Investigation of Resistive Wall Instability in the 7-GeV APS Storage Ring*

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Abstract

The Advanced Photon Source (APS) storage ring is a 7-GeV light source with 40 straight sections. Intense x-ray beams will be delivered by 34 insertion devices installed in these straight sections. The vacuum chamber for the insertion devices has an elliptical cross section with the gap equal to 8 mm. With this narrow gap, we estimate that the transverse impedance of the ring at the revolution frequency could be as high as 36 $M\Omega$ from the resistive wall. By increasing the (unnormalized) chromaticity to 7, we cure the head-tail modes of order up to m=1 for all 60 coupled bunch mode patterns around the ring. Tracking results show that the increased sextupole strength resulting from a higher chromaticity does not significantly reduce the dynamic aperture. Since increased chromaticity alone cannot cure all the head-tail modes, the APS storage ring will have a feedback system to damp the rigid-bunch modes.

Introduction Resistive wall impedance can cause the coupled bunch instability due to the peak near the origin (long range wakefield or multi-turn effects) as well as the higher-order head-tail modes via the broad-band tail (short range wakefield or single-turn effects). Since the growth rate from the resistive wall instability is in general slow, the strategy is to damp the fastest growing mode of the coupled bunch oscillation by adjusting the chromaticity slightly above zero, causing the unstable head-tail modes to become stabilized by the radiation damping and/or Landau damping.

However, we found that this is not the case for the APS storage ring.

Resistive Wall Impedance R. Gluckstern, J. Zeitzs and B. Zotter [1] have derived expressions for the longitudinal and transverse resistive wall coupling impedance for a beam pipe of arbitrary cross section in the ultra-relativistic limit. Explicit results for the transverse impedance for the beam pipe of elliptic cross section with the major axis a and the minor axis b may be written

$$\begin{aligned} Z_{x,y}(\omega) &= (1+j)\frac{Z_0 \delta L}{2\pi b^3} F_{x,y}(q) \\ &\equiv Z_{\perp,circular}(b,\omega) F_{x,y}(q), \end{aligned} \tag{1}$$

where $Z_{\perp,circular}(b,\omega)$ is the transverse impedance for the cylindrical beam pipe of radius b and $F_{x,y}(q)$ is the form factor expressed in terms of "nome" $q=(a-b)/(a+b)$. The subscripts x and y denotes the horizontal and vertical impedance, respectively. Denoting Z_y as $Z_\perp$ and using the fact that the vertical form factor, $F_y(q)$ is bounded by 0.8 and 1.0 for the entire range of q, we may approximate Eq. (1) as

$$Z_\perp(\omega) \simeq Z_{\perp,circular}(b,\omega). \tag{2}$$

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Then, Eq. (2) may be rewritten as

$$Z_\perp(\omega) = (sign(\omega)+j) Z_\perp(\omega_0)\sqrt{\frac{\omega_0}{\omega}}, \tag{3}$$

where $Z_\perp(\omega_0)$ is the impedance evaluated at the revolution frequency. The APS storage ring consists of 34 straight sections for insertion devices (IDs) with half gap, b, equal to 4 mm and length equal to 6.5 m per each straight section and the remaining sections with b equal to 2 cm. Then the resistive wall impedance, $Z_\perp(\omega_0)$, from the 34 ID vacuum chambers and the remaining sections are 34.5 $M\Omega/m$ and 1 $M\Omega/m$, respectively. We estimate that the total impedance for the APS storage ring due to the resistive wall is 36 $M\Omega/m$. In the estimation we used the resistivity of Al at room temperature is $3\cdot 10^{-8}$ Ωm, and the skin depth at revolution frequency of 0.2715 MHz is 168 μm.

Rigid Bunch Case Consider a single rigid bunch is circulating in the ring. The "rigid" bunch means no internal motion inside the bunch, and the bunch can be approximated as a macro particle with charge Q. The equation of motion including the wakefield effects may be written as

$$\frac{d^2y}{dt^2} + \omega_\beta^2 y = \frac{eQ}{\gamma m_0}\sum_{k=1}^{\infty} y(t-kT_0)\frac{W_\perp(kT_0)}{2\pi R}, \tag{4}$$

where ω_β is the free betatron oscillation frequency, T_0 is the revolution period, Q is the total charge of a bunch, and m_0 is the rest mass of a particle. The contributions from all previous turns are included as a sum in the equation.

Equation (4) may be solved by assuming that y varies harmonically as $e^{j\Omega t}$. The resulting coherent frequency shift may be written as

$$\Delta\omega_c = \Omega - \omega_\beta = jC_T \sum_{p=-\infty}^{+\infty} Z_\perp(p\omega_0+\omega_\beta), \tag{5}$$

where

$$C_T = \frac{cI_0}{4\pi\nu_\beta E/e},$$

c is the velocity of light, I_0 is the average current of a single bunch, E is the total energy of a particle, and ν_β is the vertical betatron tune.

Substituting the resistive wall impedance into Eq. (5),

$$\tau^{-1} \equiv -Im\Delta\omega_c = -C_T Z_\perp(\omega_0)\sqrt{\omega_0}\sum_p \frac{sign(p\omega_0+\omega_\beta)}{\sqrt{|p\omega_0+\omega_\beta|}}. \tag{6}$$

Separating the tune into the integral and fractional parts denoted as $\nu_\beta = n_\beta + \Delta_\beta$ and absorbing the integral part into the summation indices, we find that

$$\tau^{-1} = -C_T Z_\perp(\omega_0) G(2\pi, \Delta_\beta), \tag{7}$$

where $G(2\pi, \Delta_\beta)$ is the familiar Courant-Sessler bunch function [2]. For M evenly spaced bunches, we replace I_0 by MI_0 and the mode frequency changes to $\omega_p = (Mp + n)\omega_0 + \omega_\beta$ including the coupled bunch mode number n ranging from 0 to $M - 1$. The resulting expression for the growth rate is

$$\tau^{-1} = -C_T Z_\perp(\omega_0)\sqrt{M} G(2\pi, mod(\frac{n + \nu_\beta}{M})). \qquad (8)$$

The growth rate is positive when $\frac{n+\nu_\beta}{M}$ lies between an integer and next lower half-integer, and negative in the other half-interval. We can easily show that, if M is even, half the coupled bunch modes are stable and the other half are unstable.

When we applied the above formula to the APS storage ring, we assumed $I_0 = 5\ mA$, $\nu_\beta = 14.3$, and $E = 7\ GeV$ and used $Z_\perp(\omega_0) = 36\ M\Omega$. We found that

$$\tau^{-1} = -42.84\sqrt{M} G(2\pi, mod(\frac{n + \nu_\beta}{M})).$$

Assuming sixty bunches circulating in the ring, the growth rate for the fastest growing mode of n=45 becomes

$$\tau^{-1} \simeq 3072 s^{-1} > \tau_R^{-1} = 106 s^{-1},$$

where τ_R^{-1} is the (synchrotron) radiation damping rate. Hence we conclude that some of the coupled bunch modes grow indefinitely resulting in possible beam loss. However, by assuming rigid bunch we ignored an important stabilizing mechanism due to the internal motion of particles inside the bunch. This is the subject of the next section with application to the APS storage ring.

Non-Rigid Bunch Case

The general expression for the coherent frequency shift [3] without considering mode coupling may be written as

$$\Delta\omega_{m,n} = \frac{j}{1+m} C_T M \frac{\sum Z_\perp(\omega) H_m(\omega_p - \omega_\xi)}{B \sum H_m(\omega_p - \omega_\xi)}, \qquad (9)$$

where

$$\begin{aligned} \omega_p &= (Mp + n)\omega_0 + \omega_\beta + m\omega_s, \\ n &= \text{(coupled) bunch mode number}, \\ m &= \text{head-tail mode number}, \\ \omega_\xi &= \text{chromaticity frequency } (\xi\omega_0/\eta), \\ \xi &= \text{chromaticity } (\Delta\nu_\beta/(\Delta p/p)), \\ B &= \text{bunching factor (bunch length/circumference)}, \\ H_m(\omega) &= \text{self-power density } (|\lambda(\omega)|^2). \end{aligned}$$

For a beam with Gaussian distribution, we take Hermitian line-density mode with the Fourier transforms

$$\lambda_m(\omega) = C_m j^{-m} (\omega\sigma_\tau)^m exp(-\omega^2\sigma_\tau^2/2), \qquad (10)$$

where σ_τ is the rms bunch length in time. The factor C_m may be determined such that the denominator in Eq. (9) becomes unity, i.e. $B \sum H_m(\omega_p - \omega_\xi)$. The summation was evaluated by Zotter [4] who found that

$$C_m^2 = \frac{2\pi}{r\Gamma(m + \frac{1}{2})}, \qquad (11)$$

where

$$r = \frac{\tau_L}{\sigma_\tau} = \begin{cases} 4 & \text{for protons} \\ 3\sqrt{\pi/2} \approx 3.76 & \text{for electrons,} \end{cases}$$

and τ_L is the baseline bunch length. Then the normalized power spectrum has a peak at $\omega = \sqrt{m}/\sigma_\tau + \omega_\xi$ with values of 0.943, 0.694, 0.681, and 0.676 for $m = 0$, 1, 2, and 3, respectively. We note that these are equivalent to the maximum values of Sacherer's form factor [3] for the electron beam.

Denoting ω_{p0} as the lowest mode frequency and $\omega^\pm = \omega_{p0} \pm M\omega_0$, we may rewrite Eq. (9) as

$$\begin{aligned} \Delta\omega_{m,n} \;=\; & \frac{j}{1+m} C_T \Big(\underbrace{\frac{1}{\omega_0}\int_{-\infty}^{\omega^-}(\cdots) + \frac{1}{\omega_0}\int_{\omega^+}^{+\infty}(\cdots)}_{\text{“single-turn effect”}} \\ & + \underbrace{M Z_\perp(\omega_{p0}) H_m(\omega_{p0} - \omega_\xi)}_{\text{“multi-turn effect”}} \Big) \end{aligned}$$

If the chromaticity is zero, the growth rate from the rigid bunch approximation and the growth rate from the multi-turn effect should be close to each other. It turns out τ^{-1} =3072 sec^{-1} and 3021 sec^{-1}, respectively, showing that the single spectrum line located nearest the origin has the dominant effect. The numerical results were obtained by using the program BBI [5].

The stabilization of the lowest mode (m=0) can be achieved by adjusting the chromaticity to the value greater than zero. The single-turn effect provides a large damping effect. It is shown in Fig. 1, where the chromaticity is equal to 1. The maximum growth rate is still greater than the radiation damping rate.

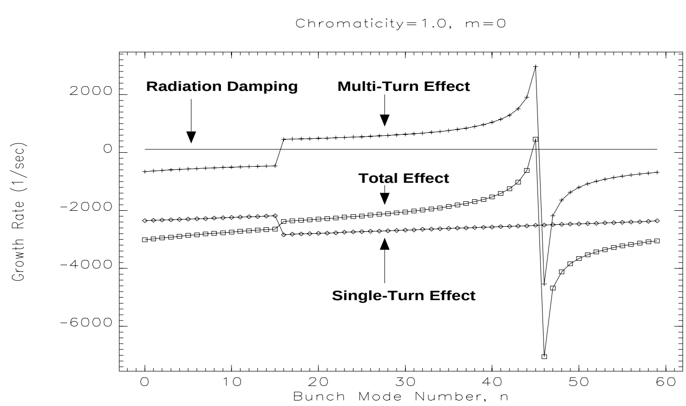

Figure. 1. Single- and Multi-Turn Effects on $m = 0$ Mode at $\xi_y = 1.0$.

Thus, we further increase the chromaticity up to 7 in order to stabilize both the m=0 and m=1 head-tail modes. The results are shown in Fig. 2.

However, at chromaticity equal to 7 we found that the higher head-tail modes become unstable. Figure 3 shows the unstable modes, $m = 3$ and $m = 4$, together with the damped modes, $m = 0$ and $m = 1$.

Even though the excitation of such high head-tail modes was never observed in the electron storage ring, it is prudent to cure the instability using the feedback-damper system. The feedback system has to damp the $m = 0$ growing mode with the chromaticity adjusted at zero, where all higher modes, $m \geq 1$, are naturally damped.

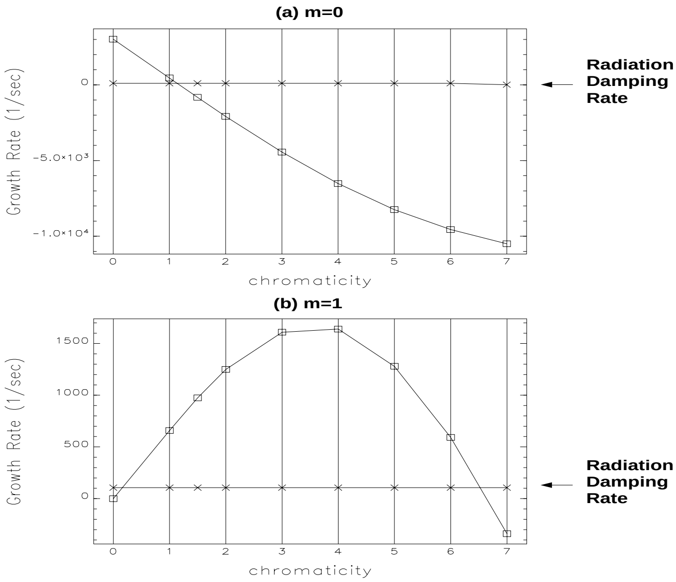

Figure. 2. Growth Rate vs. Chromaticity for $m = 0$ (a) and $m = 1$ (b) Head-Tail Modes.

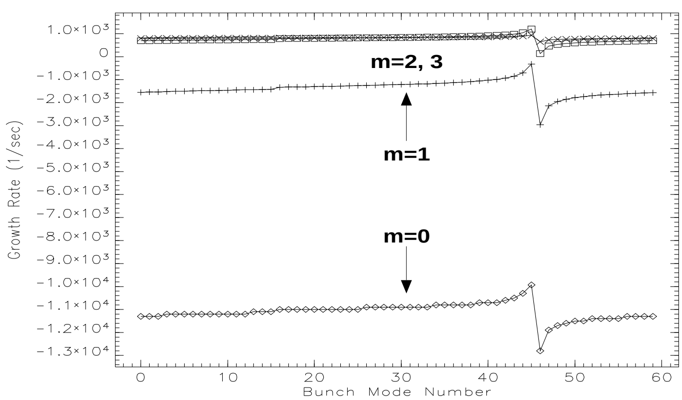

Figure. 3. Growth Rate for Various Head-Tail Modes at $\xi_y = 7.0$.

During early operation of the APS storage ring when there are only 16 narrow ID chambers instead of 34, we may not need the feedback-damper system because we found that we can stabilize all head-tail modes up to a beam current of 100 mA by shifting ξ_y to ~ 0.32.

Dynamic Aperture With the provision of operating the ring in high chromaticity, we need to make sure that the ring has large dynamic aperture. The chromaticity-correcting sextupoles in the APS storage ring can adjust the chromaticity anywhere in the triangular region bounded by the three vertices, namely $(\xi_x, \xi_y) = (0,0)$, $(20,0)$ and $(0,16)$, including the diagonal line up to $\xi_x = \xi_y = 10$. The original design value of chromaticity for the APS storage ring is equal to zero. The nominal strengths of the horizontally focusing sextupole (SF) and defocusing sextupole (SD) are

$$\frac{B''l}{(B\rho)} = 4.6\ m^{-2}\ \text{(SD)},\ 4.2\ m^{-2}\ \text{(SF)}.$$

In order to obtain a chromaticity equal to 7 in both horizontal and vertical planes, we need to increase the strengths by 14% in the SF and 10% in SD.

Dynamic aperture reduction due to increased sextupole strengths, shown in Fig. 4, is not much even without readjusting the strength of the harmonic-correction sextupoles.

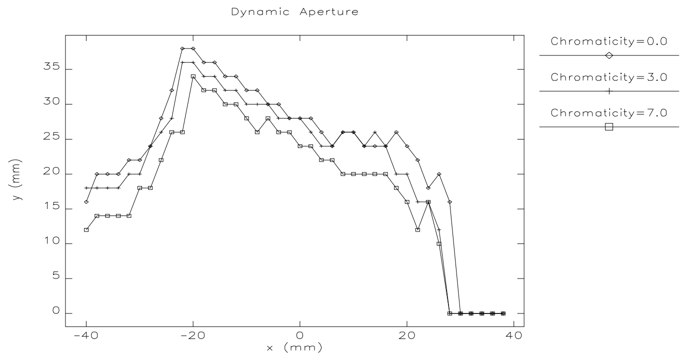

Figure. 4. Dynamic Aperture at Various Chromaticities.

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