Absolute Bunch Length Measurements at the ALS by Incoherent Synchrotron Radiation Fluctuation Analysis

Fernando Sannibale

Lawrence Berkeley National Laboratory
Based on the method described in Zolotorev, Stupakov, SLAC-PUB 7132 (1996)

The team for the ALS experiment:
Fernando Sannibale, Max Zolotorev (LBNL), Daniele Filippetto (INFN-LNF), Gennady Stupakov (SLAC).

Acknowledgements:
J. M. Byrd, S. De Santis, J. Frisch, ....
Moving charged particles can radiate photons by synchrotron radiation, Cerenkov radiation, transition radiation, etc. For all such processes, the incoherent component of the radiation is due to the random distribution of the particles along the beam.

Example: "Ideal" coasting beam moving on a circular trajectory with the particles equally separated by a longitudinal distance \( d \):

No synchrotron radiation emission for frequencies with \( \lambda > d \).

The interference between the radiation emitted by the evenly distributed electrons produces a vanishing net electric field.

In a more realistic coasting beam, the particles are randomly distributed causing a small modulation of the beam current. The interference is not fully destructive anymore and the beam radiates also at longer wavelengths.
If the particle turn by turn position along the beam changes (longitudinal dispersion, path length dependence on transverse position),
the current modulation changes and
the radiated energy and its spectrum fluctuate turn by turn.

By averaging over multiple passages, the measured spectrum converges to the characteristic incoherent spectrum of the radiation process under observation.
(synchrotron radiation in the example).

In the case of bunched beams, a strong coherent component at those wavelengths comparable or longer than the bunch length shows up
But the higher frequency part of the spectrum remains unmodified.
The electric field associated with the radiation emitted by the beam at the time $t$ is:

$$E(t) = \sum_{k=1}^{N} e(t - t_k)$$

where $e$ is the electric field of the electromagnetic pulse radiated by a single particle and $t_k$ is the randomly distributed arrival time of the particle (Poisson process).

In the frequency domain:

$$\hat{E}(\omega) = \int_{-\infty}^{\infty} E(t) e^{i \omega t} dt = \hat{e}(\omega) \sum_{k=1}^{N} e^{i \omega t_k}$$

And for the radiated power per passage:

$$P(\omega) \propto |\hat{E}(\omega)|^2 = |\hat{e}(\omega)|^2 \sum_{k,l=1}^{N} e^{i \omega (t_k - t_l)}$$

The previous quantity fluctuates passage to passage, and the average radiated power from a beam with normalized distribution $f(t)$ is:

$$\langle P(\omega) \rangle \propto |\hat{e}(\omega)|^2 \sum_{k,l=1}^{N} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} dt_k dt_l f(t_k) f(t_l) e^{i \omega (t_k - t_l)} = |\hat{e}(\omega)|^2 \left[ N + N(N-1)|\hat{f}(\omega)|^2 \right]$$

Incoherent term  Coherent term
The energy $W$ radiated per passage by incoherent radiation can be obtained by integrating $P$ over $\omega$ neglecting the coherent contribution.

It can be shown that the relative variance for $W$ is given by:

$$
\delta^2 = \frac{\sigma_w^2}{\langle W \rangle^2} = \int_{-\infty}^{\infty} \left| \hat{e}(\omega) \right|^2 \left| \hat{f}(\omega - \omega') \right|^2 d\omega d\omega' \int_{-\infty}^{\infty} \left| \hat{e}(\omega) \right|^2 d\omega
$$

The shape of $e(\omega)$ is defined by the radiation mechanism properties or by the frequency acceptance of the system used for the measure of $\delta$.

If we use a bandpass filter with gaussian transmission curve with rms bandwidth $\sigma_\omega$ and the bunch is gaussian with rms length in time units $\sigma_\tau$, we can integrate the above expression and obtain:

$$
\delta^2 = \frac{1}{\sqrt{1 + 4\sigma_\tau^2 \sigma_\omega^2}}
$$

Possibility of absolute bunch length measurements!
A Simple Physical Interpretation

For $\sigma_\tau >> 1/2\sigma_\omega$

\[ \delta^2 \equiv \frac{1}{2\sigma_\tau \sigma_\omega} \]

When the bandwidth $\sigma_\omega$, is fixed, the uncertainty principle defines the coherence length $\sigma_{tc}$. For the gaussian case:

\[ \sigma_{tc} \sigma_\omega = \frac{1}{2} \]

The electric field of photons radiated within the coherence length $\sigma_{tc}$ and within the bandwidth $\sigma_\omega$ adds coherently. $\sigma_{tc}$ defines a radiation "mode".

\[ \delta^2 \equiv \frac{\sigma_{tc}}{\sigma_\tau} = \frac{1}{M} \]

The previous equation shows that in a bunch of length $\sigma_\tau$, there are $M = \sigma_\tau / \sigma_{tc}$ independent modes radiating simultaneously within the bandwidth $\sigma_\omega$.

Each mode shows 100% intensity fluctuation, and the variance of the combined intensity scales as $1/M$ ($M$ combined Poisson processes).

2007 Particle Accelerator Conference, Albuquerque, NM USA, June 24, 2007
The previous expressions have been obtained for gaussian beams. In the general case:

The filter form factor can be measured but the bunch longitudinal distribution is generally unknown.

The table shows that by using the expression for gaussian beams for different distributions, the consequent error is at the few % level for most cases, as long as the distributions are represented by their rms length and do not include microstructures with characteristic length $\ll \sigma_z$. 

\[ \sigma_z = F_{\text{Filter}} \frac{F_{\text{Dist.}}}{\delta^2} \]

<table>
<thead>
<tr>
<th>Distribution</th>
<th>Form Factor $F_{\text{Dist.}}$</th>
<th>Error Assuming Gaussian</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gaussian</td>
<td>$\frac{1}{2\sqrt{\pi}} \approx 0.2821$</td>
<td>$0.0 %$</td>
</tr>
<tr>
<td>Rectangular</td>
<td>$\frac{1}{2\sqrt{3}} \approx 0.2887$</td>
<td>$-2.3 %$</td>
</tr>
<tr>
<td>Triangular</td>
<td>$\sqrt{\frac{2}{27}} \approx 0.2722$</td>
<td>$+3.6 %$</td>
</tr>
<tr>
<td>Saw-Tooth</td>
<td>$\frac{4}{3\sqrt{18}} \approx 0.3143$</td>
<td>$-10.2 %$</td>
</tr>
</tbody>
</table>
Transverse Beam Size Effects

In the previous derivation, a beam with no transverse size was assumed.

Analogously to the longitudinal case, the finite transverse size introduces additional independently radiating transverse modes \((M_x, M_y)\).

The resulting intensity fluctuation variance becomes:

\[
\delta^2 \approx \frac{1}{M} \times \frac{1}{M_x} \times \frac{1}{M_y}
\]

For example, for the full gaussian case one obtains (with \(\sigma_x\) and \(\sigma_y\) the rms transverse beam sizes):

\[
\delta^2 = \left(1 + \frac{\sigma_r^2}{\sigma_{tc}^2}\right)^{-\frac{1}{2}} \left(1 + \frac{\sigma_x^2}{\sigma_{xc}^2}\right)^{-\frac{1}{2}} \left(1 + \frac{\sigma_y^2}{\sigma_{yc}^2}\right)^{-\frac{1}{2}}
\]

The transverse coherence lengths \(\sigma_{xc}\) and \(\sigma_{yc}\) are defined by the radiation mechanism and include diffraction effects introduced by any limiting apertures.

\(\sigma_{xc}\) and \(\sigma_{yc}\) can be analytically calculated in the simpler cases or numerically evaluated (SRW, ...)

2007 Particle Accelerator Conference, Albuquerque, NM USA, June 24, 2007
BL7.2 collects the synchrotron radiation from a dipole magnet.

The limiting apertures were defined by the beamline acceptance 5.5/2.8 mrad (H/V)

BP filter: gaussian filter 632.8 nm, 1nm FWHM

The signal from the avalanche photo-diode (APD) was amplified and sent to a digital scope for data acquisition and analysis. (LeCroy Wavepro 7300 A)

The setup allowed for comparison with streak camera measurements.
The scope was set to measure the areas of the signal between the points A and B ($S_{AB}$) and between C and D ($S_{CD}$), and their statistical moments.

$S_{AB}$ is proportional to the pulse energy convoluted with some electronic noise. $S_{CD}$ is a measure of such a noise.

$$\Rightarrow \delta_{M}^{2} = \frac{\sigma_{S_{AB}}^{2} - \sigma_{S_{CD}}^{2}}{\langle S_{AB} \rangle - \langle S_{CD} \rangle}$$

A complete 5 ksample measurement required ~ 1 minute
The number of photons impinging on the APD is finite. Additionally, APDs exploit stochastic processes for the photon-to-electron conversion and for the amplification.

All this effects generate extra-fluctuations (shot noise) that need to be accounted.

\[
\sigma^2 = \frac{1}{4\sigma^2} \left[ \left( \delta^2_M - \kappa^2 \right)^{-2} \left( 1 + \frac{\sigma^2}{\sigma^2_{xc}} \right)^{-1} \left( 1 + \frac{\sigma^2_y}{\sigma^2_{yc}} \right)^{-1} - 1 \right]
\]

The term \( \kappa \) represents the total photon shot noise and accounts for all the terms above mentioned.

\( \kappa \) needs to be measured once forever, and this can be easily done by performing 2 or more measurements of \( \delta^2_M \) for the same bunch length for different number of photons impinging on the APD (using neutral density filters for instance).
Remarkably good agreement with streak-camera data.
No parameter has been adjusted to fit the data.

It can be shown that the statistical contribution to the error is given by (~ 2% with 5 ksamples):

\[ \frac{\Delta \sigma_\tau}{\sigma_\tau} = \sqrt{\frac{2}{N_{\text{Samples}}}} \]

The typical rms difference between the streak camera and the fluctuation data was ~ 4 %. The extra error is probably associated with the shot noise term that in our measurements was comparable to \( \delta^2 \).
Frequency domain versions of the method have been already successfully used.

They require a more complex scheme with a photon spectrometer, but potentially allow for single shot measurements.

P. Catravas et al., PRL 82, Number 26, June 99

V. Sajaev, EPAC 2000, p. 1806

V. Sajaev, BIW 04 p. 74

M. Yabashi et al., PRL 97, 084802 (2006)

First measurement using the spectrometer technique with undulator radiation at ATF.

Measurements using the spectrometer technique with undulator radiation at LEUTL, Argonne. Phase retrieval techniques.

First measurement using X-rays. Important for SASE applications!
Conclusions

We have demonstrated an absolute bunch length measurement technique based on the analysis of the fluctuations in the incoherent part of the radiation emitted by a particle beam.

The scheme is non-destructive, shows a remarkable simplicity and can be applied in both circular and linear accelerators including cases where the very short length of the bunches makes difficult the use of other techniques.
By splitting the signal in two branches with filters with the same bandwidth but with different central frequencies, it is possible to discriminate between the transverse and longitudinal fluctuation contributions by exploiting the fact that the longitudinal term depends only on the bandwidth while the transverse ones depend only on the central wavelength. Such a capability allows removing the dependence on the transverse plane and can be useful when the transverse beam size changes during operation.

We want also test the system by coupling the light from the source into an optical fiber. This will allow having the measurement setup separated from the source area for an easy accessibility and tuning of the system.