Stochastic Cooling of High Energy Bunched Beams
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- History
- The RHIC System
- Results for Longitudinal Cooling and Comparison with Simulations
- Plans for Transverse Cooling
History

Herr and Mohl reported cooling bunched beams in ICE (1978)
Chattopadhyay develops bunched beam cooling theory (1983)

\[ \theta - \omega_0 t = \varphi(t) \approx a \sin[\omega_s(a)t + \psi_0] \]

Stochastic cooling considered for SPS, RHIC and Tevatron (80s).
Unexpected RF activity swamps the Schottky signal (85s).
Cooling rate scales as 1/N, Z=79 for Au
Cooling of long bunches in FNAL recycler.
Operational cooling of gold in RHIC (2007).
RF Activity (anomalous high frequency power)

Two distinct types:
1) Strong revolution lines
2) Strong signals associated with synchrotron motion

Heavy ions are “rebucketed” to shorten the bunch and combat IBS

\[ h = 360 \rightarrow h = 2520 = 7 \times 360 \]
Low Level Drive for Halo Cooling

For cooling we need a force that reduces the energy error. The lattice mandates filter cooling. Delay is 2/3 of a turn so we can run fiber-optic in the tunnel.

\[ S(\omega) = G[1 - \exp(i\Delta \omega T_{\text{rev}})]^n \exp(i\Delta \omega T_{\text{delay}}) \]

Au, 100GeV/A

Stochastic Confinement

![Graph showing cooling force vs. frequency difference](image)
Signal Suppression

current at pickup due to voltage at kicker

\[ I_1(\omega) = B(\omega)V_K(\omega) \propto qN \]

total current at pickup

\[ I_P = I_1 + I_S, \quad I_S \propto q\sqrt{N} \]

voltage at kicker due to current at pickup

\[ V_K = -I_P Z_T \]
\[ = -(I_1 + I_S)Z_T \]
\[ = -B V_K Z_T - I_S Z_T \]

net voltage at kicker due to Schottky current

\[ V_K = \frac{-I_S Z_T}{1 + BZ_T} \equiv -Z_D I_S \]

Ip is suppressed by the same factor.

Optimal cooling gain for

\[ BZ_T \approx 1 \]
Voltage considerations

For 5-8 GHz need 3 kV rms which is large by stochastic cooling standards
Bandwidth-Voltage product sets the cost scale
Bunches are 5 ns long spaced by 100 ns
The value of the kicker voltage matters only when the bunch is present!!!
Bunched Beam Simulations (thpas090 for algorithms)

Time domain model of filter cooling.
Very similar to coherent stability problem.

For cooling

\[ T_{cool} \approx 2N_{true} \frac{M}{W} \]

In a Simulation \( N_{macro} \ll N_{true} \)

So,

\[ T_{sim} = \frac{N_{macro}}{N_{true}} T_{cool} \ll T_{cool} \]
Bunched Beam Simulations II

Dealing with intra-beam scattering

1) Start with Piwinski’s formulae
\[
\frac{1}{\sigma_p^2} \frac{d\sigma_p^2}{dt} = \alpha_{p0}
\]

2) Correct for coupling
\[
\alpha_{\perp 0} = (\alpha_x 0 + \alpha_y 0)/2
\]

3) Correct for number of macro-particles
\[
\alpha_{p1} = R \alpha_{p0}
\]

4) Correct for non-gaussian profile
\[
F(t) = I(t) \sigma_t 2 \sqrt{\pi} / Q
\]

5) Langevin kick
\[
\Delta p = \sigma_p \sqrt{\alpha_{p1} T_0 F(t) \chi}
\]
Data vs. Simulation

No cooling
5 hour store

![Graph showing data vs. simulation with no cooling for 5 hours store.](image)
Data vs. Simulation

Gain calibration in the simulation

Figure 3: Measured and simulated signal suppression at 6 GHz. The data are the top two traces and the simulation the bottom 2.
Data vs. Simulation

Cooling ON
Simulation has losses out of rf bucket
Lifetime of cooled beam shows marked increase
Transverse Cooling system

Similar cavities. Low level requires a notch filter (R&D). 40 Watt amplifiers are sufficient. 5-8 GHz keeps aperture reasonable.
Transverse Cooling Simulations

$$H_s(\epsilon, \tau) = \frac{T_0 \eta}{2\beta^2 E_0} \epsilon^2 - \int_0^\tau dt q V_{rf}(t)$$

Check of scaling, no ibs or longitudinal cooling

Figure 5: Transverse cooling rate versus the value of the longitudinal Hamiltonian. Similar results are shown in [6, 7]
An easy system to get

Cooling in both transverse dimensions and slight increase in longitudinal cooling.

Reduced transverse emittance leads to longitudinal growth.
Expected improvement with fixed lattice

Luminosity from central bucket \( Q = I_{peak} \tau_{fwhm} \), only debunching losses

central lumi, constant beta* = 80 cm
Dynamic beta function

![Graph showing luminosity over time for different beam sizes and cooling scenarios.](image)
Improved RF

Increased storage voltage from 3 to 5 MV
Clean rebucketing
This helps but we still get satellites.

Figure 8: Simulated longitudinal profiles over 5 hours with two different transverse cooling gains and 1/6th turn delay. The transverse gain of 0.25 utilized only a single one turn delay in the longitudinal cooling system, while the gain of 0.5 used the same cascaded delays we use now.
So, why the satellites?

Simplest diffusion model.

\[ \frac{\partial F(\varepsilon, \tau, n)}{\partial n} + \frac{\partial H_s}{\partial \varepsilon} \frac{\partial F}{\partial \tau} - \frac{\partial H_s}{\partial \tau} \frac{\partial F}{\partial \varepsilon} = \frac{\partial}{\partial \varepsilon} \left( \frac{\varepsilon F}{T_{cool}} + D \frac{\partial F}{\partial \varepsilon} \right) \]

One constant solution?!

\[ F(\varepsilon, \tau) = F_0 \exp(-H_s(\varepsilon, \tau)/H_0), \quad H_0 \propto DT_{cool} \]

Interplay between the longitudinal potential well and collective effects

central lumi, dynamic beta, good rebucketing, 5MV, 1/6th turn delay

\[ \begin{array}{c}
\text{current (Amps)} \\
\text{time (ns)}
\end{array} \]

\[ \begin{array}{c}
\text{95\% normalized emittance} \\
\text{time (hours)}
\end{array} \]
Conclusions
For ion beams in RHIC
1) Longitudinal stochastic cooling worked.
2) Lifetime was improved.
3) Transverse cooling looks straightforward.
4) Expect a big payoff from transverse cooling if the lattice is OK.
Voltage and Power continued

Take 16 cavities, 5-8 GHz bandwidth 40 Watts/cavity (10 K each) 
R/Q=100Ω, 10 MHz FWHP bandwidth, R ≥ 50 kilo-Ohm

gives 1 to 1.4 kV rms per cavity, or 5.6 kV total

Cavity drive signal needs to be roughly sinusoidal for R (not R/Q) to matter

Suppose $S_0(t)$ is the drive signal for a broad band kicker (like a resistor).

Periodically extend $S(t) = \sum_{k=0}^{N-1} S_0(t - k\tau_b)$

This creates a signal with 10 MHz $(1/T_b)$ wide peaks, spaced by 200 MHz $(1/\tau_b)$.

Split and pass through 100 MHz filters, centered on cavity resonance, before power amps. In this way each amplifier sees a piecewise sinusoidal input.

Combination of transmission lines and fiber optic technology for the delay line (traversal) filter.
Error Limit Simulations

Took conservative errors.
- 2 ps timing error
- 20% amplitude errors
- 2 MHz cavity frequency errors

Desired cooling voltage is modeled as band limited noise.
System is well behaved with these errors.
Only had 5 branches this run.