Tomography for beams with intense space charge

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Motivation

The requirement: Many applications need well regulated beams with high intensity and low emittance

The challenge: Beams are born at low-energies where space charge forces are an issue

- Can cause emittance growth
- Halo formation possible
- Propagating waves
Approach

- To understand space charge we need an accurate phase space diagnostic.
- Tomography is a good candidate, but to date, has been used only for beams with little space charge.
- This study further develops and uses tomography for beams with intense space charge.
Outline

1. Example
2. History/Overview
3. Extension to Beams with Space Charge
4. Simulation/Validation of Tomography
5. Experimental Results
Motivation Example

- Multi-Beamlet Merger

Haber, Kehne, Reiser and Rudd, Physical Review A (1991)

- What happens in phase space?
Importance of Phase Space

- Initial distribution
- Downstream

Simulation

- Homogenization of a beam is different in configuration and phase space
Importance of Phase Space

- Initial distribution
- Downstream

- Homogenization of a beam is different in configuration and phase space
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Simulation

Experiment

• Homogenization of a beam is different in configuration and phase space
Tomography (CAT Scan)

- Tomography is the technique of reconstructing an image from its projections.

http://www.sv.vt.edu/
http://universe-review.ca
Tomography Algorithm

Fourier Slice Theorem

Fourier transform of a parallel projection is equal to a slice of the two-dimensional Fourier transform of the original object.

Kak and Slaney, Principles of Computerized Tomographic Imaging (1988)
Tomographic Examples

- Multi-screen method
  - Sander et al. 1979
  - Minerbo et al. 1981
  - Honkavaara et. al. 2005
  - Holder et. al. 2006
- Multi-turn
  - Hancock et al. 1999
  - Connolly et al. 2000
- Multi-slit
  - Raparia et. al. 1997
  - Adachi et. al. 1998
  - Anderson et. al. 2002
  - Friedman et. al. 2004
- Cherenkov Radiation
  - Chen et al. 2003
- Quad-scan Method
  - Fraser et al. 1979
  - McKee et al. 1995
  - Sawamura et al. 1998
  - Geitz et al. 1999
  - Brunken et al. 2000
  - Yakimenko et. al. 2003
  - Montag et. al. 2004
  - Ohgaki et. al. 2004
  - Li, PhD Dis. et al. 2004
  - Zhou et al. 2006
Quad-Scan Tomography

Quadrupole Lens  Screen

- Quadrupoles rotate the phase space distribution

- Single particle: \( x'' = -\kappa x + F_{sc} \)

- No SC: \( x'' = -\kappa x \)

\[
\begin{pmatrix}
    x \\
    x'
\end{pmatrix} =
\begin{pmatrix}
    \cos \sqrt{\kappa_x} z & \frac{1}{\sqrt{\kappa_x}} \sin \sqrt{\kappa_x} z \\
    -\sqrt{\kappa_x} \sin \sqrt{\kappa_x} z & \cos \sqrt{\kappa_x} z
\end{pmatrix}
\begin{pmatrix}
    x_0 \\
    x'_0
\end{pmatrix}
\]

\[
\begin{pmatrix}
    x \\
    x'
\end{pmatrix} =
\begin{pmatrix}
    \cos \theta & \sin \theta \\
    -\sin \theta & \cos \theta
\end{pmatrix}
\begin{pmatrix}
    x_0 \\
    x'_0
\end{pmatrix}
\]

- With SC: Very complicated! Need approximations

\[
\kappa = \frac{qB}{\gamma m a v}
\]
Beam Tomography with space charge

- Single particle equation:
  \[ x'' = -\kappa x + F_{SC} \]

- Assume linear forces:
  \[ x'' = -(\kappa_{x,0} - \frac{2K}{X(X+Y)})x \]

- Find X, Y by solving envelope equations:
  \[ X'' + \kappa_x X - \frac{2K}{X+Y} - \frac{\epsilon_x^2}{Y^3} = 0 \]
  \[ Y'' + \kappa_y Y - \frac{2K}{X+Y} - \frac{\epsilon_y^2}{Y^3} = 0 \]

- Get transport matrix

Geitz et al. PAC 1999
- UMERA is serving as a low-cost model of high intensity accelerators

\[ K = \frac{qI}{2\pi \varepsilon_0 m(c\beta\gamma)^3} \]

\[ \chi = \frac{1}{1 + \frac{\beta\gamma I_0}{2I} \left( \frac{\varepsilon_n}{a^2} \right)} \]

Kishek (TUZBAB03) – talk
Walter (TUPAS047), Bernal (THPAS030) - posters

<table>
<thead>
<tr>
<th>Energy</th>
<th>10 keV</th>
</tr>
</thead>
<tbody>
<tr>
<td>Current range</td>
<td>0.6-100 mA</td>
</tr>
<tr>
<td>rms Emittance, norm</td>
<td>0.2-3 ( \mu )m</td>
</tr>
<tr>
<td>Zero-Current Tune</td>
<td>7.6</td>
</tr>
<tr>
<td>Depressed Tune</td>
<td>1.5 – 6.5</td>
</tr>
</tbody>
</table>
• Four magnets where employed for the tomographic recovery of the phase space
Tomography Simulation/ Validation

- Reconstructed phase space by Tomography is compared to that generated directly by WARP.

Non-uniform spatial distribution

Experiments with Intense Beams

- **Experiment 1:**
  Uniform beam evolution (19mA, $\chi=0.85$).

- **Experiment 2:**
  Nonuniform beam evolution (26mA, $\chi=0.91$).
Experiment 1: Single Beamlet Transport

XX’ Reconstruction

IC2 0.76m  RC3 3.16m  RC6 5.08m  RC7 5.72m  RC9 7.0m  RC13 9.56m

10 mrad  10 mm

Haber (THPAS031) - poster
Experiment 1: Single Beamlet Transport

YY’ Reconstruction

- Space Charge Dominated Beam (19mA, $\chi=0.85$)
Experiment 2: Multibeamlet Transport

XX' Reconstruction

IC2

RC3

RC6

RC7

RC13

y

x

x'

10 mrad

10 mm

Simulation

E. Gun

IC2

Injection

RC3

RC13

RC6

RC7
Experiment 2: Multibeamlet Transport

Experiment

YY' Reconstruction

Simulation
Conclusions

- Extended Tomography to beams with space charge
- Simulation validated accuracy of technique
- Experimental measurements reveal evolution of beam halo and multi-beamlet merger

Acknowledgments

Not shown (but thanks, too):

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- Dr. D. Grote
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- Dr. V. Yakimenko
- Dr. H. Li

UMER group
- Dr. D. Sutter
- K. Tian
- B. Beaudoin
- M. Holloway
- C. Wu
Backup

25 mm

UMER
E-Gun
Filtered Backprojection Algorithm (FBA)

- A simple weighting in the frequency domain is used to take is projection and estimate a pie-shaped wedge of the object’s Fourier transform.
- We multiply the value of the Fourier transform of the projection and multiply it by the width of the wedge at that frequency
- Apply inverse Fourier Transform of the filtered projections
## Quad Scan Tomography

<table>
<thead>
<tr>
<th>Article</th>
<th>Beam/Facility</th>
<th>Energy / Current</th>
<th>G. Perveance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Brunken et al. 2000</td>
<td>S-DALINAC</td>
<td>8MeV/ 10^{-6} A</td>
<td>2.5 10^{-14}</td>
</tr>
<tr>
<td>McKee et al. 1995</td>
<td>Duke</td>
<td>44MeV / 0.2A</td>
<td>3.5 10^{-11}</td>
</tr>
<tr>
<td>Ohgaki et al. 2004</td>
<td>KU-FEL</td>
<td>10 MeV/ 0.1A</td>
<td>1.3 10^{-9}</td>
</tr>
<tr>
<td>Yakimenko et al. 2003</td>
<td>Brookhaven</td>
<td>50 MeV/ 100A</td>
<td>1.2 10^{-8}</td>
</tr>
<tr>
<td>Zhou et al. 2006</td>
<td>Brookhaven</td>
<td>60 MeV/ 266A</td>
<td>1.9 10^{-8}</td>
</tr>
<tr>
<td>Montag et al. 2004</td>
<td>RHIC</td>
<td>54 MeV/ 330A</td>
<td>3.2 10^{-8}</td>
</tr>
<tr>
<td>Sawamura et al. 1998</td>
<td>JAERI</td>
<td>16 MeV/ 100A</td>
<td>3.4 10^{-8}</td>
</tr>
<tr>
<td>Geitz et al. 1999</td>
<td>TeslaTF</td>
<td>16 MeV/ 200A</td>
<td>7.0 10^{-7}</td>
</tr>
<tr>
<td>Fraser et al. 1979</td>
<td>LAMPF</td>
<td>0.75-100MeV/ 18mA</td>
<td>10^{-5} -10^{-8}</td>
</tr>
<tr>
<td>Li H. PhD Dis. 2004</td>
<td>UMER</td>
<td>10 keV/ 0.007A</td>
<td>1.0 10^{-4}</td>
</tr>
</tbody>
</table>

For UMER: G. Perveance 10^{-6} to 10^{-3}
Beam Envelope from G000

Q1 = -1.67A  
Q2 = -1.49A  
Q3 =  3.5A