COLLECTIVE BEAM-BEAM INSTABILITIES OF BUNCHES WITH TUNESPADS

D.V. Pestrikov, Budker Institute of Nuclear Physics, 630090 Novosibirsk, Russian Federation

Abstract

We discuss effects of Landau damping on the stability of coherent oscillations of short identical colliding bunches. Near the sum-type resonances \( n/(2m) \), where \( n \) and \( m \) are integers, these oscillations are unstable. Comparing the stopbands calculated for monochromatic and non-monochromatic bunches, we have found that the beam-beam tunespreads increase the widths of the stopbands of coherent modes thus, resulting in Landau anti-damping. These features mean that together with the damping of unstable modes of the monochromatic colliding bunches the tunespreads result in the instabilities for the regions of tunes where the modes of the monochromatic bunches were stable – i.e. in the Landau anti-damping. Such an anti-damping is a specific feature for resonant instabilities of coherent oscillations near the sum-type coupling resonances [7].

THE MODEL

We consider collisions of two identical short counter-moving relativistic electron and positron bunches, which move in separate storage rings and interact head-on at a single interaction point (IP). In our calculations we assume a zero dispersion function at the IP. Incoherent e.g. the horizontal oscillations of particles in the bunches are described using \( x = \sqrt{\beta} \cos \psi \) and \( p_x = p_0' / R_0 = -p \sqrt{J/\beta} \sin \psi \). Here, \( I = p J / 2 \) and \( \psi = (\theta + 2 \pi) = \psi(\theta) + 2 \pi \nu_2 \) are the action-phase variables of the unperturbed incoherent oscillations, \( \Pi = 2 \pi R_0 \) is the perimeter of the closed orbit, \( s = R_0 \theta \) is the path along the closed orbit, \( \nu_1 \) prime means that the bunches move in the rings with identical lattices, \( p_0 \epsilon f(1,2) \) fast-oscillating terms, we find that the combinations

\[
 f_{m}^{(\pm)} = f_{m}^{(1)} \pm f_{m}^{(2)}
\]

\[
 = e^{-2p\epsilon f(1,2)} x_{m/2} \sum_{n=-\infty}^{\infty} e^{-i(n+\nu-n)}\theta
\]

decribe normal modes of identical colliding bunches [8]. Since the functions \( f_{m}^{(1,2)} \) are linear combinations of the
modes \(f_m^{(±)}\), coherent oscillations of identical colliding bunches are stable only in the regions, where both modes \(f_m^{(+)}\) and \(f_m^{(-)}\) are stable. For small values of the beam-beam parameter \(\xi = Ne^2/(2\pi e\epsilon c)\) (e.g. \(B = 2\pi \xi < 1\)) and near the resonances \(\nu_x = n/(2m)\), we can neglect in equations for \(f_m^{(±)}\) the contributions of non-diagonal in \(|m|\) modes. Resulting equations for \(p_m^{(±)}(z_1, x)\) read (more details in Ref.[6], \(m > 0\))

\[
\frac{d}{dx} \left[ x^{m+1} \frac{dp_m^{(±)}(x)}{dx} \right] = \pm e^{-x} x^m V(x) p_m^{(±)}(x). \tag{2}
\]

These equations should be solved with the boundary conditions \(p_m^{(±)}(z_1, 0) = 1\), and \(dp_m^{(±)}(z_1, 0)/dx = \pm V(0)/(m + 1)\). The dispersion equations of the problem read

\[
p_m^{(±)}(z_1, \infty) = 0. \tag{3}
\]

Here, \(V(x) = 2\delta(x)/(z_1^2 - \delta^2(x)) (\Im z_1 > 0)\), and

\[
z_1 = \frac{1}{m \xi} \left( \nu - \frac{n}{2} \right), \quad \delta(x) = \frac{1}{\xi} \left( \nu_x(x) - \frac{n}{2m} \right), \tag{4}
\]

\(n\) is the azimuthal number of the resonance harmonic.

**RESULTS**

Near the resonances \(\nu_x \approx n/(2m)\) Eqs.(2) and (3) always have solutions with the eigenvalues \(\Re(z_1) = 0\) and \(\Im(z_1) \neq 0\) [2]. Since \(p_m^{(±)}(z_1, \infty)\) are functions of \(z_1^2\), these solutions describe unstable modes. The values of the increments of unstable modes as well as the widths of relevant stopbands and their positions in \(\nu_x\) are found solving Eqs.(2) and (3) numerically. In these calculations we also took into account self-consistent variations of the oscillation tunes and of \(\beta\)-functions by the beam-beam interactions: \(\cos \mu_0 = \cos \mu_x + B \sin \mu_x, \beta \sin \mu_x \beta = \beta_0 \sin \mu_0\), where \(\mu = \pi \nu x\), the subscript 0 marks bare values. Below, we neglect possible flip-flop spitting of the betatron functions. Using simulations, we have found that in our model the tunes depend on \(x\) according to

\[
\nu_x(x) = \nu_x - \Delta \nu_x(0) \left( 1 - \frac{1 - e^{-x}}{x} \right), \tag{5}
\]

where \(\Delta \nu_x(0) = \nu_x - \nu_0\) is the linear beam-beam tuneshift (e.g. in Fig. 1) Calculating increments for the modes (±) with \(1 \leq m \leq 5\) and \(1 \leq n \leq 4\), we find the stopband depicted in Fig. 2. Although only the segment \(0 \leq \nu_x \leq 1/2\) is shown in Fig. 2, the stopbands for higher, or lower values of \(\nu_x\) appear periodically with a period in \(\nu_x\) of 1/2. For dipole oscillations \(m = 1\) we have found no roots of the dispersion equation (3) for \(0 \leq \nu_x \leq 1/2\). This result means that only (−) dipole mode has the stopband below the resonance \(\nu_x = 1/2\). The stopband for the mode (−, 2) starts slightly below the resonance \(\nu_x = 1/4\). The lower ends of all other found stopbands are found to be close to the corresponding resonant tunes \(\nu_x = n/(2m)\). Numerical values of the maximum increments and of the widths of the stopbands for modes (−) and \(1 \leq m \leq 3\) in Fig. 2 are in general agreement with similar results reported in Ref.[3] and obtained using a different approach. We note that the values of the maximum increments for the modes \(m \geq 3\) are a bit smaller in the region \(\nu_x < 1/4\) than that above \(\nu_x = 1/4\). The maximum increments of the modes near the resonances \(\nu_x = 1/(2m)\) tend to zero, when \(\nu_x\) approaches the border value

\[
(\nu_x)_{\text{min}} = \frac{1}{2\pi} \arccos \frac{1 - B^2}{1 + B^2}. \tag{6}
\]

This fact is in a general agreement with experimental observations of the beam-beam instabilities in the electron-positron colliders. Although it is not shown here, a decrease in \(\xi\) results in decreases in the values of the mode increments (\(\Im \nu\)) and in the narrowing of the widths of the stopbands in \(\nu_x\). However, the ratios \(\Im \nu/\xi\) and of these widths to \(\xi\) remain the same.

Outside the spectrum of incoherent oscillations (e.g. \(z_1^2 > \delta^2(0)\)) Eqs.(2) and (3) may have solutions with
Re\(z_1\) \neq 0 and \(\text{Im}(z_1) = 0\). To avoid the mode interference, coherent tuneshifts of such stable solutions should not enter the stopbands of unstable modes.

To figure out effects of Landau damping on the stability of coherent beam-beam oscillations we compared the stopbands calculated for monochromatic and non-monochromatic bunches. For monochromatic bunches the roots of the dispersion equations corresponding to the largest increments are calculated using ([6]) \(z_1^2 = \delta^2 \pm 2\Lambda_m\delta\), where the sign + is taken for the modes (−) and

\[
\Lambda_m = \frac{4}{m(m+2)} \left(\frac{1}{m+1}\right)^{m+1}.
\]

For the dipole mode (−) Landau damping results only in minor changes of the stopband of the monochromatic bunches (Fig. 3). The maximum values of the increments almost coincide, Landau damping suppress the instability within a narrow band A′A and decreases the oscillation increments within the segment AB. Within the segment BC the beam-beam tunespread slightly increases the increments of unstable modes hence, resulting in some Landau anti-damping. Stronger Landau anti-damping indicates the stopbands of the modes with higher betatron multipole numbers \((m \geq 2\), e.g. in Fig. 4 and 5\). For such modes, the beam-beam tunespread although decreases the values of the maximum increments, moves the lower border of the stopbands of the modes (−) towards the resonant tune \(n/(2m)\) and substantially increases the widths of the stopbands. Except for the case \(m = 2\), the stopbands of the multipole modes (+) are placed above the resonant tunes almost entirely. Hence, the beam-beam tunespread suppressing the modes of the monochromatic colliding bunches opens new wide regions of the tunes \(v_x\) where the oscillations become unstable. The described Landau anti-damping of the coherent oscillations of the colliding bunches is a generic phenomenon for the coherent beam-beam interactions. These instabilities occur due to the coupling of the modes \(m\) and \(-m\) near the sum-type resonance \(m(\nu_x^{(1)} + \nu_x^{(2)}) = 2m\nu_x = n\). According to general properties of such instabilities [7] any damping can stabilize coupled coherent modes only in the case, when both coupled modes are damped sufficiently strongly. Otherwise, the oscillations become unstable.

Replacing \(\nu_x\) by \(\nu_x + a\nu\), we can also inspect some effects of the octupole lattice non-linearity on the stability of coherent beam-beam oscillations. According to data depicted in Figs. 6 and 7, the octupole fields do not cancel the described Landau anti-damping. However, it can decrease the strength of the instability provided that the sign of the non-linearity is correct.

Additional suppression of the strength of the coherent beam-beam instability can occur in collisions of long bunches due to hour-glass effect [9]. In the simplest case and provided that the disruption parameter of the bunches \(4\pi\sigma_x/\beta\) is small, the stopbands of the betatron modes of the bunches with the lengths \(\sigma_x\) comparable to \(\beta\) can be calculated using Eqs.(2) and (3) after a reduction in Eq.(2) of the function \(V(x)\) by the suppressing factor times. According to data depicted in Fig. 8, the hour-glass suppres-
CONCLUSIONS

Using the simplifying model, we have studied the influence of tunespreads on the stability of coherent oscillations of short, identical colliding $e^+e^-$ bunches. Comparing the spectra of coherent oscillations which are calculated taking into account and/or ignoring the tunespreads we have found out the Landau anti-damping of coherent oscillations of colliding bunches. Namely, together with the damping of unstable modes of the monochromatic colliding bunches the tunespreads result in the instability of coherent oscillations in the regions of betatron tunes $\nu_x$ where coherent oscillations of monochromatic bunches were stable. Effects of this anti-damping increase with an increase in the value of the betatron multipole number $m$. It is almost negligible for the dipole modes, but for the modes with $m \geq 2$ the calculations ignoring the beam-beam tunespread result in strong underestimation of the widths of the stopbands of coherent oscillations of the colliding bunches as well as in wrong positions of these stopbands relative the resonant values of the tunes $\nu_x$. Generally, effects of the tunespreads decrease the maximum values of the oscillation increments as compared to those calculated for monochromatic bunches. Octupole fields do not cancel Landau anti-damping, but can decrease increments of unstable modes. Initial estimations also show, that the hour-glass reductions do not eliminate Landau anti-damping of the coherent beam-beam modes.

We simplified our half-analytic calculations ignoring possible effects of incoherent beam-beam resonances on the stability of collective beam-beam modes assuming that only small amount of the bunch particles are captured in the resonance buckets. If the incoherent resonances are strong and/or are wide enough in $\nu_x$, the described calculations may predict the results which are not reliable (see, e.g. in Ref. [10]). In such cases, the stability of collective beam-beam modes should be studied using numerical simulations.

REFERENCES