In order for oscillation to occur, the power gained by the electromagnetic wave in one pass through the wiggler must exceed the sum of all power losses in the electron beam, the device is an oscillator. If the beam current is high enough, the spontaneous emission can cause bunching and thus enhanced emission in a single pass, which is called superradiance. Although all three types of FELs are possible in principle with RF linac drivers, the power source for high energy, high-gradient accelerators can be much higher than other radiation sources.

Until recently, FEL development focused on understanding the physics of the device itself and improving its performance. RF linacs, including related accelerator types such as recirculating linacs and microtrons, are superior in many respects to other beam sources for FELs. A unique feature of RF linacs is the time structure of the electron beam — a train of pulses of a few picoseconds duration. The FEL radiation will preserve the time structure, which is ideally suited for "pump-probe" type experiments.

Elements of FEL Physics

Energy is extracted from the electron beam in an FEL by the oscillation of the beam in a periodic magnetic field, as indicated in Fig. 1. In a planar wiggler (which is the only type we will consider), the magnetic field is of the form

\[
B_x = B_0 \cos \frac{2\pi x}{\lambda_w} \sin \frac{2\pi y}{\lambda_w} \\
B_y = B_0 \sin \frac{2\pi x}{\lambda_w} \cos \frac{2\pi y}{\lambda_w} \\
B_z = 0
\]

where \(\lambda_w\) is the wiggler period. Electrons traversing in this field in the midplane \((y = 0)\), moving in the \(x\) direction, will oscillate in the \(x\) direction, causing the emission of radiation, predominantly in the \(z\) direction. The radiation emitted by one electron during successive periods of the oscillatory motion will be coherent at the optical wavelength \(\lambda\) when the transit time, \(\tau\), of the electron in traversing a distance \(\lambda_w\) in the \(z\) direction is \(\tau = (\lambda_w + n\lambda)/c\), where \(n\) is an integer. For \(n = 1\), this leads directly to the FEL fundamental resonance condition

\[
\lambda = \frac{\lambda_w}{2} \left( 1 + k^2 \right)
\]

where \(\gamma\) is the electron total energy in units of rest energy \((mc^2)\), and

\[
k = \frac{|e| B_0 \lambda_w}{2\sqrt{2} \pi mc^2} = 660B_0(T)/\lambda \omega_m
\]
Beam Quality Effects

Performance of an FEL is critically dependent on the quality of the electron beam. The most important accelerator parameters are energy spread and energy fluctuations, emittance, and timing jitter.

As previously mentioned, the beam-energy spread, Δε/γ, should be small compared to 1/2N. Actually, it is the longitudinal component of the velocity which enters into the resonance condition. However, it has been shown that the transverse emittance produces a change in the longitudinal velocity and thus in the "longitudinal energy," approximately given by

\[ (\Delta \epsilon / \gamma)_L = \epsilon_L \delta r^2 / 2, \]

where \( \epsilon_L \) is the electron beam radius (in the wiggler) and \( \delta r^2 \) is the normalized envelope emittance of the beam. A second effect on the longitudinal energy arises from the small beam radius, \( \epsilon_L \), term in the wiggler field, \( B_\perp \), given in equation (1), and the finite y dimension of the beam. It is

\[ (\Delta \epsilon / \gamma)_y = (y_{\text{max}} / 2) (\sigma_y / \lambda_y)^2 K^2 / (1 + K^2). \]

where \( y_{\text{max}} \) is one half of the total y-dimension size of the beam. Another effective energy spread arises from the beams electromagnetic self-energy which increases as the beam bunches (at the optical wavelength). It is given by

\[ (\Delta \epsilon / \gamma)_S = \epsilon_S / \gamma_A. \]

This effect is important in the high-current, high-gain regime for relatively low-energy beams. The effective energy spreads given by equations (7), (8), and (9), plus the real energy spread, all added in quadrature, should be small compared to 1/2N.

Emittance affects the FEL performance by a direct transverse effect in addition to the longitudinal energy-spread effect. The small signal gain, equation (4), is valid when the electron beam is much smaller than the optical mode size in the wiggler. If this is not true, the gain is decreased by a factor which, to lowest order of approximation, is \( 1 / [1 - (\epsilon / \gamma_A)^2] \), where \( \epsilon \) is the mean square radius of the transverse mode, averaged over the length of the wiggler.

Timing jitter is important because of the typically short pulse lengths (10-100 ps). In the absence of jitter, the optical cavity length must be precisely adjusted to the optical length of the pulse. However, the electron pulse will lose synchronism with the wiggler if the overlap is perfect at the wiggler entrance, the pulses will no longer overlap when \( \tau = N \pi / c \), where \( \tau \) is the electron pulse length. This condition is an important limitation on wiggler length for long-wavelength FELs. Clearly, any timing jitter between electron beams will make this problem worse. The timing jitter must be substantially less than \( \tau \). Since the beam current is inversely proportional to \( \tau \), small \( \tau \) is often required, and timing jitter must therefore be very small. One potentially-important source of timing jitter can arise from pulse-to-pulse energy fluctuations, if the transport system from the accelerator to the FEL is non-isochronous.

The fractional energy width of the gain curve is \( \sim 1/2N \). When an electron has lost \( \sim 1/2N \) of its initial energy it will no longer contribute to the gain. Thus, the extraction efficiency, \( \eta \), for a cold beam FEL with a uniform wiggler is \( 1/2N \). If electrical efficiency is important, the wiggler can be tapered by varying \( \lambda_y \) or \( \epsilon_L \) to maintain the resonance condition, equation (2), as the beam loses energy. However, the taper will reduce the small signal gain and may prevent oscillation. To obtain good extraction efficiency and high gain simultaneously, high beam current is essential.

The power in the outcoupled radiation beam is given by

\[ P_0 = P_e k \sqrt{k+2A}, \]

where \( P_e \) is the electron beam power, \( k \) the outcoupling factor (typically a few percent), and \( A \) the power absorption coefficient of each of the two (assumed identical) cavity mirrors. In equation (5) we ignore diffraction losses and other normally small, effects. The power absorbed on each mirror is

\[ P_A = P_e A \epsilon / (k+2A). \]

Mirror damage due to either the peak or average absorbed power is a serious concern in high powered FELs. For a given output power, the power absorbed in the mirror will be minimized by making \( k/2A \) as large as possible. Since \( k + 2A < G \) is required to sustain oscillation, mirror damage is reduced (for a given \( P_e \)) by having high gain and high outcoupling. This consideration also demands high beam current.

The tradeoff between gain and efficiency can be avoided with a CW beam, which can be obtained from a superconducting linac or a racetrack microtron. The oscillator starts with a uniform wiggler, and when saturation is achieved, the wiggler taper is adjusted while the FEL is lasing, until the extraction efficiency is maximized. The wiggler is tapered in practice by adjusting the gap dimension (for a permanent magnet or hybrid wiggler), or by adjusting the exciting current of an electromagnetic wiggler.

one round trip through the optical cavity. The losses are due to imperfect reflection from the mirrors, diffraction, and the power coupled out for some, presumably useful, purpose. Depending on the operating wavelength (and beam the availability of mirrors), and the outcoupling desired, gains as small as 10^-3 can result in successful oscillation, but gains of at least a few percent are certainly preferred. The small signal gain, in the low gain limit, is given by

\[ G = \frac{[J_0(b)^2 - J_1(b)^2]}{1 / 2 \epsilon_0 (1/4 A)^{1/2} \lambda_y K^2 N^2}. \]

Here \( J_0 \) and \( J_1 \) are bessel functions of argument \( b = [K^2/(1+K^2)] \), \( \epsilon_0 \) is the root-mean-square radius of the electromagnetic wave in the wiggler. \( \lambda_y \) is the instantaneous beam current, and \( I_A = 17000 \) Amperes is the Alfv\'en current. Equation (4) is valid for a cold beam: zero energy spread, zero emittance, and size small compared to the optical beam. The importance of these effects will be discussed later.

When an FEL oscillator starts up, the optical wave will grow with successive passes until strong bunching of the beam occurs. The gain will then decrease until the power gained from the beam exactly balances the power loss. The system is then in a saturated equilibrium. The build-up time to saturation is typically some microseconds so that short pulse linacs are not suitable FEL drivers.

The power absorbed on each mirror is

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where \( P_e \) is the electron beam power, \( k \) the outcoupling factor (typically a few percent), and \( A \) the power absorption coefficient of each of the two (assumed identical) cavity mirrors. In equation (5) we ignore diffraction losses and other normally small, effects. The power absorbed on each mirror is

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PAC 1987
Linacs for FELs

Many of the performance criteria for accelerators for use with FELs are similar to requirements for other uses. Among the most stringent requirements are those which are nearly unique for FEL applications, such as high peak current and small emittance. The only other type of linac that has similarly stringent requirements on peak current and emittance is the linear collider. In this application, only single micropulses are needed, whereas the FEL requires that the micropulse repetition period be equal to (or a multiple of) the round-trip time of the optical cavity. Typically, this corresponds to a micropulse frequency in the range of 10-100 MHz. The FEL can operate with every RF bucket of the linac filled, but because of the high-peak current requirement, this can lead to an unnecessarily high-average current and beam power. Thus, most FEL-driver linacs operate with a subharmonic injector filling only a small fraction of the RF buckets. Long macro-pulses (tens of microseconds or more) are needed because of the finite start-up time of the FEL oscillator, so that if every bucket is filled, the macro-pulse repetition rate should be quite low to keep the average power reasonable.

A sampling of RF accelerators used as FEL drivers is given in Table I. These include conventional pulsed RF linacs at Boeing (Seattle), Los Alamos, and Stanford University, as well as the superconducting recyclertron at Stanford, the pulsed microtron at Bell Labs, and the CW racetrack microtron at NBS. Table II gives parameters of the FELs used with the accelerators listed in Table I.

In order to obtain uniform brightness, the cathode must be of high-brightness, and care must be taken to control any effects which tend to increase the emittance. The normalized RMS emittance of a beam is defined to be

$$\epsilon_{\text{RMS}} = \left( \langle x^2 \rangle - \langle x \rangle^2 \right)^{1/2}$$

where $x$ is the coordinate of a particle in the beam, $p_x$ is the particle's momentum component in the $x$ direction, and $\langle \rangle$ indicates averaging over the entire beam. It can easily be shown that for a uniform circular cathode of radius $R$

$$\epsilon_{\text{RMS}} = \frac{R (kT/mc^2)^{1/2}}{2}$$

where the transverse temperature of the beam is defined to be

$$kT = \langle p_x^2 + p_y^2 \rangle/m$$

In some laboratories it is conventional to use the envelope emittance of the beam. For the $K-V$ distribution, the envelope emittance is $\epsilon_N = 4 \epsilon_{\text{RMS}}$. For a phase-space distribution which is Gaussian in all coordinates, $x, p_x, y, p_y$, $\epsilon_{\text{RMS}}$ is the "20" emittance which contains 91% of the total current if the measurement of, say, the $x,p_x$ plane phase space includes particles with all values in the $y,p_y$ plane. With this definition,

### Table I Accelerator Parameters

<table>
<thead>
<tr>
<th>Laboratory</th>
<th>Boeing</th>
<th>Los Alamos</th>
<th>Stanford &quot;Mk III&quot;</th>
<th>Superconducting</th>
<th>Pulsed microtron</th>
<th>CW RTM</th>
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<td>SW linac</td>
<td>TW linac</td>
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<td>9</td>
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*The NBS and Boeing numbers are design values. All others are measured except parenthetic entries for the superconducting recyclertron, which are design values for December 1987 upgrade.

### Table II FEL Parameters

<table>
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<tr>
<th>Laboratory</th>
<th>Boeing</th>
<th>Los Alamos</th>
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<th>Superconducting</th>
<th>Pulsed microtron</th>
<th>CW RTM</th>
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<td>operation</td>
<td>commissioning</td>
</tr>
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</table>
where \( I_0 \) is the cathode current and \( J \) the cathode current density. To obtain good emittance, a low-temperature, high-current density cathode is needed.

Most RF accelerators use thermionic cathodes with \( J < 40 \text{ A/cm}^2 \) and \( kT \ll 1 \text{ eV} \), leading to

\[
\varepsilon_H = 2 \left( \frac{kT}{mc^2} \right)^\frac{1}{2}
\]

and

\[
\varepsilon_R = 2 \left( \frac{I_0}{nJ} \right)^\frac{1}{2} \left( \frac{kT}{mc^2} \right)^\frac{1}{2}
\]

where \( \varepsilon_H \) and \( \varepsilon_R \) are the harmonic and the usual buncher emittances, respectively.

When a high-current, low-emittance pulse has been launched from the cathode, the design of the early stages of acceleration are crucial in preserving the beam quality. There is a strong interplay between transverse and longitudinal effects, especially at low energies when space charge forces are most important.

These effects cannot be compensated in any straightforward way and thus result in increased energy spread and emittance. The wakefield effect can be reduced by lowering the accelerating frequency, thus increasing the aperture of the structure and decreasing the number of RF cavities. Future linacs for high-gain FELs will probably be operated at RF frequencies around 500 MHz, instead of the L-band (1300 MHz) and S-band (3000 MHz) frequencies currently in favor. Superconducting accelerators may prove to be of value for wakefield reduction because, at a given RF frequency, the aperture of a superconducting cavity can be much larger than a normal-conducting cavity without seriously decreasing the performance of the system.

The optimization of injected beam quality results in a pulse length which does not yield the desired peak current, further bunch compression can be obtained after acceleration (to some intermediate energy or the final energy). Magnetic bunch compression requires a linear phase-energy correlation in the bunch, which can easily be achieved by phasing of the accelerator. The amount of energy spread needed to achieve bunching is small compared to the energy spread requirement of the FEL.
BBU is a major problem when the average current is high. The most important form of BBU for FEL drivers is the steady-state cumulative effect due to the ability of the accelerating structure to support higher order RF modes (e.g., TM_{110}) which may deflect the beam even if it is centered in the structure, and can extract energy from the beam when it is off center. In a long machine, the amplitude of the transverse motion will grow exponentially with distance. The exponent is proportional to the macropulse-average beam current and the coupling impedance of the mode, and inversely proportional to the accelerating gradient. Most RF structures have many modes of this type. Each micropulse will encounter the deflecting forces at different phases and thus the time-averaged emittance will increase. In extreme cases, part of the beam is lost to the walls of the structure. There is also a smaller effect within each micropulse due to the finite bunch length and the fact that the blowup mode frequencies are usually higher than the accelerating frequency. The BBU effect can be reduced by increasing the accelerating gradient (which may be impractical for other reasons), reducing the coupling impedances of the most important modes, or "stagger tuning" the modes. Stagger tuning has been applied successfully to normal conducting structures. The idea is to distribute the blowup mode frequencies of the accelerating cells over a fractional frequency range large compared to 1/Q. This can be done without changing the accelerating frequency, or significantly affecting other important properties of the structure.

In superconducting structures, coupling impedances of blowup modes have been reduced by several orders of magnitude by a combination of cavity shaping and the use of higher-order-mode couplers which remove power from the system only at frequencies above the accelerating frequency.\(^1\) It is not clear to what extent this technique may be applicable to normal-conducting structures.

BBU is a much more serious problem in recirculating accelerators than in one-pass linacs because the accelerated beam communicates with the injected beam through the re-entered RF structure. If all of the BBU reduction techniques available for single pass linacs are employed, one can still expect the maximum current of a recirculating accelerator to be reduced by a factor approximately equal to the number of passes of the beam through the common structure. Thus, recirculating accelerators are not suitable for high average-power operation. (This is of course, a relative conclusion — the NBS RTM should be capable of electron beam power in excess of 100 kW at energies up to 105 MeV.)

Beam energy recovery has been discussed as a means of increasing the overall electrical efficiency of an FEL system. Energy recovery is essential for FELs driven by electrostatic accelerators, because of the limited charging current of this type of machine. At Los Alamos, energy recovery while lasing has been demonstrated in an RF system.\(^1\) Two separate structures are used for accelerating and decelerating the beam, primarily to reduce the possibility of BBU. Energy recovery directly in the primary accelerating structure may be the best approach to achieving high power FELs using superconducting RF accelerators because of the difficulty of introducing large amounts of RF power to superconducting cavities from external sources. For normal conducting systems the value of energy recovery appears to be primarily a cost issue. At NBS, we have looked at the possibility of beam energy recovery in an RTM-driven FEL. It does not appear practical because the admittance of the RTM in longitudinal phase space is too small to permit efficient recovery of a beam where energy spread has been increased by the FEL interaction. Furthermore, the BBU current limit would be lower than for the no-recovery case.

The RMS radius of the optical mode in the FEL optical cavity is given by

\[ q_R(z) = a_0 [1 + (z/z_0)^2]^{\frac{1}{2}} \]  

where the "waist size," \( a_0 \), is the minimum value of \( q_R \). The "Rayleigh length," \( z_0 \), defined to be the distance from the waist at which the optical mode area doubles, is given by

\[ z_0 = \frac{a_0^2}{\lambda} \]  

Note that equations (14) and (15) would describe the RMS radius of an electron beam of (unnormalized) RMS emittance \( \epsilon_{RMS} = \lambda/\pi \).

The small signal gain, equation (4), will be maximized when \( \omega_s^2 \), averaged over the length, \( l \), of the wiggler is minimized. For the low-gain regime, the beam waist, \( z = 0 \), is located at the center of the wiggler, and the Rayleigh length is set to \( z_0 = l/2 \). It is easy to show that the focal length of the cavity mirror needed to achieve this condition in the symmetric case is:

\[ f = \frac{z_m^2}{2} \]  

where \( z_m \) is the distance from the waist to the mirror (equal to one-half the optical cavity length).

In the absence of focusing by the wiggler field, the electron beam could be confined within the optical mode by quadrupole focusing located in front of the wiggler, producing an electron beam waist at the wiggler center, provided that \( \epsilon_{RMS} < \lambda/\pi \). However, the \( B_2 \) component of the wiggler field, equation (1), provides focusing in the y direction. In the usual case \( 2\pi/\lambda > 1, \), \( 2\pi/x < 1 \), the linearized vertical motion is given by

\[ \begin{bmatrix} y \\ y' \end{bmatrix} = \begin{bmatrix} \cos(z/\delta_c) \\ -\frac{L}{\delta_c} \sin(z/\delta_c) \end{bmatrix} \begin{bmatrix} \delta_c \sin(z/\delta_c) \\ \cos(z/\delta_c) \end{bmatrix} \begin{bmatrix} y \\ y' \end{bmatrix} \]  

where \( \delta_c = x\lambda/2\pi \).

The matched y-dimension beam size is given by

\[ y_{max} = \sqrt{\delta_c} = \left( \frac{e^2}{\delta_c} \right) \]  

where we have again taken \( e_0 = \delta_c\epsilon_{RMS} \) and \( y_{max} = 2\sqrt{\gamma} \).

The y-dimension beam size \( y \) is typically of order the wiggler length (exact when \( K = 1, y = N \), since \( N = 1 \)). If the accelerator has different emittances in the two transverse directions, as a microtron typically does, the wiggler should be oriented so that the y direction corresponds to the larger emittance so that the wiggler focusing helps to keep the beam size small in that direction. It is possible to add x-direction focusing to a wiggler, at the expense of y-direction focusing strength, by using "canted poles.\(^{19} \) Another possibility is to provide x-direction focusing by parabolic shaping of the wiggler poles,\(^{20} \) which does not affect first order y-focusing.
At extremely high current, the resonant interaction between the radiation field and the electron beam can result in radiation focusing (optical guiding). If the electron beam centroid is displaced, the radiation field, under certain conditions, will follow and be steered by the electron beam. When the guiding is significant, the optical mode will remain small over many Rayleigh lengths, which can strongly increase the gain in long wigglers.

References


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