EFFECT OF MAGNETIC MULTipoles ON THE ALBA DYNAMIC
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Abstract

For modern synchrotron light sources the main limitation of dynamic aperture is due to the strong chromatic sextupoles. However, small multipole errors in magnetic elements can reduce the original dynamic aperture by generating of high order resonances at the aperture boundary. For the ALBA synchrotron light source a dynamic aperture in the presence of magnetic multipoles in the main magnets was simulated by tracking code. Both systematic and random magnetic errors were taken into account. In this paper we report on the results of our considerations.

INTRODUCTION

The following definition of the magnetic field multipoles is used below:

\[ B_n + iB_s = \sum_{n=1}^{\infty} (ia_n + b_n) (x + iz)^{n-1}, \]

where \( b_n \) and \( a_n \) are the normal and the skew components respectively.

According to (1) every multipole produces the magnetic field with the amplitude at the given radius \( r_0 \)

\[ \Delta B_n (r_0) = b_n \cdot r_0^{-n-1} \quad \text{and} \quad \Delta a_n (r_0) = a_n \cdot r_0^{-n-1}. \]

This field provides an additional kick to the beam (for simplicity let us speak about the horizontal motion and the normal component)

\[ \Delta x' = \Delta B_n L / B \beta, \]

where \( B \beta \) is the magnetic rigidity and \( L \) is the magnet length. The kick (2) increases the particle betatron amplitude by

\[ \Delta x = \beta x \Delta x', \]

and if this additional displacement exceeds the stable motion boundary (caused by other sources, say, strong sextupoles) the particle will be lost. For \( n \) magnets with statistically independent field errors one can estimate

\[ \Delta x \sim \sqrt{n} \cdot B \beta \sigma_n L / B \beta \]

and if we are concerned about the dynamic aperture reduction by 1mm with \( B \beta = 10 \text{T-m} \) (@3GeV), \( n = 100, L = 0.5 \text{m} \) and \( \beta = 10 \text{m} \), the level of the rms field error required for it can be estimated as \( \sigma_n \sim 2 \cdot 10^{-4} \text{T} \).

In spite the above calculation is very rough and naive the order of magnitude seems to be correct: for a reasonable decreasing of dynamic aperture we should take into account such a value of the multipole field errors, which yields the magnetic field amplitude \( \sigma_n \sim 1 \cdot 10^{-4} \text{T} \) at the border of the dynamic aperture (\( r_0 \sim 20 \pm 30 \text{mm} \)).

Now we can formulate the following practical conclusion for the multipoles scaling: multipoles with different \( n \) reduce dynamic aperture by around the same factor if they produce the same amplitude of magnetic field at the boundary of a stable motion.

More realistic illustration of the DA reduction by small magnetic errors is given in Fig.1.

![Figure 1: The 6-order resonance produced by the error field remains stable up to the certain level at which the resonance is destroyed and the dynamic aperture reduces from ~30 mm to ~25 mm.](image)

FIELD ERRORS AND SIMULATION

RESULTS

By the moment the bulk of the ALBA magnets is manufactured and measured, and the measurement results are used here for the simulation.

Two kinds of errors can be mentioned: (a) the systematic one relates to the particular magnet design and (b) the random one shows quality of the magnet production. Both error types were extracted from the measurements and normalized to the main multipole of the magnet. The random errors were distributed in the magnets according to the Gaussian distribution truncated at \( \pm 2 \sigma \) to avoid unrealistically large field errors. Five seeds for every random error level were used to calculate the average and rms DA values. As the DA reduction due to the multipole errors is small, the scaling factors \( (N = 1e, 5e, 10e, 20e, 50e) \) were used to look at the sensitivity of the DA to the error magnitude.

As it is difficult to consider and interpret all possible kind of magnetic errors, we tried to classify them on their potential danger for the beam dynamics.

Bending Magnet

The sextupole component is considered as the main error of the ALBA bending magnet. The measured profile of the sextupole component (Fig.2) along the reference orbit demonstrates two peaks at the both magnet ends and some low level of the residue sextupole inside the magnet. The peaks are attributed to the non perpendicular central trajectory in the focusing fringe field of a rectangular magnet and should not be confused with the 1D pseudo-sextupole term in the dipole end-field region \( \sim d^2B_1 / ds^2 \) [3]. The integral of this term along the orbit is equal to zero while the integral of the peaks in Fig.2 is around +3 T/m (which is still by order of magnitude less than the field integral for the chromatic sextupole).
According to the results in Fig.2, the systematic sextupole component was inserted in all the bending magnets and the dynamic aperture was defined by tracking (Fig.3).

One can observe decrease of the -3\% off-energy DA which seems not so critical. The sextupole in the bending magnets provides also the residue chromaticity (mainly the vertical one) but its value is also negligible and re-optimization of the chromatic sextupoles is not needed.

**Quadrupole Magnet**

Quadrupole and sextupole magnets have been measured at BINP by the rotation coil technique. The measurement accuracy is ±10^{-4} for the low harmonic numbers and it falls down to ±0.5·10^{-4} for the high harmonic numbers.

The measured field harmonics taken as 1-error levels for the quadrupoles are listed in Table 1. The amplitude for other measured harmonics (up to n = 20) is <10^{-4} and in this study we use 1-e sys = 0 and 1-e ran = 1.

<table>
<thead>
<tr>
<th>n</th>
<th>Normal ×10^4</th>
<th>Skew ×10^4</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1-e sys</td>
<td>1-e ran</td>
</tr>
<tr>
<td>2</td>
<td>10000</td>
<td>10000</td>
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<td>3</td>
<td>0</td>
<td>3</td>
</tr>
<tr>
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<td>0</td>
<td>2</td>
</tr>
<tr>
<td>6</td>
<td>-1</td>
<td>1</td>
</tr>
<tr>
<td>10</td>
<td>-3</td>
<td>1</td>
</tr>
</tbody>
</table>

At first all the errors were considered separately to study the influence of every individual error type to the DA, then a combined effect of all the multipoles was studied. Fig.4 shows the on-energy DA shrink caused by 10\(e\) of different magnetic multipoles, from the single one \(b_6\) to the joint action of all the normal and skew multipoles distributed in the ALBA quadrupole magnets.

The DA decrease resulted from the action of all the multipoles as a function of the error scale factor is shown in Fig.5.

One can see that up to the level of ~5÷10\(e\) (in the units of Table 1) the reduction of the aperture seems acceptable.

**Sextupole Magnet**

The field of high harmonics measured by rotating coil at \(r_0 = 25\) mm and referred to the sextupole field has the same magnitude as for the quad: 10^4 · \(\Delta B_n / B_2\) = 1 + 4. But taking into account weaker sextupole field and less total length of the sextupoles compare to the quadrupoles, it
seems one may neglect the influence of these high field components in the sextupole magnet to the dynamic aperture.

Figure 3: Sextupole harmonics with the horizontal steering coil switched on (at the maximum current 5 A) and off

More serious effect comes from correction coils (dipole and skew-quadrupole) wound at the sextupole yoke (Fig.3). While switched at the maximum current of 5 A, the coils produce at 25 mm radius the field amplitude ~0.003 T (with the field of the main sextupole coil is ~0.2 T), which is by factor 10 higher than $\sigma_5 = 2 \cdot 10^{-4}$ T estimated above. Corresponding field components ($b_5$ for the vertical correction field, $a_5$ for the horizontal one and $a_4$ for the skew-quad coil) were extracted from the magnetic measurements and used for the DA estimation. As the realistic distribution of these fields depends on the COD and the value of the linear coupling, which at the moment is unknown, we have spread the relevant harmonics randomly in the relevant sextupoles and simulate the dynamic aperture. The results are given in Fig.4.

Figure 4: ALBA DA reduction due to the multipole errors in the sextupole magnets with 5 A maximum current in the correction coils

One can see from Fig.4 that the main source of the aperture limitation is the skew-octupole term parasitically produced by the skew-quadrupole correction coils. As the maximum current of 5 A seems overestimated for the regular machine operation, we have repeated the calculation for 1 A maximum current (Fig.5).

Figure 5: DA reduce due to the sextupole correction coils energized by 1 A of maximum current

CONCLUSIONS

Results of the systematic study of the ALBA light source dynamic aperture caused by different sorts of magnetic field imperfections are presented. The sextupole component in the bending magnets as it seems does not provide serious effect on the dynamic aperture. Multipole field components in the quadrupoles also seem not too dangerous up to the relative field error $10^4 \cdot \Delta B_n / B_n \approx 5$. In this connection it is worth to note that a tendency in the magnet design for the modern storage rings to make the tolerance of the multipole error as tough as $10^4 \cdot \Delta B_n / B_n \approx 1$, and even better, seems a bit excessive.

Steering and skew-quadrupole correction coils been installed in the sextupoles to save the room at the ring should be considered as a possible source of rather strong magnetic imperfection and studied carefully from the viewpoint of dynamic aperture deterioration.

REFERENCES