SLED II: A NEW METHOD OF RF PULSE COMPRESSION*

P. B. Wilson, Z. D. Farkas, and R. D. Ruth
Stanford Linear Accelerator Center, Stanford University, Stanford, CA 94309

Introduction

In the SLED\textsuperscript{1} method of RF pulse compression, two high \(Q\) resonators store energy from an RF source for a relatively long time interval (typically 3 to 5 \(\mu\)sec). Triggered by a reversal in RF phase, this stored energy is then released during a much shorter interval equal to the filling time of the accelerating structure. A peak power gain on the order of three and a compression efficiency on the order of 60\% are typically attained. The shape of the output pulse is, however, a sharply decaying exponential. In SLED-II the two cavities are replaced by two lengths of resonant line, forming a Resonant Line SLED (RELS) and resulting in a flat output pulse. Therefore, RELS stages can be cascaded to give a greater peak power gain. Using two stages, a peak power gain greater than ten can be achieved with a reasonable compression efficiency. Unlike the BEC\textsuperscript{2}, the RELS compression factor per stage is not limited to two, albeit at the expense of intrinsic efficiency. Like the BEC, it uses long lines rather than short cavities.

Theory

A resonant line is a transmission line terminated in a short circuit and connected to an input transmission line via a coupling network as illustrated in Fig. 1. The distance between the coupling network and the short must be an integral multiple of half guide wavelengths. For the RELS, the losses are characterized by the line attenuation \(\tau\) (rather than by \(Q\)) and the coupling is characterized by \(s\), the reflection coefficient when the line is infinitely long (rather than by \(Q\)).

The RELS theory has been discussed\textsuperscript{3,4} Here we expand on the theory using a somewhat different approach. Let \(s\) be the reflection coefficient of the coupling network when the line is terminated in a matched load, \(D/2\) its time delay and \(\tau\) its attenuation in nepers. After turning on an incident field of amplitude \(E_t\), the emitted field after \(nD\) time intervals is given by\textsuperscript{5}

\[
E_c(0) = 0,
\]

\[
E_c(n) = E_t (1 - s^2)^{e^{-2\tau n}} \cdot [1 + s e^{-2\tau} + s^2 e^{-4\tau} + \ldots + s^{(n-1)} e^{-(n-1)2\tau}],
\]

\[n = 1, 2, 3, \ldots\] \hspace{1cm} (1)

\[
E_c(n) = E_t \frac{(1 - s^2)^{e^{-2\tau}}}{1 - s e^{-2\tau}} \cdot [1 - s^n e^{-n2\tau}],
\]

\[n = 0, 1, 2, 3, \ldots\] \hspace{1cm} (2)

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Fig. 1. Resonant line fields.

Fig. 2. Discrete and continuous emitted field vs time. The steady state emitted field is

\[
E_c(n = \infty) = E_{ef} = E_t \frac{(1 - s^2)^{e^{-2\tau}}}{1 - s e^{-2\tau}}. \hspace{1cm} (3)
\]

If \(\tau = 0\) then \(E_{ef} = 1 + s\). As the maximum possible value of \(s\) is 1, the maximum emitted field is 2, the same as for SLED.

Let \(t_n\) be the beginning of each \(D\) interval, \(t_n = nD\). Substituting \(n \equiv t_n/D\) and \(s \equiv e^{lns}\),

\[
E_e(t_n) = E_{ef} \left[1 - s^{t_n/D} \exp \left(-\frac{t_n2\tau}{D}\right)\right]
\]

\[= E_{ef} \left[1 - \exp \left(-\frac{t_n}{D} (2\tau - \ln s)\right)\right]. \hspace{1cm} (4)
\]

Typical continuous and discrete emitted fields are plotted in Fig. 2.

The continuous emitted field of a resonant cavity is

\[
E_c(t) = \alpha \left[1 - \exp \left(-\frac{\omega t}{2} \left(\frac{1}{Q_o} + \frac{1}{Q_e}\right)\right)\right]. \hspace{1cm} (5)
\]

Comparing the above two expressions, we obtain the relationship between attenuation and unloaded \(Q\), and between reflection coefficient and external \(Q\):

\[
E_{ef} = \alpha, \hspace{1cm} \frac{\tau}{D} = \frac{\omega}{4Q_o}, \hspace{1cm} \frac{-\ln s}{D} = \frac{\omega}{2Q_e}. \hspace{1cm} (6)
\]
The first stage output field—assuming single stage modulation—of a RELS with a line of length \( D \). We then divide interval, Fig. 3.

For any piecewise variable input function \( E_i \), as long as the changes in \( E_i \) occur at \( nD \) intervals, where \( n \) is an integer, the emitted field during each \( nD \) time interval, by analogy with a resonant cavity, is

\[
E_e(n) = E_{ef} - [E_{ef} - E_{st}]s^n e^{-n^2r},
\]

\[ n = 0, 1, 2, 3, \ldots . \]

Here, \( E_{st} \) is the emitted field at the start of each interval. For the first interval \( E_{st} \) is zero, and for the subsequent intervals it is the same as the value at the end of the previous interval. Using superposition, the reverse field is

\[
E_r(n) = E_e(n) - E_i s.
\]

The normalized reverse power during each time interval is

\[
P_r(n) = E_r^2(n).
\]

The amplitude of the first step of \( E_r \) is \( E_i s \). We can obtain the steady state value of the emitted field by measuring \( E_r \) the instant after turning off the RF, after it has been on long enough to reach steady state, and obtain \( \tau \) by solving Eq. (3).

**Single-Stage RELS**

We can separate the incident from the reflected field by placing two resonant lines at the two isolated outputs of a 3 dB coupler, or by making it a part of a resonant ring. We then have a single stage RELS. Let the incident pulse width \( T_k = n_o D \). At time \( t = (n_o - 1)D \)—as with SLED—we change the phase of the incident field by \( \pi \), the incident field \( E_i = 1 \) from zero to \( (n_o - 1)D \), and \( E_i = 1 \) from \( (n_o - 1)D \) to \( n_o D \). \( E_i \) changes sign instantaneously but—unlike with SLED—the emitted field remains constant for a duration \( D \). The reverse field during this interval is

\[
E_r(n_o - 1) \equiv E_p = E_e(n_o - 1) + E_i s.
\]

Here, \( E_p \) is constant for a duration \( D \) and is the output pulse. We choose \( D \) to equal the accelerator section fill time, \( T_f \). Here also, as with SLED, the maximum value of \( E_p \) is three.

**Table I**

One-Stage RELS Power Gain and Efficiency as a Function of Compression Factors and REL Attenuation.

<table>
<thead>
<tr>
<th>( n_o )</th>
<th>( s )</th>
<th>( P_g )</th>
<th>( \eta )</th>
<th>( P_g )</th>
<th>( \eta )</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>0.549</td>
<td>2.66</td>
<td>89</td>
<td>2.57</td>
<td>86</td>
</tr>
<tr>
<td>4</td>
<td>0.610</td>
<td>3.44</td>
<td>86</td>
<td>3.29</td>
<td>82</td>
</tr>
<tr>
<td>5</td>
<td>0.651</td>
<td>4.02</td>
<td>80</td>
<td>3.81</td>
<td>76</td>
</tr>
<tr>
<td>6</td>
<td>0.685</td>
<td>4.49</td>
<td>75</td>
<td>4.21</td>
<td>70</td>
</tr>
</tbody>
</table>

The power gain, compression factor, and compression efficiency are

\[
P_g = E_p^2, \quad C_f = T_k/D = n_o, \quad \eta = P_g/n_o.
\]

The power gain and compression efficiency as a function of compression factor, with optimized \( s \), are listed in Table I. The values agree with those given in Table 1 in Ref. 3. The output field and output power of a single stage RELS as a function of time in units of \( D \) for \( n_o = 5 \), are plotted in Figs. 3.

**Two-Stage RELS**

Unlike SLED where the exponential output pulse shape makes cascading of SLED stages impractical, the RELS output pulse is rectangular and therefore RELS can be cascaded.

**Definitions of parameters for a two-stage RELS:**

<table>
<thead>
<tr>
<th>( D )</th>
<th>Last stage RL round-trip delay</th>
</tr>
</thead>
<tbody>
<tr>
<td>( n_o )</td>
<td>Last stage input pulse length in units of ( D )</td>
</tr>
<tr>
<td>( D_1 )</td>
<td>First stage RL round-trip delay</td>
</tr>
<tr>
<td>( n_1 )</td>
<td>First stage input pulse length in units of ( D_1 )</td>
</tr>
<tr>
<td>( T_k )</td>
<td>Length of RELS input pulse</td>
</tr>
<tr>
<td>( C_f )</td>
<td>Overall compression factor</td>
</tr>
</tbody>
</table>

The input pulse to the last stage has to be the output pulse of the first stage. Hence,

\[
D_1 = n_o D.
\]

The length of the input pulse (to the first stage) and the compression factor are

\[
T_k = n_1 D_1 = n_1 n_o D,
\]

\[
C_f = T_k/D = n_1 n_o.
\]

The modulation for a two-stage RELS can be obtained as follows. Divide the input pulse \( T_k \) into \( C_f \) intervals of duration \( D \). Let \( E_{i1} = -1 \) from zero to \( (C_f - n_o)D \) and let \( E_{i1} = 1 \) from \( (C_f - n_o)D \) to \( C_f D \). Let \( E_{i0} = 1 \) when \( n \) is not an integral multiple of \( n_o \) and \( E_{i0} = -1 \) when \( n \) is an integral multiple of \( n_o \). The incident field amplitude at each interval is

\[
E_i(n) = \begin{cases} 1 & \text{when } n \text{ is not an integral multiple of } n_o, \\ -1 & \text{when } n \text{ is an integral multiple of } n_o. \end{cases}
\]

We calculate the power gain and compression efficiency as a function of \( C_f \) as follows. First we obtain the first stage output field—assuming single stage modulation—for a RELS with a line of length \( D_1 \). We then divide...
Each constant amplitude interval $D_1$ of the first stage output wave into $n_o$ intervals and modulate each interval $D_1$. We apply Eqs. (7),(8) to each of the $n_o$ intervals in succession and obtain the reverse field as a function of $n$. The first stage input and output field waveforms for $n_1 = 5$ and $n_o = 3$ are plotted in Fig. 4. The second stage input field amplitude (first stage output), the output field, and the output power are plotted in Fig. 5.

The power gain and compression efficiency are:

$$P_g = E_o^2(C_f - 1), \quad \eta = \frac{P_g(C_f - 1)}{C_f}.$$ \hfill (15)

The values of power gain and efficiency for several combinations of $n_1$ and $n_o$ are tabulated in Table II.

The compression factor is the product $n_1 n_o$, and does not depend on the relative values of $n_1$ and $n_o$. The power gain and compression efficiency also depend only on the product of $n_1 n_o$. Therefore, if $n_1 \neq n_o$, then $n_1$ should be chosen to be greater than $n_o$ if we wish to minimize line length, and less than $n_o$ if we wish to minimize switching transitions.

**Practical RELS**

A two-stage RELS is shown in Fig. 6. Assume for the NLC a two-stage RELS with $n_1 = 5$ and $n_o = 3$. The pulse length required by the NLC is 80 ns; thus $D = 80$ ns. Use a WC281 guide operating at 11.4 GHz in the $TE_{01}$ mode with an attenuation of 0.1 nepers/µs. Because the short is not perfect, the attenuation must include the power absorbed by the short. The length of the first stage line is $D_1 = n_1 D = 3D = 240$ ns. To obtain the length in meters multiply by the RL length by the group velocity which is, in this case, 0.267 m/ns. The RL lengths are 32.1 m for stage one, and 10.7 m for stage two. The klystron pulse length $T_k = 5(240) = 1.2 \mu s$. We expect a gain of about nine (line three of Table 2).

![Fig. 4. First stage RELS input and output fields.](image1)

![Fig. 5. RELS Second-Stage output field and power.](image2)

![Fig. 6. Two-stage RELS.](image3)

### Table II

Two-Stage RELS Power Gain and Efficiency as a Function of Compression Factors and Line Attenuation.

<table>
<thead>
<tr>
<th>$n_o$</th>
<th>$s_o$</th>
<th>$n_1$</th>
<th>$s_1$</th>
<th>$C_f$</th>
<th>$\tau$</th>
<th>$P_g$</th>
<th>$\eta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>0.549</td>
<td>3</td>
<td>0.549</td>
<td>9</td>
<td>0.000</td>
<td>7.05</td>
<td>78</td>
</tr>
<tr>
<td>3</td>
<td>0.549</td>
<td>3</td>
<td>0.549</td>
<td>9</td>
<td>0.005</td>
<td>6.58</td>
<td>73</td>
</tr>
<tr>
<td>5</td>
<td>0.651</td>
<td>3</td>
<td>0.549</td>
<td>15</td>
<td>0.005</td>
<td>9.60</td>
<td>64</td>
</tr>
<tr>
<td>3</td>
<td>0.549</td>
<td>6</td>
<td>0.685</td>
<td>18</td>
<td>0.000</td>
<td>11.0</td>
<td>61</td>
</tr>
<tr>
<td>3</td>
<td>0.549</td>
<td>6</td>
<td>0.685</td>
<td>18</td>
<td>0.005</td>
<td>10.4</td>
<td>58</td>
</tr>
<tr>
<td>6</td>
<td>0.685</td>
<td>3</td>
<td>0.549</td>
<td>18</td>
<td>0.005</td>
<td>9.60</td>
<td>54</td>
</tr>
<tr>
<td>5</td>
<td>0.651</td>
<td>5</td>
<td>0.651</td>
<td>25</td>
<td>0.005</td>
<td>14.0</td>
<td>56</td>
</tr>
<tr>
<td>6</td>
<td>0.685</td>
<td>6</td>
<td>0.685</td>
<td>36</td>
<td>0.005</td>
<td>16.5</td>
<td>46</td>
</tr>
</tbody>
</table>

References