A Vlasov-Maxwell Solver to Study Microbunching Instability in the FERMI@ELETTRA First Bunch Compressor System

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1. Self Consistent Vlasov-Maxwell Treatment
2. Field Calculation
3. Self Consistent Monte Carlo Method
4. Microbunching Instability Studies

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Self Consistent Vlasov-Maxwell Treatment I

\[ E_\parallel = 0 \]
\[ B_Y = 0 \]
Self Consistent Vlasov-Maxwell Treatment II

Wave equation in lab frame with “2D” planar source:

\[ (\partial^2_Z + \partial^2_X + \partial^2_Y - \partial^2_u)E = H(Y)S(R, u), \quad E(R, Y = \pm g, u) = 0. \]

where \( u = ct \), \( E(R, Y, u) = (E_Z, E_X, B) \), \( R = (Z, X) \).

Vlasov equation in beam frame:

\[ f_s - \kappa(s) x f_z + F_z f_{pz} + p_x f_x + [\kappa(s) p_z + F_x] f_{px} = 0 \]

where

\[ F_z = \frac{e}{\bar{v}E} \mathbf{V} \cdot \mathbf{E}, \]

\[ F_x = \frac{e}{E\beta^2} \left[ -\tilde{X}'(s) E_Z + \tilde{Z}'(s) E_X + \bar{v}B \right], \]

and \( \mathbf{V} = \bar{v}(t(s) + p_x \mathbf{n}(s)), \mathbf{E} = (E_Z, E_X) \) and \( B \)

are evaluated at \( R = \bar{R}(s) + x \mathbf{n}(s) \) and \( u = (s - z)/\beta \).
Field Calculation (Lab Frame)

\[ \mathcal{E}(\mathbf{R}, u) := \langle \mathcal{E}(\mathbf{R}, \cdot, u) \rangle = \int_{-g}^{g} H(Y) \mathcal{E}(\mathbf{R}, Y, u) dY. \]

averaged field computed much more quickly

\[ \mathcal{E}(\mathbf{R}, u) = -\frac{1}{2\pi} \sum_{k=0}^{\infty} (-1)^k (1 - \frac{\delta_{k0}}{2}) \int_{-\infty}^{u-kh} dv \int_{-\pi}^{\pi} d\theta S(\hat{\mathbf{R}}, v, k) \]

where \( \hat{\mathbf{R}} = \mathbf{R} + \sqrt{(u - v)^2 - (kh)^2}(\cos \theta, \sin \theta) \).

Issues

- localization in \( \theta \) (angular size of the beam) for \( v \ll u - kh \) and in \( v \)
- delicate calculation (must be done cum grano salis)

\( \theta \) integration: superconvergent trapezoidal rule
\( v \) integration: adaptive Gauss-Kronrod rule
Beam to Lab Charge/Current Density Transformation

- To solve Maxwell equations in lab frame must express lab frame charge/current density in terms of beam frame phase space density
- To a good approximation lab frame charge/current densities are

\[
\rho_L(R, Y, u) = H(Y)\rho(r, \beta u), \quad J_L(R, Y, u) = \beta c H(Y)[\rho(r, \beta u) t(\beta u + z) + \tau(r, \beta u) n(\beta u + z)],
\]

\[
\rho(r, s) = Q \int dp_z dp_x f(\zeta, s), \quad \tau(r, s) = Q \int dp_z dp_x p_x f(\zeta, s),
\]

where \( \zeta = (z, p_z, x, p_x) \)

**Remark:** subtlety in the change of independent variable \( u = ct \rightarrow s \)

Derivation to be published in a forthcoming paper
Self Consistent Monte Carlo Method

Outline and comparison with PIC for Vlasov-Poisson (VP) system from $s$ to $s + \Delta s$

- From scattered beam frame points at $s \rightarrow$ smooth/global Lab frame charge/current density via a 2D Fourier method (Charge deposition (+ filtering) in VP PIC).

1D Example:

1D orthogonal series estimator of $f(x)$, $x \in [0, 1]$

$$f_J(x) := \sum_{j=0}^{J} \theta_j \phi_j(x), \quad \theta_j = \int_{0}^{1} \phi_j(x)f(x)dx, \quad \phi_0(x) = 1, \phi_j(x) = \sqrt{2}\cos(\pi j x), j = 1, 2, \ldots$$

According to the fact that $f(x)$ is a probability density

$$\theta_j = E\{I_{\{X \in [0,1]\}} \phi_j(X)\}, \quad \text{therefore a natural estimate is} \quad \hat{\theta}_j := \frac{1}{N} \sum_{n=1}^{N} I_{\{X_n \in [0,1]\}} \phi_j(X_n)$$

- Calculate fields at $s$ from history of Lab Frame charge/current density using our field formula (Solve Poisson Equation in VP PIC)
- Use fields at $s$ to move the phase space points to $s + \Delta s$ (Same in VP PIC)
Microbunching can cause an instability which degrades beam quality.

This is a major concern for free electron lasers where very bright electron beams are required.

FERMI@ELETTRA first bunch compressor system proposed as a benchmark for testing codes at the September’07 workshop on microbunching instability in Trieste.

See https://www.elettra.trieste.it/FERMI/index.php?n=Main.MicrobProgram
## FERMI@ELETTRA First Bunch Compressor Parameters

![Layout of the first bunch compressor system](image-url)

Table 1: Chicane parameters and beam parameters at first dipole

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Energy reference particle</td>
<td>$E_r$</td>
<td>233</td>
<td>MeV</td>
</tr>
<tr>
<td>Peak current</td>
<td>$I$</td>
<td>120</td>
<td>A</td>
</tr>
<tr>
<td>Bunch charge</td>
<td>$Q$</td>
<td>1</td>
<td>nC</td>
</tr>
<tr>
<td>Norm. transverse emittance</td>
<td>$\gamma \epsilon_0$</td>
<td>1</td>
<td>$\mu$m</td>
</tr>
<tr>
<td>Alpha function</td>
<td>$\alpha_0$</td>
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<td></td>
</tr>
<tr>
<td>Beta function</td>
<td>$\beta_0$</td>
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<td>m</td>
</tr>
<tr>
<td>Linear energy chirp</td>
<td>$u$</td>
<td>-27.5</td>
<td>1/m</td>
</tr>
<tr>
<td>Uncorrelated energy spread</td>
<td>$\sigma_E$</td>
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<td>KeV</td>
</tr>
<tr>
<td>Momentum compaction</td>
<td>$R_{56}$</td>
<td>0.0025</td>
<td>m</td>
</tr>
<tr>
<td>Radius of curvature</td>
<td>$\rho_0$</td>
<td>5</td>
<td>m</td>
</tr>
<tr>
<td>Magnetic length</td>
<td>$L_b$</td>
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<td>m</td>
</tr>
<tr>
<td>Distance 1st-2nd, 3rd-4th bend</td>
<td>$L_1$</td>
<td>2.5</td>
<td>m</td>
</tr>
<tr>
<td>Distance 2rd-3nd bend</td>
<td>$L_2$</td>
<td>1</td>
<td>m</td>
</tr>
</tbody>
</table>
Initial charge density in norm. coordinates for $A=0.05$, $\lambda = 300\,\mu m$. Init. phase space density $= (1 + A \cos(2\pi z/\lambda)) \mu(z) \rho_c(z, p_z) g(x, p_x)$. 
Gain factor := | b(k_f, s_f)/b(k_0, 0) |, where b(k, s) = \int dz \exp(-i k z) F(z, s) and k_f = k_0/(1 + u R_{56}(s_f)) for a given initial wavelength \( \lambda = 2\pi/k_0 \).
Here the compressor factor \( C = 1/(1 + u R_{56}(s_f)) = 3.54, \) \( s_f = 8m \).
Fermi@Elettra First Bunch Compressor II

Mean power

Path length (m)

Mean power

A=0
A=0.05, \lambda=300\mu m
x-emittance

A=0
A=0.05, $\lambda=300\mu$m

$A=0$: 1.48 mm-mrad
$A=0.05$, $\lambda=300\mu$m: 1.50 mm-mrad
Charge density in norm. coord. at $s = 8m$ for no initial modulation ($A=0$).
Charge density in normalized coordinates at $s = 8m$ for $\lambda = 600\mu m$
Charge density in normalized coordinates at $s = 8\text{m}$ for $\lambda = 300\mu\text{m}$
$E \cdot t$ in normalized coordinates at $s=8m$ for no initial modulation.
$E \cdot t$ in normalized coordinates at $s=8\text{m}$ for $\lambda = 600\mu\text{m}$. 
$E \cdot t$ in normalized coordinates at $s=8m$ for $\lambda = 300\mu m$. 
$E \cdot t$ in normalized coordinates at $s=4m$ for $\lambda = 600\mu m$. 
$E \cdot t$ in normalized coordinates at $s=4m$ for $\lambda = 300\mu m$. 
$E \cdot t$ in normalized coordinates at $s=5m$ for $\lambda = 600\mu m$. 
$E \cdot t$ in normalized coordinates at $s=5m$ for $\lambda = 300\mu m$. 
Main Issues and Accomplishments

- FERMI@ELETTRA microbunching studies:
  Creation of modulations in $E \cdot t$ for the $\lambda = 300 \mu$m case but no detrimental effect on the charge density
  Simulations done at the HPC at UNM and on NERSC at LBNL, typical runs on NERSC: $N \text{ procs} = 200-700$, $N \text{ particles} = 2 \times 10^7$, few hours of CPU time

- Storage/computational cost very important
  - As much analytical work as possible
  - State of the art numerical techniques: integration, interpolation, density estimation, quasirandom generator (see Warnock et.al. TUPP109 Tuesday)
  - Parallel computing, parallel I/O

- Delicacy of field calculation, support of charge/phase space density
Future Work

- Study wavelengths shorter than $\lambda = 300\mu m$ and different amplitudes of the initial modulation

- Complete studies for benchmark microbunching instability including RF cavities

- Results will be presented at the next Microbunching Instability Workshop at LBNL
Charge density in normalized coordinates at $s=4m$ for $\lambda = 200\mu m$. 
$E \cdot t$ in normalized coordinates at $s=4m$ for $\lambda = 200 \mu m$. 