PLASMA WAVE WIGGLERS FOR FREE ELECTRON LASERS

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ABSTRACT

We explore the possibility of using relativistic plasma density waves as wigglers for producing free electron laser radiation. Two possible wave and beam geometries are explored. In the first, the wiggler is a purely electric wiggler with frequency ω_p (plasma frequency) but (approximately) zero wavenumber k_p. If an electron beam is injected parallel to a wide plasma wave wavefront, it is wiggled transversely with an apparent wiggler wavelength $\lambda_w = 2\pi c/\omega_p$. In the second scheme, the electron beam is injected down the axis between two narrow plasma waves propagating opposite to the beam. The electrons are wiggled by the radial fields of the waves and are guided (focused) by the waves' ponderomotive force. With either scheme, effective wiggler wavelengths of order 50-100 μm may be obtained in a plasma of density $10^{17}(\mathrm{cm}^{-3})$, thereby permitting generation of short wavelength radiation with modest energy beams. The effective wiggler strengths $a_w = eA/mc^2 \approx 0.5$ can be extremely large. We discuss the excitation methods for such wigglers and examine these with PIC computer simulations.

I. INTRODUCTION

There is a great deal of interest in developing short wavelength wigglers for free electron lasers and synchrotrons. Unfortunately, the high magnetic field strengths needed for such wigglers are not attainable with present technology for conventional static magnetic wigglers with wavelengths less than 1 cm. Operation at wavelengths shorter than this, therefore, would require a large increase in the number of wiggler periods and thus result in a very small extraction efficiency as well as require a very small electron beam energy spread and emittance. In this paper, we explore the suitability of two possible electric wiggler schemes with effective wiggler wavelengths in the 100 μm range. In our proposal these purely electric wigglers are relativistic plasma density waves. Such wigglers, because of their extremely short wavelengths, allow the use of a rather modest energy electron beam for FEL action in the visible or VUV range.

In the first plasma wiggler scheme, the wiggler consists of a purely electric field oscillating perpendicular to the electron beam with a frequency ω_p but with no spatial dependence ($k_{px} \approx 0$) (see Fig. 1). The amplification of radiation by such a field in vacuum has been modelled recently by Y. T. Yan and J. M. Dawson[1], and the use of the plasma wave was proposed in Ref. 2.

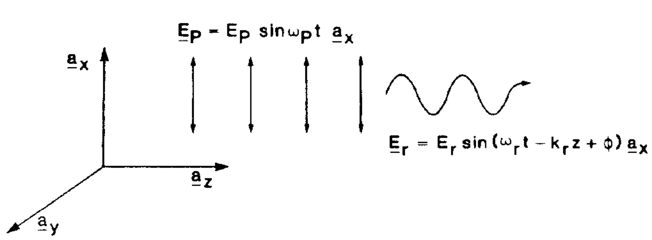

Fig. 1. Geometry of plasma wiggler FEL scheme 1 ($\vec{v} \perp \vec{k}_p$).

In the second plasma wiggler scheme, the wiggler consists of two narrow plasma waves propagating anti-parallel to the electron beam (see Fig. 2). The two waves are chosen to be π out of phase so that their radial fields add and wiggle the electron beam transversely as it passes between the waves.

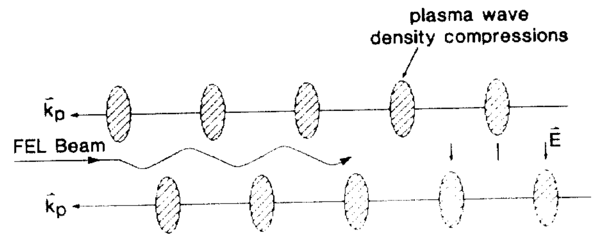

Fig. 2. Geometry of plasma wiggler FEL scheme 2 ($\vec{v}_b \parallel -\vec{k}_p$).

Using a plasma wave as a purely electric wiggler is attractive for two reasons: First, the effective wiggler wavelength ($\approx 2\pi c/\omega_p$, typically of order 100 μm) is shorter than that available with conventional magnet wigglers and second, the effective wiggler strength can be extremely large (equivalent to 1 megagauss wiggler fields at 100 μm wavelength). However, when a plasma medium is used for exciting the purely electric wiggler, the FEL electron beam can be strongly influenced by it. The limitations to FEL gain imposed by the plasma medium were discussed in Ref. 2. In this paper we discuss how plasma density waves suitable for FEL actions can be excited and present 2-D particle-in-cell computer simulations of wave excitation and test particle trajectories of the FEL beam.

II. FEL MECHANISM IN A PLASMA WIGGLER

The equivalence between magnetic, electromagnetic and the two purely electric (plasma) wigglers schemes can most easily be seen from the point of view of the electrons. Although the four cases appear quite different in the laboratory frame, in the electron frame all appear to be electromagnetic waves. Here, ω_o and ω_p represent the frequencies of the electromagnetic and electrostatic (plasma) waves respectively, and k_o, k_w and k_p ($\approx \omega_p/c$) represent the wavenumbers of the electromagnetic, magnetic and plasma wave wigglers respectively in the laboratory frame. In scheme 1, the purely electric wiggler has zero k parallel to the beam velocity (v_b) whereas the purely magnetic wiggler has a zero ω. In the electron frame the momentum four-vectors are

$$\begin{pmatrix} k'c \\ i\omega' \end{pmatrix} = \begin{bmatrix} \gamma & i\beta\gamma \\ -i\beta\gamma & \gamma \end{bmatrix} \begin{pmatrix} k_o \\ i\omega_o \end{pmatrix}_{\text{electromagnetic}} \quad \begin{pmatrix} k_w \\ 0 \end{pmatrix}_{\text{purely magnetic}} \quad \begin{pmatrix} 0 \\ i\omega_p \end{pmatrix}_{\text{purely electric}} \text{Scheme 1} \quad \begin{pmatrix} -k_p \\ i\omega_p \end{pmatrix}_{\text{purely electric}} \text{Scheme 2} \tag{1}$$

For $\beta \approx 1$, the transformed quantities satisfy $\omega' = k'c$ for all the cases except their wavelengths are different; $\lambda' = \lambda_w/\gamma$ (magnetic), $\lambda_o/2\gamma$ (electromagnetic), $2\pi c/\omega_p\gamma$ (purely electric, $\vec{v}_b \perp \vec{k}$) and $\pi c/\omega_p\gamma$ (purely electric, $\vec{v}_b \parallel -\vec{k}$). The choice of plasma density determines the effective wiggler wavelength in the plasma schemes:

$$\lambda_w = 2\pi c/\omega_p \approx 3\times10^6/\sqrt{n_o(\mathrm{cm}^{-3})} \qquad (\vec{v}_b \perp \vec{k})$$
$$\lambda_w = \pi c/\omega_p \qquad (\vec{v}_b \parallel -\vec{k})$$

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Thus for plasma densities in the range 10^{13} to 10^{18} cm^{-3}, λ_w ranges from 1 cm to 15 μm.

Each electromagnetic wave has a wiggler strength $a_w = eE'/m\omega'c = eA/mc^2$ equal to its corresponding value, $eB\lambda_w/2\pi mc^2$ (magnetic), $eE_o/m\omega_o c$ (electromagnetic) and $eE_p/m\omega_p c$ (purely electric or plasma) in the lab frame. The effective maximum undulator strength parameter a_w of the plasma wiggler can be easily estimated from 1-D Poisson's equation: $\underline{\nabla}\cdot\underline{E} = -4\pi e n_1$, where n_1 is the perturbed electron density associated with the plasma wave. The maximum density rarefaction occurs when $n_1 = n_o$, known as the cold plasma wavebreaking limit. In this limit, $|ik_pE_{p,max}| \approx 4\pi e n_o$ or $eE/m\omega_p c = 1$ for $\omega_p/k_p c = 1$. Thus the maximum value of E_p attainable in the plasma corresponds to $a_w = 1$, independent of the plasma density. The effective maximum magnet strength of a plasma wave is

$$B_{eff}\,(\text{gauss}) = \frac{2\pi mc^2}{e\lambda_p}\,a_w \le \frac{2\pi mc^2}{\lambda_p} \quad 3\times10^{-3}\sqrt{n_o(\text{cm}^{-3})}$$

It can be seen that effective magnetic field strengths of order 1 megagauss are possible in a plasma of density 10^{17} cm^{-3}.

In the plasma wigglers, as in the other two cases, the noise that seeds the lasing process comes from Compton scattering, with the radiated frequency in the exact forward direction $\omega_r \approx 2\gamma^2\omega_p$ $(\vec{v}_b \perp \vec{k})$ or $\omega_r \approx 4\gamma^2\omega_p$ $(\vec{v}_b \,||\, -\vec{k})$. Once, there is spontaneous emission at ω_r, there is a resonant ponderomotive force on the radiating electrons. This force is $\underline{F} = -e\underline{v}_w \times \underline{B}_r/c$, where v_w is the wiggle velocity and B_r is the magnetic field of the spontaneous or stimulated radiation.

The first plasma wiggler Scheme has been described in some detail in Ref. 2. Here we turn to scheme 2. For the geometry of our second plasma wiggler scheme we propose two narrow plasma waves with potentials of the form

$$\phi_1 = \phi_p \sin(\omega_p t + k_p z)\, e^{-y^2/w^2} \quad \text{and} \tag{2}$$

$$\phi_2 = \phi_p \sin(\omega_p t + k_p z + \pi)\, e^{-(y-\Delta)^2/w^2}\,.$$

These represent two waves of width w travelling in the $-\underline{a}_z$ direction separated (radially) by an amount Δ. If the width of the waves (w) is made comparable ot $k_p^{-1} = c/\omega_p$, it is easy to see by differentiating the potentials with respect to y or z that the radial field $E_y = -\partial\phi/\partial y$ will be comparable in magnitude to the longitudinal field $E_z = -\partial\phi/\partial z$[9]. The maximum wiggler field (E_y) between the two waves is approximately $1.6(k_p\phi_p)$ or 1.6 times the maximum longitudinal field of each wave separately and is optimized for $\Delta = \sqrt{2}w$. The phase factor π in ϕ_2 assures that the wiggler fields add rather than cancel on axis ($y = \Delta/2$). The longitudinal field does not play much of a role in this geometry since it is parallel to $\vec{v}_b$. Furthermore, it vanishes on axis.

At this point we comment on the need for two waves, since one wave would appear to suffice to wiggle the electron beam. The reason that one wave does not make a good wiggler is that the electrons would be blown radially outward by the wave's so-called ponderomotive force. This force arises from the transverse gradient of the time averaged longitudinal field (i.e., $F = (-e^2/m\omega_p^2)\partial_y \langle E_z^2\rangle_t$). It can be recognized as the divergence of the electromagnetic stress tensor (also as the radiation pressure of the wave). With two waves present this force cancels on axis and provides a focusing force for particles off axis.

III. PLASMA WIGGLER EXCITATION

The relativistic plasma waves, used for wigglers, can be excited by two schemes known as laser beat wave excitation[3] and wakefield excitation[4]. Both are actively being investigated for accelerator applications in which the electrons are injected in the same direction as the phase velocity of the wave. If instead, the electrons are injected perpendicular to the plasma wave (parallel to the wavefronts), they are wiggled transversely causing them to radiate.

In the beat wave excitation scheme, two laser beams (ω_o, k_o) and (ω_1, k_1) are copropagated into a homogeneous plasma such that their frequency difference is the plasma frequency. Under such resonant excitation, the radiation pressure of the lasers drives up the plasma wave, which eventually saturates due to relativistic effects[4]. The peak amplitude at saturation, assuming a square shaped laser pulse, is given by

$$a_w^{plasma} = \left[\frac{16}{3}\, a_w^o\, a_w^1\right]^{1/3} < 1 \tag{3}$$

where superscripts refer to the plasma wave and the two lasers, and $a_w^j = eE^j/m\omega^j c$ as before. Note that, once again the saturation amplitude is independent of the plasma density. The growth rate of the plasma wave, however, does depend on the plasma density. The excitation of plasma waves by this technique has been demonstrated in a series of experiments at UCLA[6]. Using a CO_2 laser, with a modest intensity of 2×10^{13} W/cm^2, operating on 10.6 μm and 9.6 μm lines, a plasma wave with $a_w \approx 0.03$–0.1 was excited in a 10^{17} cm^{-3} density plasma. This corresponds to a 105 μm period wiggler with an equivalent magnetic field of ≈ 32 – 95 kG. Using a more intense laser should allow even higher wiggler strengths to be generated.

For scheme 1 we need a wide plasma wave which has E_z nearly constant over 100 or so wiggle periods. The actual length of the wave (in the direction of the driving lasers) need only be 2 or 3 wavelengths with the central potential trough used for wiggling the electron beam. The laser beam pump depletion length is given by[7].

$$L_d \ge \frac{\omega_o^2}{\omega_p^2}\,\frac{c}{\omega_p}\,\frac{4}{a_w^{plasma}}$$

This assumes that the maximum effective length of the laser pulse is determined by relativistic detuning[4]. For typical parameters, $\omega_o/\omega_p \ge 10$ and $a_w^{plasma} \approx 0.5$, L_d can be hundreds of plasma wavelengths long. This is appropriate for scheme number 2 $(\vec{v}_b \,||\, -\vec{k}_p)$. By using small F number cylindrical focus optics which produce a line focus that is greater than 100 λ_w wide but only has a Rayleigh length of about 3 λ_w, it may be possible to create the geometry appropriate for scheme number 1 $(\vec{v}_b \perp \vec{k}_p)$. 2-D simulations for beat wave excitation have been presented elsewhere[3,7].

We now discuss an alternate method of excitation of the plasma wiggler which uses a high-current but low-voltage electron beam as the free energy source. This is known as the wake field excitation scheme. To distinguish this electron beam from the radiating beam we shall call this the driver bunch (or bunches). As shown by Chen et al.[4], when such a driver bunch is shot through a high density plasma, the space charge force of the bunch displaces the plasma electrons and leaves behind a wake of plasma oscillations. The phase velocity of the wake, like that of a wake behind a boat, is tied to the velocity of the driving bunch, which is almost c. Once excited, the plasma wave in this scheme is exactly the same as in the laser beat wave scheme. In order to obtain a large a_w, one has to use a dense driving bunch that is shorter than λ_p (say $\pi c/\omega_p$) or is ramped and cut off sharply. For a_w of up to 0.5, provided that the above conditions are met, then $a_w \approx n_b/n_o$ where n_b is the driving beam density. The required beam current is $I_b\,(\text{amps}) = 10^3 N(n_b/n_o)$ if the driving bunch is assumed to be like a football, Nc/ω_p wide in one dimension and c/ω_p wide in the other.

As an example consider a plasma wake field wiggler excited by a 5 ps long bunch from a linac. The wiggler wavelength and strength turn out to be 3 mm and 15 kG, respectively, if the plasma density is 10^{14} cm^{-3}. To excite a wave $1c/\omega_p$ wide (appropriate for scheme 2) requires 500 A current.

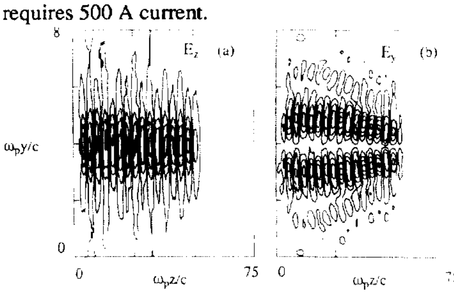


Fig. 3
Wakefield excited plasma wiggler.

Two-dimensional simulations of the wake-field excitation of a plasma wave wiggler have been carried out[9]. In Fig. 3 contour plots of constant E_z and E_y are shown for a 10 MeV driving beam moving in the z-direction. The driving beam was gaussian in z of length $\pi c/\omega_p$ and square in y of width $2c/\omega_p$. Note that the radial field (E_y) vanishes on axis as expected (e.g., by differentiating one of Eqs. (3)). Fig. 4 illustrates the real space trajectories of counterstreaming test particles (γ = 1.5) in the single driving beam wake-fields of Fig. 3. The ponderomotive blowout described in section IIB is clearly visible.

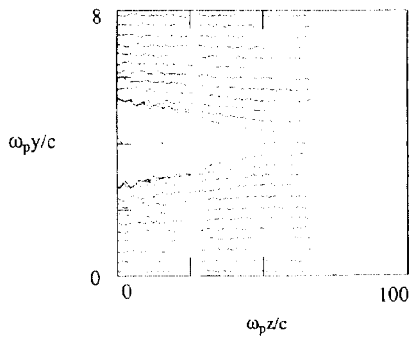
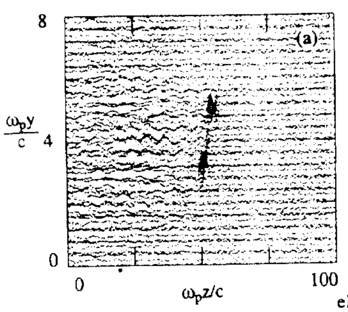


Fig. 4. Ponderomotive blowout of test particles moving right to left in the fields of Fig. 3.

Figure 5 illustrates the double plasma wave wiggler solution to the ponderomotive problem (scheme 2). Fig. 5a shows location of the two driving beams and Fig. 5b and 5c show slice plots of the resulting wiggler field (E_y) and longitudinal field (E_z). Note that E_y is maximum down the axis of symmetry and that $-\partial_y E_z^2$ provides a focussing ponderomotive force. Fig. 6 shows the phase space and real space trajectories of test particles in the fields of Fig. 5. Particles in the central region are clearly focussed to the axis and wiggled.



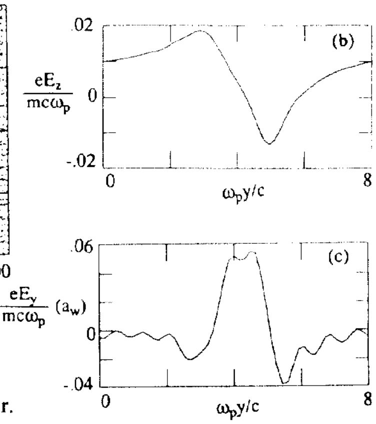


Fig. 5. Double plasma wave wiggler.

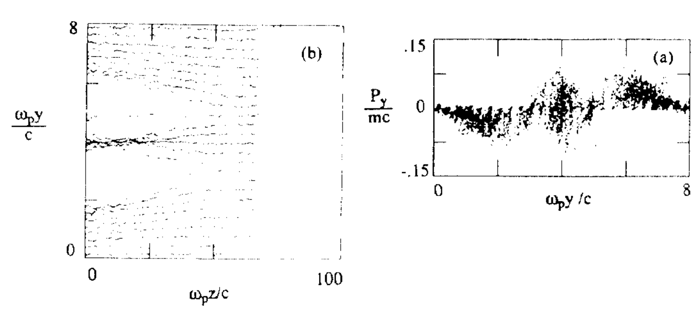


Fig. 6. Phase space and real space trajectories in the double plasma wave wigglers.

Detailed calculations and simulations of the plasma wiggler FEL which take into account the variation in a_w across the beam cross section, edge effects, beam energy spread, emittance and plasma wave harmonics are being carried out presently.

IV. PROSPECTS

In this paper we have described two concepts for using plasma waves as wigglers, which simultaneously offer the prospect of a small effective wiggler wavelength and a high wiggler strength. We have described how such wigglers may be produced and modelled the trajectories of test particles in the wigglers with computer simulations. There is a considerable on-going effort on generating coherent relativistic plasma wave wavefronts for high energy accelerator applications A natural extension of this work will lead to characterization of these waves for wiggler application. In this paper the effect of the plasma medium on the FEL process has been neglected. It was shown in Refs. 2 and 11 that plasma instabilities provide a limit on the effective length of the FEL beam and that plasma wakefields may induce unacceptable energy spread on the beam unless beam shaping or precursor beams are employed. Extremely high quality electron beams are required for FEL action in the visible or VUV region using any kind of wiggler[10], including a plasma wave wiggler. Finally, although lasing may not be possible at even shorter wavelengths than this, the strong a_w and λ_w of a plasma wiggler may enable significant spontaneous emission (say of γ rays) to be obtained by using a modest energy electron beam (such as that from a compact GeV storage ring).

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