can be obtained.

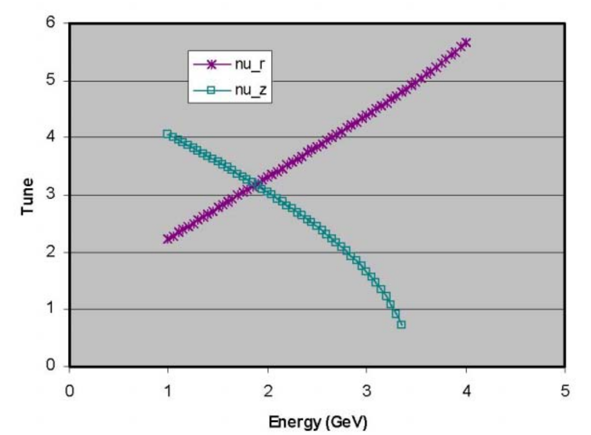

Figure 1: Tunes ν_r, ν_z in a 4 GeV radial-sector cyclotron.

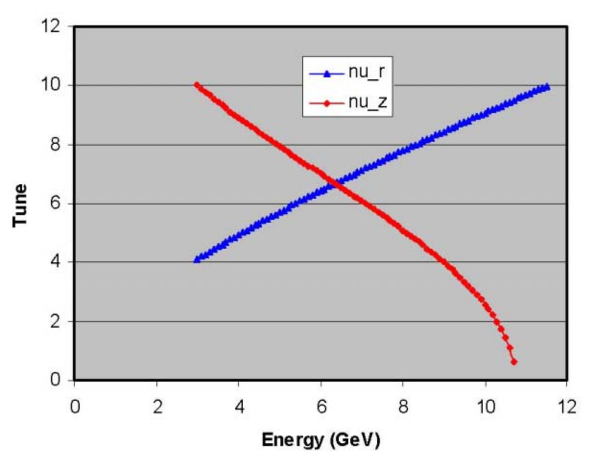

Figure 2: Tunes ν_r ν_z in a 13 GeV radial-sector cyclotron.

In the second case, $\gamma_m = 15$ leads to $N = 45$, $h = 0.533^{\cdot}$, $F^2 = 224$, $B_c = 0.022^{\cdot}$ T, and $R_c = 140.83$ m. The tracking results (Fig. 2) showed similar behaviour to those for the low-energy ring, with ν_z dropping to zero at 10.8 GeV. In both rings the radial tune ν_r increases $\approx \gamma$.

The radii for both rings are of course, considerably larger than those for their spiral-sector counterparts (10 m and 41 m respectively), and in practice would be enlarged further by the inclusion of drift spaces for the rf cavities.

FFAG STUDIES USING CYCLOPS

F0D0-2:

Our first test of CYCLOPS on an FFAG lattice was made with F0D0-2, designed by J.S. Berg[8] for accelerating muons from 10 to 20 GeV. Both the positive-bending D and negative-bending F are sector magnets, in which the field magnitudes decrease outwards with constant gradient. The CYCLOPS results agreed very closely with Berg's for all the parameters examined (orbit radius, beta functions, tunes, and orbit time – the latter two being shown in Figures 3 & 4). Note that this linear non-scaling design does not aim to be perfectly isochronous.

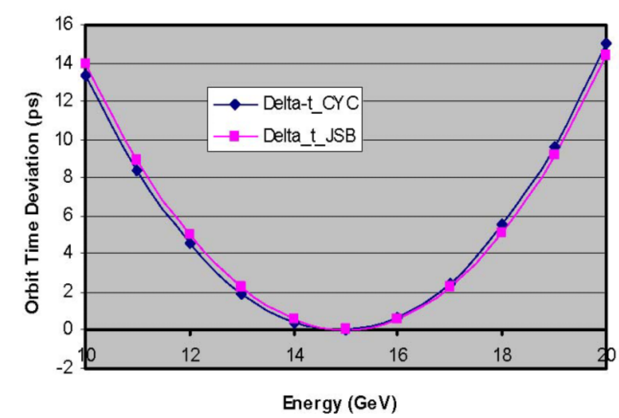

Figure 4: Orbit time deviation in the F0D0-2 FFAG: JSB – Berg; CYC – CYCLOPS.

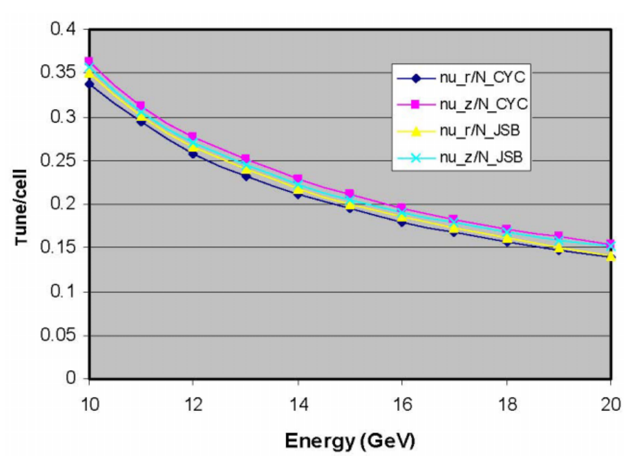

Figure 3: Tunes in the F0D0-2 FFAG: JSB – Berg; CYC – CYCLOPS.

IFFAG:

Because of its greater complexity, this lattice (described in the Introduction above) presented a greater challenge to producing an adequately detailed field map. To represent the hard-edge magnets with sufficient accuracy probably requires a finer mesh with more grid points than the current version of CYCLOPS is able to digest. The symptoms of this were a strong dependence of tune on mesh size at some energies, and poorer than usual closure of the orbits. Nevertheless, the tune results (see Figure 5) show general agreement with Rees's values, confirming his success in creating a magnet arrangement that can maintain positive axial focusing in an isochronous field at very high γ values. They also confirm Teng's suggestion in 1956 that high-order terms in the series expansions for the tunes in isochronous machines may drive ν_r away from γ [9].

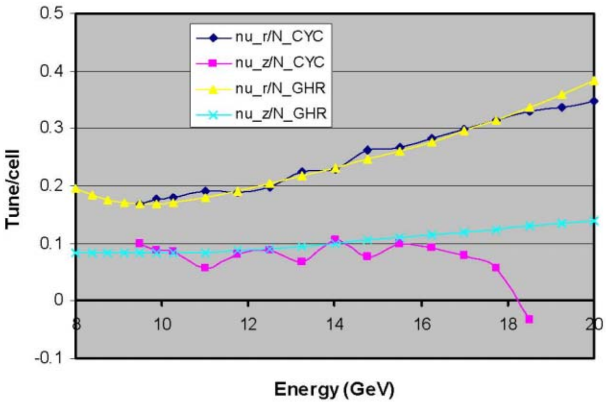

Figure 5: Betatron tunes in the isochronous IFFAG: GHR – Rees; CYC – CYCLOPS.

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