PARAMETER OPTIMIZATION OF A RECTANGULAR DIELECTRIC BASED WAKEFIELD ACCELERATING STRUCTURE

S. Baturin, A. Altmark, I.L. Sheynman, St.Petersburg Electrotechnical University “LETI”, Saint-Petersburg, Russia
A. Kanareykin, St.Petersburg Electrotechnical University “LETI”, Saint-Petersburg, Russia and Euclid Techlabs LLC, Gaithersburg, MD USA

Abstract
In this talk, we present the algorithm and simulation results of a single mode wakefield parametric study of the rectangular dielectric based wakefield accelerating structure. Analytical solutions of wakefield generation in rectangular dielectric structures have been studied in order to achieve optimal relations between the geometrical parameters and dielectric constant of the structure, and the drive beam parameters like bunch charge and bunch length. Optimization has been carried out for maximization of the accelerating gradient in the single LSM11 mode approximation.

INTRODUCTION
In this paper we consider dielectric based wakefield acceleration technology [1-4] as one of the most promising for the development of the high gradient accelerating structures to be used for the next generation of linear colliders [5] and future X-ray FELs [6].

It is assumed that dielectric lined structures will be excited by a high intensity electron beam (e.g., the CLIC collider project [7,8]) for generating high power X-band, mm-wave or THz radiation. This type of electromagnetic wave generation is essential for avoiding the need to develop high power upper GHz/THz sources and coupling components that are able to sustain and transmit into the structure GW-level power in the same frequency ranges [2-4].

An accelerating structure with dielectric loading is a dielectric waveguide with an axial vacuum channel for beam propagation. The dielectric is surrounded by a conducting metal wall [1-4]. Recent experiments have proved that the dielectric based structures can sustain accelerating gradients in excess of 100’s of MV/m [8,9] and GV/m [3] in the upper GHz and THz frequency ranges respectively.

For example, a high current (up to 100 nC) short (1–2 mm) relatively low energy (15–100 MeV) electron bunch in this type of a structure can generate Cerenkov wakefields with a longitudinal field magnitude up to 100-300 MV/m in the X - Ka band frequency ranges [9]. A 3 nC charge and 30 μm long bunch from the 23 GeV FACET/SLAC accelerator generates 1-10 GV/m wakefields in the THz range [3,10]. These wakefields are used for accelerating a less intense but high energy electron bunch propagating behind the drive bunch at a distance corresponding to the accelerating phase of the $\hat{E}_z$ field [11]. Dielectric based structures provide in addition to a high accelerating gradient the control over the frequency spectrum of the structure by introducing additional ferroelectric layers [12,13] as well as the possibility of using new promising microwave/THz materials (such as diamond and sapphire) with unique breakdown strength and thermoconductive properties [14]. Cerenkov radiation generated by a relativistic electron bunch in a rectangular waveguide with a transverse, inhomogeneous dielectric layers has been analyzed in [15], where a modification of the transverse operator method was used. In [15], the Sturm–Liouville second order operator with an alternating sign weight function was considered. This approach makes it possible to obtain a complete analytic solution for eigenmodes and to solve the problem of Cerenkov wakefield generation in a rectangular accelerating structure with a composite dielectric loading in the most general form [15].

Figure 1: A rectangular dielectric based accelerating structure.

Below, we present the analytical solution for wakefield generation in the rectangular dielectric structure to optimize relations between both the geometrical parameters and dielectric constant of the structure, and the beam parameters for two specific user facilities, the Argonne Wakefield Accelerator (AWA) and the FACET facility at SLAC.

ANALYTICAL APPROXIMATION
We consider an ultrarelativistic Gaussian electron bunch passing along the central axis of a rectangular DLA (dielectric loaded accelerating) structure presented in Fig.1.

Let us take into account first the symmetric accelerating LSM mode ($H_y = 0$), which corresponds to the first asymmetric eigenfunction of the transverse $\hat{T}_e$ operator of the wave equation [15]. We introduce dimensionless parameters as follows:
Here $R_w$ is the outer half height of the waveguide, $R_c$ is the vacuum gap half height, and $w$ is the width of the waveguide.

We assume here that the condition $\omega^2 << \gamma^2 \beta c (\pi / w)^2$ is fulfilled for the frequency of the mode under consideration. In this case the dispersion relationship for the LSM symmetric modes presented in [15] can be written as:

$$\cot(X) = X \frac{\tanh(\pi p_2)}{\varepsilon(p_1 - 1)\pi p_2}. \quad (2)$$

The axial electric field $E_z$ for this approximation is given by the following expression:

$$E_z = \frac{4\pi Q}{w^2} \frac{\pi^2 p_2}{S(p_1, p_2)^2 + (\pi p_2)^2} \frac{1}{\varepsilon - 1} \frac{1}{p_1 - 1} \times$$
$$\times \left[ \cos(X) \right]^2 \left[ \sinh(\pi p_2) \right] \frac{4X}{\sin(2X) + 2X} \times$$
$$\times \exp \left\{ -\frac{\sigma_z^2}{2w^2} S(p_1, p_2)^2 \right\}. \quad (3)$$

Here

$$S(p_1, p_2) = \frac{1}{\sqrt{\varepsilon - 1}} \left[ \frac{X^2}{(p_1 - 1)^2} + (\pi p_2)^2 \right]. \quad (4)$$

Where $\sigma_z$ is the bunch length, and $Q$ is the bunch charge.

The mode frequency can be calculated as

$$f = \frac{c}{2\pi w} \frac{S(p_1, p_2)}{p_2}. \quad (5)$$

Now we maximize $E_z$ using the expression (3) by varying $p_1$, $p_2$ and $w$ for the given beam parameters and dielectric constant value $\varepsilon$. The optimal structure dimensions have been simulated for the FACET and AWA accelerator parameters. The corresponding parameter optimization results are presented below.

**OPTIMIZATION FOR THE AWA BEAM**

Let us consider the AWA beam parameters: bunch length $\sigma_z = 1.5$ mm, bunch charge $Q = 20 nC$, and transverse bunch size $\sigma_r = 0.5$ mm. The vacuum gap half-height has to satisfy $R_c > 3\sigma_r$ to be able for the bunch to pass through the structure with no significant beam loss.

An additional analysis that has been carried out (Fig. 2) shows that the optimal dielectric constant of the dielectric loading material should be within the range that corresponds to the diamond dielectric constant. This leads to additional terms for the maximization procedure for $R_c > 1.5$ mm. We chose the dielectric constant as $\varepsilon = 5.7$, which corresponds to a diamond like material that exhibits specific advantages by comparison with the cordierite and alumina ceramics previously used for DLA applications [4,16].

Maximization of $E_z$ according to expression (3) for the given parameters leads to:

$$R_c = 1.5 \text{ mm}, \quad R_w = 2.33 \text{ mm}, \quad w = 11.1 \text{ mm}. \quad (6)$$

Optimization of the AWA structure geometry was carried out numerically. The wakefield magnitude was found $E_z = 24.3 \text{ MV/m}$ or $1.22 \text{ MV/m/nC}$ for the accelerating mode frequency $f = 36.2$ GHz. The maximum charge of a single AWA bunch can be up to $120 \text{ nC}$ [2,9] giving the upper gradient limit of ~ $150 \text{ MV/m}$ in the single mode approximation.

![Figure 2: $E_z$ field spectrum distribution for the AWA bunch parameters and structure geometry corresponding to expression (6).](image)

The corresponding amplitude-spectrum distribution for the parameters (6) is presented in the Fig. 2. One can see that the AWA bunch wakefield is nearly monochromatic for the optimized structure parameters. This gives an additional opportunity for the more precise bunch phasing in multibunch wakefield accelerating methods [17].
OPTIMIZATION FOR THE FACET BEAM

In this section we consider the FACET beam parameters [4,16]: bunch length $\sigma_z = 30 \, \mu\text{m}$, bunch charge $Q = 3\, \text{nC}$, transverse size $\sigma_r = 30 \, \mu\text{m}$. Following the same approach as for the AWA structure above one can use $R_c > 3\sigma_r$ or $R_c > 100 \, \mu\text{m}$. For the dielectric layer we considered diamond as in the AWA case. Due to the small transverse and longitudinal dimensions of the FACET beam it could be considered as a potential driver for a terahertz radiation source based on dielectric structure.

Maximization of (3) for the FACET beam parameters leads to:

$$R_c = 100 \, \mu\text{m}, \quad R_w = 112 \, \mu\text{m}, \quad w = 406 \, \mu\text{m}. \quad (7)$$

With the wakefield amplitude $E_z = 2.7\, \text{GV/m}$ or $0.9\, \text{GV/m/nC}$ and frequency $f = 1.32 \, \text{THz}$. The maximization was performed numerically.

CONCLUSION

We considered a new algorithm for the single mode longitudinal wakefield magnitude maximization of rectangular dielectric based wakefield accelerating structures. Analytical solution of the wakefield generated in rectangular dielectric structures are presented. The optimal relations between both the geometrical parameters and bunch charges and lengths were found. We have shown that theoretical predictions of the wakefield maxima are quite promising for both AWA and FACET beam parameters that are $1.22 \, \text{MV/m/nC}$ at $36 \, \text{GHz}$ and $0.9 \, \text{GV/m/nC}$ at $1.3 \, \text{THz}$ respectively. Multibunch acceleration schemes are to be considered as well. It should be noted that the results presented here open new possibilities for considering the DLA structures designed for the AWA and FACET beams as new narrow band high power terahertz sources.