MECHANICAL MODE DAMPING
IN SUPERCONDUCTING LOW $\beta$ RESONATORS

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Abstract
Mechanical instability is often the main obstacle to reaching high field in superconducting low beta resonators during accelerator operation. Among many types of instability, the resonant mechanical modes are particularly dangerous. This problem is being discussed in the framework of low $\beta$ quarter wave resonators, and a new solution, based on mechanical damping, is proposed. Theoretical modelling and experimental results are presented, which demonstrate that this technique could eliminate the necessity of fast tuners and related electronics in many superconducting cavities, resulting in a significant cost saving both in the construction and in the operation of superconducting low beta linacs.

1 INTRODUCTION
Superconductive resonators are characterised by very high rf quality factors and, consequently, by very small rf resonance bandwidth $\Delta f \equiv f / Q$ [1]. As an example, in a 100 MHz cavity with a quality factor of $10^9$, the bandwidth is only 0.1 Hz; this makes virtually impossible to tune and lock it to an external reference frequency in a real accelerator, since even small mechanical deformations can easily displace the centre frequency by an amount exceeding 0.1 Hz by far. In real cavities the loaded quality factor is dominated by the coupling conditions: in high current accelerators the real bandwidth is increased by many orders of magnitude by the beam loading requirements; in low current accelerators, on the other hand, overcoupling is needed just to increase the resonator bandwidth and most of the input rf power is being reflected back by the cavity and dissipated at room temperature. In existing low $\beta$ cavities working below 100 MHz, due to their large dimensions and their particular geometry, mechanical instabilities can cause frequency fluctuations up to hundreds of hertz in a few milliseconds. Since the rf power cannot be increased over some reasonable value which justifies the choice of superconductivity, in most cases this problem is being partially solved by means of fast rf tuners [2], [3], [4]. These devices, which are variable reactance coupled to the resonators, can change the centre frequency of the mode by a maximum amount proportional to $P_R / E_a^2$, where $P_R$ is
the reactive power in the fast tuner and $E_a$ the resonator accelerating field. Fast tuners can generally solve instability problems, but they introduce new complications in the rf system and significant power losses; they usually put a limit to the maximum accelerating field achievable by the resonator. Mechanical tuners with a fast response were developed as well [5], but they did not find an application in low beta cavities. Some questions arise: since the problem is serious and the existing solutions are not free of drawbacks, is it possible to use a simple, passive mechanical damping system to the resonator to improve its mechanical stability? What would be the requirements of such a device, and how far could it go in the solution of the problem?

2 MECHANICAL INSTABILITIES IN LOW $\beta$ CAVITIES

2.1 MAIN CAUSES OF MECHANICAL DETUNING
Due to the necessity of low rf frequencies, low $\beta$ resonators are generally either 2,3 or 4-gap quarter wave structures [6]. In such geometry the rf resonant frequency is determined mainly by the inner conductor length and by the accelerating gap capacitance. Changes of the geometry can be caused by variations of the liquid or gas helium pressure in the cryogenic system; by microphonic noise originated outside the cryostat and transmitted to it through the ground, the beam pipes, the helium pipes and even through the air; by mechanical movements inside the cryostat (tuners, couplers, helium boiling); and, finally, by forces
generated by the electromagnetic field [7], [8]. Deformations can be either resonant or non-resonant; the formers are generated by forces applied to the resonator at the frequency of one of its mechanical resonant modes. Non-resonant deformations depend only on the strength of the external force related to the strength of the structure; the most common ones, which can be usually compensated by means of mechanical tuners, are originated by slow fluctuations of the helium bath pressure. In resonant deformation, however, the displacement from the original shape depends mainly on the quality factor of the mechanical mode (typically between few hundreds and few thousands); even very weak forces, if applied at resonance, can cause relatively large deformations and cavity detuning. It is generally accepted that these modes do not give any problem if they are above 150 Hz; since a typical mechanical eigenfrequency in low β cavities can be as low as a few tens of Hz, this is the main reason for the necessity of fast tuners.

2.2 DANGEROUS MECHANICAL MODES IN QUARTER WAVE RESONATORS

2.2.1 Mechanical detuning observed at LNL

At LNL we have developed four different types of niobium resonators, with optimum velocity $\beta = 0.047$ and 0.055, 0.11, 0.15, working at 80, 160 and 240 MHz, respectively [9]. The inner conductor, a uniform tube made of niobium 2 mm thick with 30 mm outer radius, is welded to a very rigid, 12 mm thick niobium disk. We observed that we had two main kinds of deformations: the first one is related to the effect of the helium pressure on the disk which changes the effective inner conductor length $L$; since in quarter wave resonators $f' = \lambda / 4$, this causes a frequency shift $\delta f / f \propto \delta L / L$. In the case of the 80 MHz cavities (see fig. 1) we have about 40 Hz/µm, and the response that we have measured can vary from 0 to 1 Hz/mbar in different resonators. In a properly operating cryogenic system these pressure changes are usually slow and their effects can be compensated by means of our standard mechanical tuners.

The second, more dangerous kind of deformation is the lowest mechanical mode of vibration of the inner conductor. This motion changes the resonator geometry in the high field region around the beam ports, causing a frequency shift roughly proportional to the oscillation amplitude. No problem appeared in operating the shorter, higher β cavities. For our 80 MHz, 1m long cavity, however, we observed a 42 Hz mode, with a
mechanical quality factor of about 2000, which can give, when excited resonantly, a significant frequency shift.

2.2.2 Mathematical description of the dangerous modes

The vibration of a uniform tube of length $L$, with one end clamped and one end free, has well known properties (see, e.g., ref. [10]). The shape is given by $y(z,t) = y_0 W(z/L) \cos(\omega t + \varphi)$, where $y_0 \equiv y(L)$ is the displacement at the tube vertex and $z$ is the coordinate along the tube ($z = 0$ at the clamped end); here,

$$W(z,L) = \frac{1}{2} \left[ \cosh\left(\frac{\alpha z}{L}\right) - \cos\left(\frac{\alpha z}{L}\right) - 0.734 \left\{ \sinh\left(\frac{\alpha z}{L}\right) - \sin\left(\frac{\alpha z}{L}\right) \right\} \right]$$

(1)

and $\alpha$ is a constant characteristic of the mode (fig. 2). It should be noted that, for the $1^{st}$ mode,

![Figure 2: Shape of the 1st and 2nd models of a vibrating beam with a clamped end.](image)

The eigenfrequencies are given by

$$\int_0^L W^2(z,L) dz = \frac{1}{4} L$$

(2)

where $E$ is the Young modulus of the material, $I$ is the geometrical moment of inertia of the tube, $\mu L$ is the mass per unit length, $\alpha_1 \approx 1.875, \alpha_2 \approx 4.694$ for the two lowest modes. For a tube with inner radius $r_i$ and outer radius $r_o$, $I = \frac{\pi}{4} (r_o^4 - r_i^4)$. We can see that increasing the wall thickness by reducing the inner radius, thus preserving the rf geometry of the resonator, would lower the eigenfrequencies even more. The
calculated eigenfrequency of the first two modes of the LNL niobium cavities are shown in tab. 1; the 45 Hz
eigenfrequency calculated with tabulated mechanical parameters is very close to the 42 Hz one that we
measured in the real cavity. This mode, because to its high quality factor, could lead to a large frequency
shift and to the resonator unlocking; all other modes are above 150 Hz and this explains why they did not
cause any problem. Similar behaviour can be expected from resonators made of copper, since the Young
modulus and the density values of this material are similar to the Niobium ones.

<table>
<thead>
<tr>
<th>resonator type</th>
<th>low $\beta$</th>
<th>medium $\beta$</th>
<th>high $\beta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>optimum velocity $\beta$</td>
<td>0.047, 0.055</td>
<td>0.11</td>
<td>0.17</td>
</tr>
<tr>
<td>resonator frequency (MHz)</td>
<td>80</td>
<td>160</td>
<td>240</td>
</tr>
<tr>
<td>inner conductor length (cm)</td>
<td>93.5</td>
<td>47</td>
<td>31</td>
</tr>
<tr>
<td>1\textsuperscript{st} mechanical mode frequency (Hz)</td>
<td>45</td>
<td>179</td>
<td>412</td>
</tr>
<tr>
<td>2\textsuperscript{nd} mechanical mode frequency (Hz)</td>
<td>284</td>
<td>1123</td>
<td>2582</td>
</tr>
</tbody>
</table>

Tab. 1. Calculated resonant frequency for the lower mechanical modes in the LNL bulk niobium resonators.

Two natural solutions appear for such problem: the first one is designing resonators with mechanical modes
above 150 Hz; this could lead to modifications of the rf design of the resonator, or to the addition of reinforcing
mechanical parts made of materials with high value of the Young modulus. This method is not always
applicable to quarter wave resonators working below 100 MHz without increasing the resonator cost above
reasonable limits. The second solution is lowering of the mechanical quality factor in order to reduce the
steady state amplitude of the vibration: this method can lead to very good results.

3 QUARTER WAVE RESONATORS MECHANICAL MODES DAMPING

3.1 DISSIPATOR DESIGN CONCEPTS

In order to reduce the mechanical quality factor, a
dissipator can be introduced. Desirable characteristics are:
a) high sensitivity to micron-scale vibrations; b) strong
damping; c) no new modes introduced; d) capability to
work in liquid helium; e) no limitations to the resonator rf
performance; f) possibility to apply it to existing resonators,
and g) good predictability of its mechanical properties at
the design stage. The nature of the dangerous modes,
which spreads the tiny displacement of the inner conductor
vertex over a long distance, prevents us from using
general purpose dampers that could subtract only a small
fraction of the mechanical power from the oscillation;
moreover, absorbing materials are not very efficient at
liquid helium temperature. As a consequence, a special
device which couples strongly with the mode to be
suppressed must be used, in analogy with the higher order
rf modes damping technique; a friction damper is a natural
choice

Fig. 3 The LNL mechanical damper.
1) Reinforcing tube;
2), 3) load;
4) terminating disk;
5) centring pins
The conceptual design of the LNL damper [11] is in fig. 3. It consists of a reinforcing tube and a load. The reinforcing tube, made of stainless steel and terminated with a brass disk, is inserted in the niobium inner conductor and welded to the stainless steel flange that allows for liquid helium cooling of the resonator top plate. A load, located inside the reinforcing tube and sitting on the brass disk, is free to slide over it and kept on the niobium tube axis by means of three centring pins. The system is done in such a way that every oscillation of the inner conductor will produce a slide of the load, and corresponding power dissipation.

### Tab. 2. Parameters of the dissipator tested with an 80 MHz niobium resonator.

<table>
<thead>
<tr>
<th>dissipator load</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>mass $m_l$</td>
<td>120 g</td>
</tr>
<tr>
<td>dynamic friction coeff. $D$</td>
<td>0.54</td>
</tr>
<tr>
<td>static friction coeff. $S$</td>
<td>0.45</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>reinforcing tube</th>
<th>inner conductor</th>
</tr>
</thead>
<tbody>
<tr>
<td>material</td>
<td>stainless steel</td>
</tr>
<tr>
<td>density</td>
<td>$7.8 \text{g/cm}^2$</td>
</tr>
<tr>
<td>Young modulus</td>
<td>$2.11 \times 10^{11} \text{Nm}^{-2}$</td>
</tr>
<tr>
<td>inner radius</td>
<td>20 mm</td>
</tr>
<tr>
<td>outer radius</td>
<td>24 mm</td>
</tr>
<tr>
<td>total length</td>
<td>350 mm</td>
</tr>
<tr>
<td>effective length $z$</td>
<td>330 mm</td>
</tr>
</tbody>
</table>

The LNL dissipator was developed for the niobium 80 MHz resonators [12]; the dissipator parameters (see tab. 2) were chosen so that the maximum common mode vibration amplitude would give tolerable frequency fluctuations, while keeping the resonator vibrations at low level for a wide range of external vibration power. The reinforcing tube length is determined by the necessity of having a high resonant frequency of the tube itself, and having at the same time the dissipator in a position where the inner conductor oscillation amplitude is still significant. The tube material, stainless steel, have a reasonably high Young modulus and good mechanical properties. The sliding surfaces materials, brass and stainless steel, have static and dynamic friction coefficients of similar values, favouring a smooth response of the device.

### 3.2 THEORETICAL MODEL

The stored energy of the mode can be calculated as a function of the displacement of the vertex, $y_0$. From (2),

$$U(y_0) = \int_0^L \frac{1}{2} \mu \omega^2 y_0^2 W^2(z, L) dz \equiv \frac{1}{8} \mu L \omega^2 y_0^2$$

(3)
In order to have sliding of the dissipator, the force on the load must overcome the value \( m_i g S \), where \( S \) is the static friction coefficient and \( m_i \) is the load mass. To calculate the relative forces at the position \( z \) between niobium tube and stainless steel tube before sliding, it is possible to treat the vibrating tubes as simple harmonic oscillators, defining an equivalent mass \( m(z) \) and an equivalent restoring force

\[
-k(z)\dot{y}(z) = \frac{\partial U}{\partial y(z)}
\]

concentrated at a distance \( z \) from the clamped end, so that

\[
\frac{1}{2}k(z)y^2(z) = U(y_0)
\]

and \( \omega = \sqrt{\frac{k(z)}{m(z)}} \). Consequently,

\[
k(z) = \frac{\mu L \omega^2}{4W^2(z, L)}
\]

and

\[
m(z) = \frac{\mu L}{4W^2(z, L)}
\]

By means of this simple but useful approximation it is possible to estimate the properties of the modes of two coaxial tubes clamped at one end and connected together at the point \( z \) where the dissipator is located. In the following formulas the subscript \( C \) will be related to the niobium conductor, while the subscript \( R \) will refer to the reinforcing tube.

The common mode vibration of the two tubes together will have a frequency

\[
y_{CM}(z) = \frac{2m_i g S}{k_c(z) - k_r(z + \zeta) - \omega^2_{CM} [(m_c(z) + m_i) - m_r(z + \zeta) - m_r(z + \zeta)]}
\]

where \( \zeta \) is the distance between the clamped ends of the reinforcing and the conductor tube. The sliding starts when the maximum common mode amplitude \( y_{CM}(z) \) is reached, where

At the top of the inner conductor this amplitude will be

\[
y_{0CM} = \frac{y_{CM}}{W(z, L_c)}.
\]

The power dissipated by the load located at the position \( z \) can be approximated, when \( y_0 \geq y_{0CM} \), with the following expression:

\[
P_i = 4m_i g D(y_0 - y_{0CM}) W(z, L) f \quad \text{for} \quad y_0 \geq y_{0CM}
\]

\[
P_i = 0 \quad \text{for} \quad y_0 \leq y_{0CM}
\]
where $D$ is the dynamic friction coefficient between load and terminating disk and $g = 9.8 \text{ms}^{-2}$; we have assumed that the two tubes will move together in the part of the cycle where $y_0 \leq y_{0CM}$, and that they will move independently for $y_0 \geq y_{0CM}$; if the energy loss per cycle is much lower than the total stored energy, and if the mechanical eigenfrequencies of the two tubes are very far each other, this appears to be a reasonable approximation. The dissipator will introduce in the mechanical quality factor a term

$$Q_i = \frac{\omega U(y_0)}{P_i} = \frac{\pi \mu L \omega^2}{16 m_i g D W(z, L)} (y_0 - y_{0CM}) \quad (8)$$

Since $Q_i$ is proportional to the vibration amplitude, the dissipator effect will be weaker at high amplitudes. The total mechanical quality factor is

$$Q_T = \left( Q_{M}^{-1} + Q_i^{-1} \right)^{-1} \quad (9)$$

where $Q_M$ is the mechanical quality factor of the mode in the absence of the dissipator.

3.3 THEORETICAL AND EXPERIMENTAL RESULTS

The calculated effect of the dissipator on the inner conductor oscillation amplitude as a function of the power coupled to the resonator by an external vibration source is shown in fig. 4; we have used only tabulated parameters for material density, Young modulus and friction coefficients. The experimental set-up is sketched in fig. 5. The attenuation value reaches a maximum, and then it slows down, as the oscillation becomes wider, as foreseen by our model. It is clear that early unsticking and large damping are somehow conflicting:
the heavier is the mass, the later the damping starts but the higher is the power dissipated; the same argument is valid for the static and dynamic friction coefficient values, which should be chosen as close as possible in order to obtain smooth sliding. In our case, we have set experimentally a maximum common mode amplitude of about 2 Hz peak to peak, smaller than our operation rf resonance bandwidth.

The results of our measurements, performed at room temperature and at liquid helium temperature, are shown in fig. 6. At room temperature we could excite directly the resonator by means of a mechanical vibrator, and we could verify a linear response of the mechanical mode with the excitation amplitude; at liquid helium temperature we had a much higher sensitivity but we could excite the resonator only indirectly through the cryostat, thus introducing also the cryostat response. In both cases the highest vibration-induced frequency error was measured, with and without activating the mechanical dissipator. The slope of the attenuation in the real system seems to be slightly more regular than calculated; this is an indication that also some weak dissipation mechanisms exist which are not included in our model. Moreover, for very low vibrator power the frequency error is higher with the dissipator rather than without it: this can be explained by the fact that reducing the mechanical quality factor of the cavity could make it more sensitive to white noise as a result of a wider resonance curve. However, a substantially good agreement between calculations and experiment can be observed, and a very good stabilisation of the resonator performed by the dissipator: the frequency error is attenuated from 10 Hz to 2.5, from 100 Hz to 5 and from 1000 Hz to 20. This puts a strong motivation to install a dissipator in every low beta resonator and suggests the possibility to eliminate completely the need of electronic fast tuners.

Mechanical resonance measurement setup

*Fig. 5 Experimental set-up of the dissipator measurements*
4 CONCLUSIONS

Resonant mechanical instabilities in low $\beta$ superconducting cavities, an old enemy that requires complex electronics to be contrasted, can be strongly attenuated by mechanical damping. Strong coupling to the mode to be suppressed is needed, in analogy with the technique developed for rf higher order mode damping. This technique will allow in the future, at least in some cases like in quarter wave resonators, the elimination of fast tuners which are often the limiting factor for achieving high accelerating field in superconductive low $\beta$ accelerators; in other cases this will allow the use of much cheaper and less demanding devices. Moreover, designing of superconducting cavities with very low frequency, which presented until now a very high risk of uncontrollable instabilities, could become feasible. This technique is very promising and should be extended, from the design stage, to all low $\beta$ superconductive resonators with low frequency mechanical modes.

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