SYMMETRY RESTORATION OF THE SPring-8 STORAGE RING
BY COUNTER-SEXTUPOLE MAGNETS

JASRI/SPring-8, Hyogo 679-5198, Japan
†RIKEN, SPring-8 Joint Project for XFEL, Hyogo 679-5148, Japan
‡JRI Solutions, Ltd., Osaka 550-0013, Japan

Abstract

In 2000 magnet-free long straight sections of about 30m were realized in the SPring-8 storage ring. At that time, in modifying the lattice we took care of the periodicity of cell structure, especially of sextupole field distribution along the ring. The dynamic aperture for on-momentum electrons was kept by betatron phase matching and that for off-momentum electrons was enlarged by correcting local chromaticity over the matching section. Though sextupole magnets used in this correction were weak, they lowered the symmetry of the lattice. To recover it and improve the beam performance, we recently installed “counter-sextupole” magnets to cancel non-linear kicks inside the matching section. This scheme worked well and the dynamic aperture was enlarged.

INTRODUCTION

In the SPring-8 storage ring there are four magnet-free long straight sections (LSS’s) of about 30m. In designing the storage ring, the LSS’s were considered as one of future upgrading options [1, 2], and the phase-1 lattice was constructed by repeating a double-bend cell structure 48 times and removing bending magnets from every twelfth cell. The symmetry of this phase-1 lattice was high and this ensured a large dynamic aperture of the storage ring. The symmetry was 24-fold for hybrid optics used in early stage and 48-fold for HHLV (High-Horizontal and Low-Vertical beta) optics used later [3].

In the summer shutdown period of 2000, after operating the storage ring with the phase-1 lattice for about three years, we locally rearranged quadrupole and sextupole magnets to realize phase-2 lattice with four LSS’s. Since the local rearrangement of quadrupole and sextupole magnets can easily cause lowering of the lattice symmetry, we especially took care of this point and developed a method of “quasi-transparent matching of sextupole fields” by combining two key concepts of “betatron phase matching” and “local chromaticity correction” [4]. Based on this idea, we converted three unit cells to a matching section. Owing to the “quasi-transparent matching”, the phase-2 lattice has an approximate 36-fold (=48-3×4) symmetry, and we could keep large dynamic aperture for on- and off-momentum electrons. The beam performance was sufficiently high even after introducing four LSS’s of about 30m in the storage ring.

In 2004, to suppress oscillation of a stored beam during injection, we introduced a new scheme of sextupole optimization. In this scheme the ratio of strengths of sextupole magnets in the injection bump orbit is determined so that the effect of non-linear kicks becomes minimum [5]. A periodic structure of sextupole excitation pattern was extended from one unit cell to two cells to increase the number of independent sextupole families. By tuning the sextupole strengths, we could keep a sufficient dynamic aperture for beam injection. At this time, however, the symmetry of lattice was lowered from 36-fold to 18-fold due to the two-cell structure of sextupole field distribution, and as explained, this 18-fold symmetry of lattice is an approximate one due to sextupole magnets in the matching section for local chromaticity correction.

We then tried to recover this approximate 18-fold symmetry to a complete one as much as possible to get higher beam performance. We then found that it is efficient to install additional sextupole magnets at each LSS’s to cancel non-linear kicks inside the matching section. In 2007 we installed such “counter-sextupole” magnets at each LSS’s and improved the injection efficiency and beam lifetime. In what follows we show how we have done this.

MATCHING SECTION

As explained in the previous section, three unit cells were converted to the matching section in which magnet-free space of about 30m was realized. Figure 1 shows this section. The betatron phase advance is chosen to be $2\pi n$, where $n = 2$ for the horizontal direction and $n = 1$ for the vertical direction. By this betatron phase matching, the symmetry of the ring can be kept high, being 18-fold, if no sextupole magnets are used in the matching section.

In the low-emittance ring, however, the beam is focused horizontally in the arc section by strong quadrupole magnets and the effect of local chromaticity can be large. The horizontal and vertical natural chromaticities over the matching section are $\Delta \xi_x = -3.9$ and $\Delta \xi_y = -2.4$, respectively. If the local chromaticity is uncorrected, electrons with different momenta receive different focusing force and non-negligible phase slip occurs.
Figure 1: Optical functions and magnet arrangement (blue: bending, green: quadrupole, orange: sextupole) of the matching section before installation of counter-sextupoles. Only SFL is used and other sextupoles are switched off.

The chromatic aberration generally affects betatron oscillation in both horizontal and vertical directions. However, for improving the efficiency of beam injection and Touschek beam lifetime, it is important to correct the chromatic aberration in the horizontal direction, and this allows us to lower the required strength of sextupole magnets. We hence corrected the horizontal chromaticity by using weak sextupole magnets in the arc of matching sections (SFL in Fig. 1). By carefully adjusting the strength of the sextupole magnet SFL, we could obtain sufficiently large dynamic aperture for on- and off-momentum electrons. After correcting by SFL, the local chromaticities were $\Delta \xi_x = -2.2$ and $\Delta \xi_y = -3.0$.

COUNTER-SEXTUPOLE MAGNET

Since SFL is excited in the matching section, the 18-fold symmetry of the ring is not an exact one. The symmetry however holds in an approximate manner, since the strength of SFL is weak and its field can be regarded as perturbation.

As we shall explain below, this approximate 18-fold symmetry can be recovered by using another sextupole magnet located $m\pi$ apart in betatron phase from SFL, where $m$ is an integer. Such sextupole magnets can be used to cancel the non-linear kick due to SFL, and we call such magnets as the “counter-sextupole” SCT. (Similar considerations of chromaticity correction are found in [6, 7, 8].)

Doublet Scheme

Since both of an injected beam and a scattered electron due to the Touschek effect have large oscillation amplitudes in the horizontal direction, we may first neglect vertical oscillation and try to cancel the horizontal kick due to SFL. A suitable position of SCT we found is shown in Fig. 2 where the betatron phase advances are also shown over a half of the the matching section. The horizontal phase advance between SFL and SCT is $\Delta \psi_x = 0.95\pi$.

05 Beam Dynamics and Electromagnetic Fields

Since $\Delta \psi_x \simeq \pi$ and sextupole magnets except SFL and SCT are switched off, the horizontal orbit shift $x$ at SFL and SCT are related by

$$x_{\text{SCT}} = -\left(\frac{\beta_x^{\text{SCT}}}{\beta_x^{\text{SFL}}}\right)^{1/2}x_{\text{SFL}}$$

where $\beta_x^{\text{SFL}}$ and $\beta_x^{\text{SCT}}$ are the horizontal betatron function at SFL and SCT, respectively. When the vertical oscillation is neglected, the kick generated by SFL is written as

$$\theta_x^{\text{SFL}} = -\lambda_{\text{SFL}}x_{\text{SFL}}^2.$$

where $\lambda_{\text{SFL}}$ is the strength of SFL. This kick propagates to the position of SCT and causes the following angle deviation at this point:

$$\Delta x_{\text{SCT}} = -\left(\frac{\beta_x^{\text{SFL}}}{\beta_x^{\text{SCT}}}\right)^{1/2}\theta_x^{\text{SFL}}.$$  

Then, if SCT gives a “counter-kick” $-\Delta x_{\text{SCT}}$, the kick by SFL is canceled:

$$\theta_x^{\text{SCT}} = -\lambda_{\text{SCT}}x_{\text{SCT}}^2 = -\Delta x_{\text{SCT}}^2,\, \lambda_{\text{SCT}} = \left(\frac{\beta_x^{\text{SFL}}}{\beta_x^{\text{SCT}}}\right)^{3/2}\lambda_{\text{SFL}}.$$  

From these, we can determine the strength of SCT as

$$\lambda_{\text{SCT}} = \left(\frac{\beta_x^{\text{SFL}}}{\beta_x^{\text{SCT}}}\right)^{3/2}\lambda_{\text{SFL}}.$$  

Based on this idea, we carried out particle tracking and checked the effect of counter-sextupole magnets. The results however showed that the cancellation of kicks is not enough and we cannot enlarge the dynamic aperture by this simple scheme.

Triplet Scheme

To improve the above scheme and obtain better cancellation of non-linear kicks, we partially took vertical oscillation into account by increasing the number of sextupole magnets. We selected S1L shown in Fig. 2 for this purpose because the horizontal phase advance between S1L and SFL is $\Delta \psi_x = 0.82\pi \sim \pi$, and we can expect cancellation of horizontal kicks among S1L, SFL and SCT when their strengths are properly chosen. We then have one degree of freedom that can be used for cancellation of vertical kicks.

The optimization of sextupole strengths can be done in the following way: Suppose that there are $n$ sextupole magnets located $\pi$ apart in horizontal betatron phase from each D02 Non-linear Dynamics - Resonances, Tracking, Higher Order

![Figure 1: Optical functions and magnet arrangement (blue: bending, green: quadrupole, orange: sextupole) of the matching section before installation of counter-sextupoles. Only SFL is used and other sextupoles are switched off.](image1)

![Figure 2: Sextupole magnet positions and the betatron phase advance. The counter-sextupole is indicated as SCT.](image2)
other, and an electron enters into the first magnet with the position \((x_1, y_1)\) and angle \((x'_1, y'_1)\). In the linear approximation we can write the electron’s trajectory by using the optical functions. Then, the kick generated by the \(i\)-th sextupole magnet with strength \(\lambda_i\) is written with \((x_1, y_1)\) and \((x'_1, y'_1)\), and this kick is propagated to the exit of the \(n\)-th sextupole magnet. By summing up the contribution of all sextupole magnets, we can write the deviation of position and angle at the end as

\[
\Delta X = 0 \\
\Delta X' = Ax_1^2 + By_1^2 + Cx_1y_1 + Dy_1^2 \approx Ax_1^2 \\
\Delta Y = Ex_1y_1 + Fx_1y_1' \\
\Delta Y' = Gx_1y_1 + Hx_1y_1'.
\]

In the second equation, we neglected higher order terms of vertical oscillation. The coefficient \(A\), for example, is written in the following form:

\[
A = \sum_{i=1}^{n} a_i \lambda_i \\
a_i = (-1)^{n-i+1} (\beta_x^{(i)} / \beta_x^{(n)})^{1/2} (\beta_y^{(i)} / \beta_y^{(1)}).
\]

For the horizontal direction, we require \(\Delta X' = 0\) and hence \(A = 0\). For the vertical direction, we have a choice of optimization. Here, we impose the condition that an electron entering with \(y_1' = 0\) goes out with \(\Delta Y' = 0\), and this requires \(G = 0\). The adequacy of our choice must be checked with computer simulations. For the case of \(n = 3\), the two equations \(A = 0\) and \(G = 0\) give the ratio of three sextupole magnets. In the case of Fig. 2 we have

\[
\lambda_{SIL} : \lambda_{SFL} : \lambda_{SCT} = 0.146 : 1 : 0.543.
\]

Since this ratio was obtained under some approximate conditions, we should searched an optimum solution around here with particle tracking. By fixing the absolute strength of SFL for correcting local chromaticity and re-optimizing the strengths of sextupole magnets in normal cells, we finally obtained

\[
\lambda_{SIL} : \lambda_{SFL} : \lambda_{SCT} = 0.139 : 1 : 0.354.
\]

The local chromaticities after correction are \(\Delta \xi_x = -1.2\) and \(\Delta \xi_y = -3.5\).

The dynamic apertures calculated with and without SCT are compared in Fig. 3. We see that the dynamic aperture was improved for both on- and off-momentum electrons by introducing SCT.

**BEAM PERFORMANCE**

In August 2007 installation of counter-sextupole magnets was finished and we carried out beam tuning. Since the dynamic aperture was enlarged, the injection efficiency was increased by about 5% and this agrees well with simulation results. The momentum acceptance was also increased. We checked this by measuring the Touschek beam lifetime as a function of RF accelerating voltage. The Touschek beam lifetime is an index of the momentum acceptance, and a longer beam lifetime at high RF voltages means a larger momentum acceptance [9]. The results are shown in Fig. 4. From the figure we see that when SCT is used, the beam lifetime becomes longer at high RF voltages and the momentum acceptance becomes larger. This tendency in the Touschek beam lifetime can be seen more clearly when the 05 Beam Dynamics and Electromagnetic Fields

![Dynamic aperture of the ring calculated at the beam injection point. Relative momentum deviation of electrons is indicated by \(\delta\).](image)

Figure 3: Dynamic aperture of the ring calculated at the beam injection point. Relative momentum deviation of electrons is indicated by \(\delta\).

![Touschek beam lifetime measured at 1mA/bunch. Gap of a 25m-long in-vacuum undulator (LU) is closed as shown in the same figure.](image)

Figure 4: Touschek beam lifetime measured at 1mA/bunch. Gap of a 25m-long in-vacuum undulator (LU) is closed as shown in the same figure.

The introduction of “counter-sextupole” magnets also enables us to make independent tuning of optical functions of each LSS. This is necessary for efficient use of LSS’s because insertion devices to be installed in the future will require different values of beta and/or dispersion functions optimized for each device. Studies on independent tuning of LSS optics are ongoing.

**REFERENCES**


D02 Non-linear Dynamics - Resonances, Tracking, Higher Order