

# INVESTIGATION OF INTENSE BEAM TRANSPORT ON INJECTION LINE AND INFLECTOR OF COMPACT CYCLOTRON\*

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## Abstract

In this paper we present a numerical simulation work of high intensity DC beam transport on injection line and inflector of compact cyclotron. A two-dimensional PIC/FFT method is adopted to simulate beam transport on the injection line. In this method, external applied field is dealt with single particle tracking scheme; space charge field is dealt with Particle-Mesh method, and FFT technique is utilized to calculate the space charge forces. A fully three-dimensional model is established for self-consistently simulating the DC beam transport in the spiral inflector, which is developed based upon conventional Particle-Mesh method and includes both space charge effects and image charge effects from the electrodes. Moreover, we developed two object-oriented codes CYCPIC2D (for the injection line) and CYCPIC3D (for inflector). We also present a simulation result of high intensity beam transport in the injection line and inflector of CYCIAE-100 cyclotron.

## INTRODUCTION

Along with beam intensity increase in modern cyclotron, space charge force is becoming an important factor which can heavily impact beam's behaviour. PIC method is an efficient macro-particle computer simulation method which is well developed in accelerator community along with the development of computer science during the past decades<sup>[1-3]</sup>.

In compact cyclotron with external ion source, DC particle beam emitted from ion source is injected along the axial beam line and then inflected onto the middle plane of central region using spiral inflector. In order to extract intense beam with high transmission efficiency, it is significant to carry out quantitative simulation study on space charge effects during beam transport on injection line and spiral inflector.

## INJECTION LINE

### Physical model

The general form of potential solution of 2D Poisson equation of electric field in beam rest frame can be expressed as

$$\phi(x, y) = \frac{1}{2\pi\epsilon_0} \int G(x, x', y, y') \rho(x', y') dx' dy' \quad (1)$$

Where  $(x, y)$  and  $(x', y')$  are the coordinates of target point and source point respectively,  $\rho(x', y')$  is the charge density of beam, and  $G(x, x', y, y')$  is 2D Green function of open boundary system. The computational domain including all particles is divided into a discrete grid space and the particle charge is deposited onto the grid using a CIC (Cloud-In-Cell) scheme to obtain the charge density of the grid space. Then the potential at the grid nodes can be expressed as

$$\phi_D(x_i, y_j) = \frac{\Delta x \Delta y}{2\pi\epsilon_0} \sum_{m=1}^{N_x} \sum_{n=1}^{N_y} G_D(x_i, x_m, y_j, y_n) \rho_D(x_m, y_n) \quad (2)$$

Where  $\Delta x$  and  $\Delta y$  are the mesh sizes along  $x$  and  $y$  directions respectively, and  $N_x$  and  $N_y$  are the number of nodes along  $x$  and  $y$  directions respectively.

The space-charge potential on the nodes can be calculated by solving Eq.2 using a FFT based algorithm [4]. Then the electric field is transformed into the lab frame using Lorentz transformation formula and interpolated onto each particle's location to obtain the space charge force on each macro-particle.

To integrate the movement equation of charged particle, symplectic split-operator method is adopted to separate external applied field and space charge field [5].

We implemented above model in an Object-Oriented code CYCPIC2D using FORTRAN 95 language. In our code the longitudinal coordinate is used as the independent variable.

### Application

As the first application case, CYCPIC2D was applied to simulate 8mA DC beam transport on the axial injection line of CYCIAE-100 cyclotron, which is under construction at CIAE (Chinese Institute of Atomic Energy). The arrangement of transport elements is O-BS-O-D-O-F-O (BS: magnetic solenoid, D: defocus quadrupole, F: focus quadrupole and O: drift). The parameters of each element were determined by TRACE 3-D [6].

We simulated the beam transport under different neutralized rates. The result was also verified by compared with ORBIT [1] and TRACE-3D. The RMS beam envelopes given by the three codes are shown in Fig.1. The beam envelopes from the three codes agree well with each other in low intensity conditions. CYCPIC2D and ORBIT can still give almost the same results under low neutralized rates, while the result of

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TRACE 3-D becomes divergent, as shown in Figure (b) and (c). When the beam intensity improves to 20mA, the divergency of TRACE 3-D is particularly evident, as shown in Figure (d). This phenomenon is caused by the error introduced by the truncation of space charge force to linear terms in TRACE-3D, which gets bigger and bigger along with the increase of beam intensity. In addition, the maximal value of RMS envelops for 8mA beam with neutralized rates of 90%, 50% and 0% are 1.57, 2.94 and 4.35 cm, respectively. Whereas, due to the space limit in the injection region of the machine, the designed inner radius of the tube is only 2.5 cm. Therefore, in order to achieve reasonable transmission efficiency, it is critical to keep the neutralization high enough in the tube and make the length of injection line as short as possible.

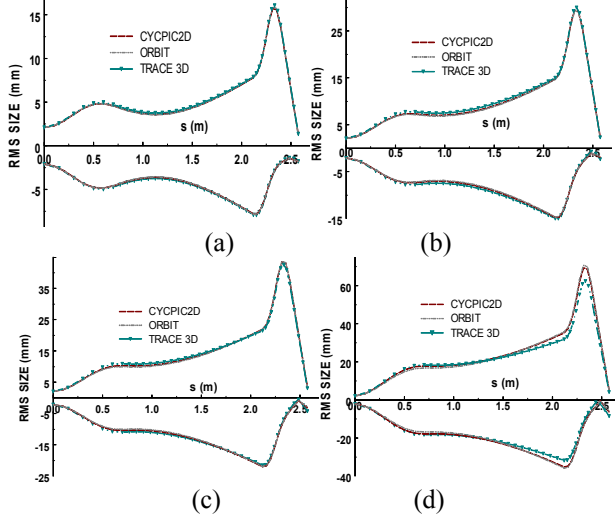


Figure 1: RMS envelops given by CYPIC2D, ORBIT and TRACE 3-D. (a)-(c) 8mA beam, 90%, 50% and 0% neutralized rates. (d) 20mA beam, no neutralization

## INFLECTOR

### Physical model

The space charge effects on the beam orbit in the spiral inflector have been studied using simplified analytical model [7-9], which shows that space charge have considerable influence on the transmission process when the intensity exceeds several mA. In their models, following two assumptions are used: 1), the beam is treated as an infinitely long straight cylindrical line with uniform particle distribution, which has explicit analytical expression of transverse space charge field; 2) the image charge effects from the electrode surface is neglected. Actually, above two assumptions can bring in considerable error for intense beam. A fully 3D self-consistent macro-particle simulation will help to give out more precise result for intense beam at several mA levels.

We developed a new iterative Particle-Mesh simulation model based on the conventional PIC to model intense particle transport in inflector. In this model, the physical computational domain  $\Omega$  which includes the entire inflector is defined as a fixed 3D rectangular and decomposed into a discrete Cartesian grid domain. All

macro-particles are tracked through the computational domain step-by-step based on the external electric field and magnetic field using integration method such as 3-D leap-frog algorithm; after each tracking step, the charge of each macro-particle is deposited onto neighbouring 8 grid points using the CIC charge distribution scheme to accumulate charge on the grid. The total space charge density at a grid point can be expressed as:

$$\rho_{D,m}(x_i, y_j, z_k) = \frac{1}{\Delta x \Delta y \Delta z} \sum_{k=1}^N q W(x_{n,m} - x_i) W(y_{n,m} - y_j) W(z_{n,m} - z_k) \quad (3)$$

$$i=0,1,\dots,n_x \quad j=0,1,\dots,n_y \quad k=0,1,\dots,n_z$$

Where  $Nstep$  is the integration step number,  $(x_i, y_j, z_k)$  are the coordinates of the  $(i, j, k)$  grid point,  $(x_{n,m}, y_{n,m}, z_{n,m})$  are the coordinates of the  $n$ th macro particle at the  $m$ th step,  $Nx$ ,  $Ny$  and  $Nz$  are the grid point number along  $x$ ,  $y$  and  $z$  directions, and  $W$  is the weighting function of charge distribution. The actual space charge density on the grid point is

$$\bar{\rho}_D(x_i, y_j, z_k) = \frac{\rho_D(x_i, y_j, z_k)}{\sum_{l=0}^{Nx} \sum_{m=0}^{Ny} \sum_{n=0}^{Nz} \rho_D(x_l, y_m, z_n)} \frac{I \cdot T}{\Delta x \Delta y \Delta z} \quad (4)$$

Where  $I$  is beam current and  $T$  is total transport time.

After all the macro-particles have passed through the computational domain, Poisson equation is solved using infinite difference successive relaxation method to obtain space charge potential distribution and then space charge field distribution can be calculated. In addition, it is worthwhile to mention that the energy of particles in inflector is typically far smaller than its rest energy, so the relativistic effects are neglected.

Thus the first iteration is over and the second iteration starts. This time all the macro-particles are re-tracked from the beginning based on both the external field and the new calculated space charge field of last iteration. Then, similarly, Poisson equation is solved to obtain new potential and field. This procedure is repeated until the maximal change of space charge potential between two sequential iterations is below the expected error tolerance value, which means the final converged solution is found.

The above iterative algorithm is implemented in an object-oriented macro-particle simulation code using FORTRAN 95, named CYCPLIC3D.

### Application

The detailed calculation of beam trajectory and optimal design of inflector structure for 100MeV cyclotron at Chinese Institute of Atomic Energy (CIAE) is being carried out using CASINO [10], INFLECTOR [11] and RELAX3D [12]. Tab. 1 shows some key parameters of injector and result data without considering the effect of space charge effect and image charge effects. The initial particle distribution in transverse 4D phase space is assumed as uniform and energy spread is zero.

$$\mathcal{E}_u = \mathcal{E}_h = 50.0 \pi \text{ mm-mrad}, \quad \beta_u = \beta_h = 0.1 \pi \text{ mm/mrad},$$

