Short Wavelength SASE FELs: Experiments vs. Theory

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INPUT (electrons)
- Momentum
- Momentum spread/chirp
- Slice emittance/ phase space distribution
- Total charge
- Long. charge profile
- Peak current
- Orbit control

OUTPUT (photons)
- Gain length
- Saturation behaviour
- Spectrum
- Harmonics
- Transverse coherence
- Pulse length
- Effective input power
- Fluctuations

Do we understand the machinery?
Point-like bunch of electrons radiates coherently $P \propto N_e^2$!

„Point“ means above all: bunch length $< \lambda_{\text{radiation}}$

Synchrotron radiation of incoherent electron distribution: $P \propto N_e$

$\rightarrow$ desired: bunch length $< \text{wavelength}$

OR (even better)
Density modulation at desired wavelength

$\rightarrow$ Potential gain in power: $N_e \sim 10^6$ !!

Idea:
Start with an electron bunch much longer than the desired wavelength and achieve bunching at the optical wavelength automatically

Basic theory of FELs

Step 1: Energy modulation

A: Electron travels on sine-like trajectory

\[ v_x(z) = c \frac{K}{\gamma} \cos \left( \frac{2\pi}{\lambda_u} z \right) \], with undulator parameter: \( K = \frac{e\lambda_u B}{2\pi m_e c} \)

B: External electromagnetic wave moving parallel to electron beam:

\[ E_x(z,t) = E_0 \cos(k_L z - \omega_L t) \]

Change of energy \( W \) in presence of electric field:

\[ \frac{dW}{dz} = \frac{q}{v_z} \vec{v} \vec{E} = -\frac{qE_0K}{\gamma\beta_z} \sin \Psi \]

with the ponderomotive phase:

\[ \Psi = (k_u + k_L) z - \omega_L t + \varphi_0 \]
Basic FEL theory

\[ \frac{dW}{dz} = -\frac{qE_{0}K}{\gamma\beta_{z}} \sin \Psi \]

The energy \( dW \) is taken from or transferred to the radiation field.
For most frequencies, \( dW/dz \) oscillates very rapidly.

Continuous energy transfer?

Yes, if \( \Psi \) constant.

\[ \rightarrow \frac{d\Psi}{dz} = 0 ! \]

\( \rightarrow \) Resonance condition:

\[ \lambda_{L} = \frac{\lambda_{u}}{2\gamma^{2}} \left(1 + \frac{K^{2}}{2}\right) \]

Same equation as for wavelength of undulator radiation!
**Basic FEL theory**

**Step 2: Current modulation**

Energy modulation by $\Delta \gamma$ leads to change of Phase $\Psi$:

\[
\frac{d\Psi}{dz} = k_u \frac{2}{\gamma_{\text{res}}} \Delta \gamma
\]

Combined with Step 1:

\[
\frac{dW}{dz} = -\frac{q E_0 K}{\gamma \beta_z} \sin \Psi
\]

yields

\[
\frac{d^2 \Psi}{dz^2} = -\Omega^2 \sin \Psi
\]

with

\[
\Omega^2 = \frac{q}{m_0 c^2} \frac{E_0 K k_u}{\gamma_{\text{res}}^2 \beta_z}
\]

like synchrotron oscillation

-- but at spatial period $\lambda_{\text{light}}$

$\rightarrow$ current modulation !!
Step 3: Radiation

Current modulation $j_{\text{Light}}$ drives radiation of light:

$$\frac{dE_{\text{Light}}}{dz} = \text{const} \cdot j_{\text{Light}}$$

System of Diff. Eqs. defines **High Gain FEL**:
(Kondratenko, Saldin 1980)
(Bonifacio, Pellegrini 1984)
Theory: High-gain FEL

Most simple case:

\[
\frac{d^3E}{dz^3} = i\Gamma^3E
\]

Ansatz: \[E = A \exp(\Lambda z)\]

\[
\Lambda^3 = i\Gamma^3 \Rightarrow \Lambda_1 = -i\Gamma; \quad \Lambda_2 = \frac{i + \sqrt{3}}{2} \Gamma; \quad \Lambda_3 = \frac{i - \sqrt{3}}{2} \Gamma
\]

\[z \gg \Gamma^{-1}: \text{exponential growth:}\]

\[
P_{\text{rad}} = \frac{1}{9} P_{\text{in}} \exp\left(\frac{Z}{L_G}\right) \quad L_G = \frac{1}{\sqrt{3}} \left(\frac{I_A \gamma^3 \sigma_r^2 \lambda_u}{4\pi \cdot \hat{I} \cdot K^2}\right)^{\frac{1}{3}}
\]

\[L_G \propto (\text{current density})^{-\frac{1}{3}}\]

\[
\rho_{\text{FEL}} = \frac{1}{4\pi \sqrt{3}} \frac{\lambda_u}{L_G} \approx 10^{-4} \ldots 10^{-2}
\]

1. Expect exponential gain with e-folding length \(L_G\)

Major additional assumption: Orbit is perfectly straight

2. Gain should saturate when modulation is complete

What do we observe?
For all experiments there exists a “reasonable” electron beam parameter set such that gain length and saturation level agree with theoretical expectations.
Exponential growth ? ✓

Reasonable gain length ? ✓

Achieve full density modulation ? ✓

But: measurement of relevant beam parameters is not precise enough to just predict gain length with reasonable precision.
Gain vs. momentum error $\eta = \frac{d\rho}{\rho}$ (momentum spread $\sigma_\eta$)

*Note:* Emittance effect similar

FEL is a narrow band amplifier

*Note:* Cannot produce few-cycle pulses!

FLASH experiment:

**Bandwidth? ✓**
FEL can also start from initial density modulation given by noise. Equivalent: starting from spontaneous undulator radiation. Self-Amplified Spontaneous Radiation (SASE) is a very robust mode of operation! Theory must model shot noise. Predicts effective “initial conditions” Critical benchmark test for numerical FEL codes, e.g. GENESIS (Reiche), GINGER (Fawley), SIMPLEX (Tanaka), FAST (Yurkov). Equivalent input energy by shot noise: 0.3 pJ.
Start-up from noise

SASE output will fluctuate from pulse to pulse, -- just as ANY part of spontaneous synchrotron radiation does! Remember: FEL is just an amplifier!

- **single Mode** (after monochromator slit)
  - $\sigma = 100\%$  
  - $M = 1$

- **short pulses**  
  - $M = 2.6$ modes
  - $\sigma = 61\%$

- **long pulses**  
  - $M = 6$ modes
  - $\sigma = 40.7\%$

![Graphs showing distributions of energy and time](image-url)
Start-up from noise

Simple 1D model: Superposition of many wavetrains with random phases

A) Short bunch << wavetrain

Large probability of destructive interference

“single Mode”

B) Bunch length >> wavetrain: ”many Modes”

\[ P(E) \, dE = \exp(-E) \, dE \]

\[ g_M(E) \, dE = \frac{1}{\Gamma(M)} (E)^{M-1} e^{-E} \, dE \]

Extract M from histogram
→ pulse length

Fluctuation properties? ✔
Pulse length

Time-domain measurement of pulse length:
not (yet) available for X-ray (established in the visible, FROG etc.)

Alternative: intensity fluctuation translates into spectral fluctuation:
Width of frequency spikes $\leftrightarrow$ length of pulse

3 single pulse
Spectra @FLASH:
  measured
  predicted

$\sim 0.4% \rightarrow \sim 25$ fs pulse duration @ 32 nm
Transverse Coherence

Emittance of a perfectly coherent ("gaussian") light beam emittance:

\[ \varepsilon_{\text{Light}} = \sigma_r \cdot \sigma_\theta = \frac{\lambda_{\text{Light}}}{4\pi} \]

→ FEL theory predicts high transverse coherence of photon beam, if electron beam emittance:

\[ \varepsilon_{\text{electrons}} < \approx \frac{\lambda_{\text{Light}}}{4\pi} \]

Observation of interference pattern at FLASH:

double slit

intensity modulation

FEL simulation

Verified @ 32nm + 13nm
Density modulation becomes anharmonic at high gain:

**3rd harmonics**

@ 4.8 nm

**5th harmonics**

@ 2.75 nm

FLASH typical pulse energies (avg.):

Fundamental (13.8 nm): 40 µJ

3rd harmonics (4.6 nm): (0.25 ± 0.1) µJ

5th harmonics (2.75 nm): (10 ± 4) nJ

Harmonics ? ✅
Most electron beam parameters relevant within slices < coherence length ~1 ... 10 fs
• relaxes requirements on beam specs
• complicates measurements and beam dynamics

Emittance: \( \varepsilon \leq \lambda/4\pi \)

Short Pulse length: \( \sigma_s = 10 – 100 \text{ fs} \)

Peak current inside bunch: \( \hat{I} > 1 \text{ kA} \)

Energy width: \( \sigma_E/E \leq \sim10^{-3} \)

Straight trajectory in undulator: \( < 10 \mu\text{m} \)

Increasingly difficult for shorter wavelength:
• longer undulator, smaller emittance, larger peak current

E. Prat: THAN 026
Beam dynamics simulation tools

ASTRA
Flöttmann

CSRtrack
sub-bunch
Dohlus

ELEGANT
heater &
cavity wake & s.c. field
Borland

GENESIS
rf-field
Reiche

130 MeV

100 MeV

dogleg
dipoles

modules
(1st & 3rd)

bc1

dipoles

linac 2

dipoles

2 GeV

17.5 GeV

main

linac

modules

quads
dipoles

quads

SASE1

undulator

(with quads)

more investigations
Longitudinal bunch compression

- Very complicated beam dynamics due to coherent synchrotron radiation
- Difficult access to relevant parameters
- Ultra-short photon pulses created ~20fs FWHM
Resolving 20 fs with LOLA

Three examples for different compressor settings:

Resolution ~20 fs

See talk by M. Röhrs
Single shot spectrum of **coherent infrared radiation** exhibits structure in the longitudinal density modulation $< 5 \, \mu m$.
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FEL machinery

Most probably yes, but we should know more details about the operator (electron beam).