Coherent electron cooling*
Free Electron Lasers and High-energy Electron Cooling**

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FELs and colliders

From the first talk on Coherent Electron Cooling at International FEL Conference, Novosibirsk, Russia, August, 2007

And so, my fellow FELers, ask not what storage rings can do for FELs; Ask what FELs can do for our storage rings?

And so, my fellow Americans, ask not what your country can do for you; ask what you can do for your country?

Measure of Collider Performance is the Luminosity

\[
\dot{N}_{\text{events}} = \sigma_{A\rightarrow B} \cdot L
\]

\[
L = \frac{f_{\text{coll}} \cdot N_1 \cdot N_2}{4\pi\beta^*\varepsilon} \cdot g(\beta^*, h, \theta, \sigma_z)
\]

Main sources of luminosity limitation

- Beam-Beam effects
- Large or growing emittance
- Long or growing bunch-length, i.e. Beam disruption and Hour-glass effect
- Crossing angle
- Beam Intensity & Instabilities

In many cases an Effective Cooling can significantly increase Luminosity

V.N. Litvinenko, 2009 Particle Accelerator Conference, Vancouver, May 8, 2009
## Examples of hadron beams cooling

<table>
<thead>
<tr>
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</tr>
</thead>
<tbody>
<tr>
<td>RHIC</td>
<td>Au</td>
<td>130</td>
<td>~1</td>
<td>20,961 ∞</td>
<td>~ 1</td>
<td>0.015/0.05</td>
</tr>
<tr>
<td>RHIC</td>
<td>p</td>
<td>250</td>
<td>~100</td>
<td>40,246 ∞</td>
<td>&gt; 30</td>
<td>0.1/0.3</td>
</tr>
<tr>
<td>LHC</td>
<td>p</td>
<td>7,000</td>
<td>~ 1,000</td>
<td>13/26</td>
<td>∞ ∞</td>
<td>0.3/&lt;1</td>
</tr>
</tbody>
</table>

### Potential increases in luminosities:

- RHIC polarized pp > 2 fold, eRHIC > 5 fold, LHC > 2 fold

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V.N. Litvinenko, 2009 Particle Accelerator Conference, Vancouver, May 8, 2009
Content

• A bit of history
• Principles of Coherent Electron Cooling (CeC)
• Analytical estimations, Simulations
• Proof of Principle test using R&D ERL at BNL
• Conclusions
History

possibility of coherent electron cooling was discussed qualitatively by Yaroslav Derbenev about 28 years ago

- Coherent electron cooling, Ya. S. Derbenev, Randall Laboratory of Physics, University of Michigan, MI, USA, UM HE 91-28, August 7, 1991
- Ya.S.Derbenev, Electron-stochastic cooling, DESY , Hamburg, Germany, 1995 .........

UM HE 91-28
August 7, 1991

CONCLUSION

The method considered above combines principles of electron and stochastic cooling and microwave amplification. Such an unification promises to frequently increase the cooling rate and stacking of high-temperature, intensive heavy particle beams. Certainly, for the whole understanding of new possibilities thorough theoretical study is required of all principle properties and other factors of the method.
Q: What's new in today's presentation?

- This is a new Coherent electron Cooling scheme and the first with complete analytical and quantitative evaluation.
- The spirit of amplifying the interaction remains the same as in 80's, but the underlying physics of interaction is different and also specific.
- ERLs and FEL did advanced in last 30 years – hence, the practicality of this scheme.
- Now we can analytically estimate and numerically calculate Coherent electron Cooling cooling decrements for a wide variety of cases.


Coherent Electron Cooling

Comprehensive option

Economic option

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Coherent Electron Cooling is nothing else but FEL based version of van der Meer’s longitudinal stochastic cooling

- It has pick-up (the modulator)
- It has an amplifier (the FEL)
- It uses time-of-flight dependence on energy of a particle
- It has a kicker (e-beam)

Main differences / advantages:
- The use of electron beam gives a lot of flexibility
- FEL amplifier has HUGE bandwidths of $10^{13}$-$10^{15}$ Hz
Longitudinal cooling, ultra-relativistic case ($\gamma >> 1$)

$$\omega_p = \sqrt{4\pi n_e e^2 / \gamma o m_e}$$

$$q = -Ze \cdot (1 - \cos \varphi_i)$$

$$q_{peak} = -2Ze$$

$$\rho_k = kq(\varphi_i); \quad n_k = \frac{\rho_k}{2\pi\beta\epsilon_k}$$

Amplifier of the e-beam modulation in an FEL with gain $G_{\text{FEL}} \sim 10^2 - 10^3$

$$A_{\perp} = 2\pi\beta_{\perp} e_n / \gamma_o$$

$$k_{\text{FEL}} = 2\pi / \lambda_{\text{FEL}}; \quad k_{\text{cm}} = k_{\text{FEL}} / 2\gamma_o$$

$$n_{\text{amp}} = G_o \cdot n_k \cos(k_{\text{cm}}z)$$

$$\Delta \varphi = 4\pi n \Rightarrow \varphi = -q_{\parallel} \cdot \cos(k_{\text{cm}}z)$$

$$\vec{E} = -\vec{\nabla} \varphi = -2\vec{E}_o \cdot X \sin(k_{\text{cm}}z)$$

$$E_o = 2G_o\gamma_o e / \beta\epsilon_{\text{in}}$$

$$X = q / e \approx Z(1 - \cos \varphi_i) \sim Z$$

 Dispersion section (for hadrons)

$$\lambda_{\text{FEL}} = \lambda_{\perp} \left(1 + \left(\frac{\bar{a}_{\perp}^2}{2\gamma_o^2}\right)\right) / 2\gamma_o^2$$

$$\bar{a}_{\perp} = e\bar{A}_{\perp} / mc^2$$

$$L_{\text{Go}} = \frac{\lambda_{\perp}}{4\pi\rho\sqrt{3}}$$

$$L_G = L_{\text{Go}} (1 + \Lambda)$$

$$G_{\text{FEL}} = e^{L_{\text{FEL}} / L_G}$$

$$\Delta \varphi = \frac{L_{\text{FEL}}}{3L_G}$$

Dispersion

$$c\Delta t = -D \cdot \frac{\gamma - \gamma_o}{\gamma_o}; \quad D_{\text{free}} = \frac{L}{\gamma}; \quad D_{\text{chicane}} = l_{\text{chicane}} \cdot \theta^2$$

Debye radii

$$R_{D\perp} >> R_{D\parallel}$$

$$R_{D\parallel,ab} = \frac{e\sigma_{\parallel}}{\gamma o} \omega_p$$

$$R_{D\perp,ab} = \frac{e\sigma_{\perp}}{\gamma o} \omega_p < \lambda_{\text{FEL}}$$

High gain FEL (for electrons)

Kicker

Modulator

Dispersion

Hadrons

Electrons

At a half of plasma oscillation
e-Density modulation caused by a hadron (co-moving frame)

Induces charge: \( q = -Ze \cdot (1 - \cos \omega_p t) \)

**Analytical:** for kappa-2 anisotropic electron plasma, 

\[
\tilde{n}(\mathbf{r}, t) = \frac{Ze \omega_p^2}{2\pi \sigma_v \sigma_v \sigma_v} \int_0^t \sin \tau \left( r^2 + \left( \frac{x - v_h \tau / \omega_p}{r_{Dv}} \right)^2 + \left( \frac{y - v_h \tau / \omega_p}{r_{Dv}} \right)^2 + \left( \frac{z - v_h \tau / \omega_p}{r_{Dv}} \right)^2 \right)^{-\frac{3}{2}} d\tau
\]

**Numerical:** VORPAL @ TechX

Parameters of the problem

\[
R_{Dv} \propto \left( |v_x| + \sigma_v \right) / \omega_p; \quad \alpha = x, y, z
\]

\[
t = \tau / \omega_p; \quad \tilde{v} = \bar{v} \sigma_v; \quad \tilde{r} = \bar{r} \sigma_v / \omega_p; \quad \omega_p = \sqrt{\frac{4\pi e^2 n_e}{m}}
\]

\[
S = r_{Dv}; \quad \xi = \frac{Z}{4\pi \bar{m} R^2 \bar{v}},
\]

\[
A = \frac{a}{s}; \quad X = \frac{x_{ho}}{a}; \quad Y = \frac{y_{ho}}{a}.
\]

Figure 3: A transverse cross section of the wake behind a gold ion, with the color denoting density enhancement.

V.N. Litvinenko, 2009 Particle Accelerator Conference, Vancouver, May 8, 2009
Numerical simulations (VORPAL @ TechX)
Provides for simulation with arbitrary distributions and finite electron beam size

VORPAL Simulations Relevant to Coherent Electron Cooling, G.I. Bell et al., EPAC'08, (2008)

\[ R = \frac{\sigma_{v \perp}}{\sigma_{v z}}; \quad T = \frac{V_{hx}}{\sigma_{v z}}; \quad L = \frac{V_{hz}}{\sigma_{v z}} \]

\[ q = -Ze \cdot (1 - \cos \omega_p t) \]
Central Section of Coherent electron Cooling

Electron density modulation is amplified in the FEL and made into a train with duration of \( N_c \sim \frac{L_{\text{gain}}}{\lambda_w} \) alternating hills (high density) and valleys (low density) with period of FEL wavelength \( \lambda_w \). Maximum gain for the electron density of High Gain FEL is \( \sim 10^3 \).

\[
\lambda_{\text{fel}} = \lambda_w \left(1 + \left\langle a_w^2 \right\rangle \right)/2\gamma_o^2 \quad L_{\text{Go}} = \frac{\lambda_w}{4\pi\rho\sqrt{3}} \quad L_G = L_{\text{Go}}(1 + \Lambda)
\]

Electron density modulation is amplified in the FEL and made into a train with duration of \( N_c \sim \frac{L_{\text{gain}}}{\lambda_w} \) alternating hills (high density) and valleys (low density) with period of FEL wavelength \( \lambda_w \). Maximum gain for the electron density of High Gain FEL is \( \sim 10^3 \).

\[
v_{\text{group}} = \left( c + 2v_{//} \right)/3 = c \left( 1 - \frac{1}{2\gamma^2} + \frac{1}{3\gamma^2} \right) = c \left( 1 - \frac{1}{2\gamma^2} \right) + \frac{c}{3\gamma^2} \left( 1 - 2a_w^2 \right) = v_{\text{hadrons}} + \frac{c}{3\gamma^2} \left( 1 - 2a_w^2 \right)
\]

Economic option requires: \( 2a_w^2 < 1 \) !!!
3D FEL response
calculated Genesis 1.3, confirmed by RON

Main FEL parameters for eRHIC with 250 GeV protons

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Energy, MeV</td>
<td>136.2</td>
</tr>
<tr>
<td>Peak current, A</td>
<td>100</td>
</tr>
<tr>
<td>Bunchlength, psec</td>
<td>50</td>
</tr>
<tr>
<td>Emittance, norm</td>
<td>5 mm mrad</td>
</tr>
<tr>
<td>Energy spread</td>
<td>0.03%</td>
</tr>
<tr>
<td>Wiggler</td>
<td>Helical</td>
</tr>
</tbody>
</table>

The amplitude (blue line) and the phase (red line, in the units of $\pi$) of the FEL gain envelope after 7.5 gain-lengths (300 period). Total slippage in the FEL is $300\lambda$, $\lambda=0.5$ µm. A clip shows the central part of the full gain function for the range of $\zeta$={50\lambda, 60\lambda}.

\[ G(\zeta) = G_o \text{Re}(K(\zeta) \cdot e^{ik\zeta}); \zeta = z - vt; k = \frac{2\pi}{\lambda} \]

\[ \Lambda_k = \iint |K(z - \zeta)|^2 d\zeta \]
Evolution of the maximum bunching in the e-beam and the FEL power simulated by Genesis. The location of the maxima, both for the optical power and the bunching progresses with a lower speed compared with prediction by 1D theory, i.e. electrons carry ~75% for the "information".

\[
V_g \approx \frac{c + 3 \langle v_z \rangle}{4} = c \left(1 - \frac{3}{8} \frac{1 + a_w^2}{\gamma_o^2}\right)
\]

Evolution of the maxima locations in the e-beam bunching and the FEL power simulated by Genesis. Gain length for the optical power is 1 m (20 periods) and for the amplitude/modulation is 2m (40 periods).
The Kicker

A hadron with central energy \( E_0 \) phased with the hill where longitudinal electric field is zero, a hadron with higher energy \( E > E_0 \) arrives earlier and is decelerated, while hadron with lower energy \( E < E_0 \) arrives later and is accelerated by the collective field of electrons.

**Analytical estimation**

\[
\Delta \varphi = 4 \pi \rho \Rightarrow \varphi = -\frac{8G \cdot Z e}{\pi \beta z \cdot k_{cm}} \cdot \cos(k_{cm} z); \quad \vec{E} = -\vec{\nabla} \varphi = -\frac{8G \cdot Z e}{\pi \beta z} \cdot \sin(k_{cm} z)
\]

**Periodical longitudinal electric field**

\[
\frac{dE}{dz} = -eE_{\text{peak}} \cdot \sin\left\{kD \frac{E - E_0}{E_0}\right\}; \quad kD\sigma_\delta \sim 1
\]

\[
\sigma_\delta = \frac{\sigma_E}{E_0}
\]

\[
\xi_{\text{CEC}} = -\frac{\Delta E}{E - E_0} \approx \frac{e \cdot E_0 \cdot l_2}{\gamma_0 m_pc^2 \cdot \sigma_\epsilon} \cdot \frac{Z^2}{A}
\]

**Simulations: only started**

Step 1: use 3D FEL code out output + tracking

First simulation indicate that equations on the left significantly underestimate the kick, i.e. the density modulation continues to grow after beam leaves the FEL

Output from Genesis propagated for 25 m

Step 2:

use VORPAL with input from Genesis, in preparation

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V.N. Litvinenko, 2009 Particle Accelerator Conference, Vancouver, May 8, 2009
Analytical formula for damping decrement

- 1/2 of plasma oscillation in the modulator creates a pancake of electrons with the charge \(-2Ze\)
- Electron clamp is well within \(\Delta z \sim \lambda_{FEL}/2\pi\)
- Gain in SASE FEL is \(G \sim 10^2\text{ to } 10^3\)
- Electron beam is wider than \(2\gamma_0 \lambda_{FEL}\); it is 1D field
- Length of the kicker is \(\sim \beta\)-function

\[
\zeta = -\frac{\Delta E_i}{E - E_o} = A \cdot \frac{L_2}{\beta} \cdot \chi \cdot \frac{\sin \varphi_3 \cdot \sin \varphi_2 \cdot (\sin \frac{\varphi_1}{2})^2}{\varphi_3 \varphi_2}
\]

\[
A = 2G_o \frac{Z^2}{A} \cdot \frac{r_p}{\varepsilon_{\perp} \sigma_\delta}; \quad \chi = k_{FEL} D \cdot \sigma_\delta;
\]

\[
\varphi_3 = k_{FEL} D \delta; \quad \delta = \frac{E - E_o}{E_o}
\]

\[
\frac{L_2}{\beta} \cdot \chi \cdot \text{sinc}(\varphi_3) \cdot \sin \varphi_2 \cdot \left(\frac{\sin \frac{\varphi_1}{2}}{2}\right) \sim 1
\]

Beam-Average decrement

\[
\int \frac{2J_1(x)}{x} e^{-x^2/2} dx = 0.889
\]

Electron bunches are usually much shorter and cooling time for the entire bunch is proportional to the bunch-lengths ratios

\[
\chi = 1
\]

\[
\langle \zeta_{\text{CeC}} \rangle = \xi \frac{\sigma_{z,e}}{\sigma_{z,h}} = 2 \frac{G_o \sigma_{z,e}}{\sigma_\delta \sigma_{z,h}} \frac{Z^2}{A} \frac{r_p}{\varepsilon_{\perp} \kappa}; \quad \kappa \sim 1
\]
Analytical formula for damping decrement

\[ \langle \xi_{CeC} \rangle = \xi \frac{\sigma_{\tau,e}}{\sigma_{\tau,h}} = \kappa \cdot 2G_o \cdot \frac{Z^2}{A} \cdot \frac{r_p \cdot \sigma_{\tau,e}}{\varepsilon_{\perp n} \left( \sigma_\delta \cdot \sigma_{\tau,h} \right)} ; \kappa \sim 1 \]

\[ \langle \xi_{CeC} \rangle \sim \frac{1}{\varepsilon_{long,h} \varepsilon_{trans,h}} \]

Note that damping decrement

a) Does not depend on the energy of particles!
b) Improves as cooling goes on

It makes it realistic to think about cooling intense proton beam in RHIC & LHC at 100s of GeV and 7 TeV energies
Even though LHC needs one more trick (back up slides)
Transverse cooling

- Transverse cooling can be obtained by using coupling with longitudinal motion via transverse dispersion.
- Sharing of cooling decrements is similar to sum of decrements theorem for synchrotron radiation damping, i.e. decrement of longitudinal cooling can be split into appropriate portions to cool both transversely and longitudinally: \( J_s + J_h + J_v = 1 \).
- Vertical (better to say the second eigen mode) cooling is coming from transverse coupling.

Non-achromatic chicane installed at the exit of the FEL before the kicker section turns the wave-fronts of the charged planes in electron beam.

\[
\delta(ct) = -R_{26} \cdot x
\]

\[
\Delta E = -e Z^2 \cdot E_o \cdot l_2 \cdot \sin \left\{ k \left( \frac{D E - E_o}{E_o} + R_{16} x' - R_{26} x + R_{36} y' + R_{46} y \right) \right\};
\]

\[
\Delta x = -D_x \cdot e Z^2 \cdot E_o \cdot L_2 \cdot k R_{26} x + \ldots
\]

\[
\xi_\perp = J_{\perp} \xi_{CeC}; \quad \xi_\parallel = (1 - 2J_{\perp}) \xi_{CeC};
\]

\[
\frac{d \varepsilon_x}{dt} = -\frac{\varepsilon_x}{\tau_{CeC\perp}}; \quad \frac{d \sigma_{\varepsilon}}{dt} = -\frac{\sigma_{\varepsilon}}{\tau_{CeC\parallel}};
\]

\[
\tau_{CeC\perp} = \frac{1}{2J_{\perp} \xi_{CeC}}; \quad \tau_{CeC\parallel} = \frac{1}{2(1 - 2J_{\perp}) \xi_{CeC}};
\]
Example: Coherent electron Cooling vs. IBS at RHIC

\[ \frac{\sigma^2_x}{\tau_{IBS//}} = \frac{N_{E}c}{2^5 \pi^3 \epsilon_x^{3/2}} \left( \frac{f(\chi_m)}{\beta v} \right) \], \quad \frac{\epsilon_{x0}}{\tau_{IBS//}} = \frac{N_{E}c}{2^5 \pi^3 \epsilon_x^{3/2}} \left( \frac{H}{\beta_y} \right) f(\chi_m); \quad K = 1

\[ f(\chi_m) = \int_{\chi_m}^{\infty} \frac{d\chi}{\chi} \ln \left( \frac{\chi}{\chi_m} \right) e^{-\chi}; \quad \chi_m = \frac{r_m c^2}{b_{max} \sigma_E^2}; \; b_{max} = n^{-1/3}; \; r_c = \frac{e^2}{mc^2}; \quad (e \rightarrow Ze; m \rightarrow Am) \]

\[ X = \frac{\epsilon_{x\perp}}{\epsilon_{x0}}; \; S = \left( \frac{\sigma_s}{\sigma_{s0}} \right)^2 = \left( \frac{\sigma_E}{\sigma_{E0}} \right)^2 \]

\[ \frac{dX}{dt} = \frac{1}{\tau_{IBS//}} \frac{1}{X^{3/2} S^{1/2}} - \frac{\xi_{\perp}}{\tau_{CeC} S} \]

\[ \frac{dS}{dt} = \frac{1}{\tau_{IBS//}} \frac{1}{X^{3/2} S^{1/2}} - \frac{1 - 2 \xi_{\perp}}{\tau_{CeC} X} \]

Statistical solution:

\[ X = \frac{\tau_{CeC}}{\sqrt{\tau_{IBS//} \tau_{IBS\perp}}} \cdot \frac{1}{\left( \xi_{\perp} (1 - 2 \xi_{\perp}) \right)^{1/2}}; \quad S = \frac{\tau_{CeC}}{\tau_{IBS//}} \cdot \frac{\tau_{IBS\perp}}{\tau_{IBS//}} \cdot \sqrt{\frac{\xi_{\perp}}{(1 - 2 \xi_{\perp})^3}} \]

\[ \epsilon_{x0} = 2 \mu m; \; \sigma_{s0} = 13 cm; \; \sigma_{E0} = 4 \cdot 10^{-4} \]

\[ \tau_{IBS//} = 4.6 \text{ hrs}; \; \tau_{IBS\perp} = 1.6 \text{ hrs}; \]

This may allow:

a) RHIC pp - keep the luminosity at beam-beam limit all the time
b) RHIC pp – reduce bunch length to few cm (from present 1 m)
   1. To reduce hourglass effect
   2. To concentrate event in short vertexes of the detectors
c) eRHIC - reduce polarized beam current down to 50 mA while keeping the same luminosity
d) eRHIC - increase electron beam energy to 20 GeV
e) Both - increase luminosity by reducing \( \beta^* \) to 5-10 cm from present 0.5 m
Effects of the surrounding particles

Each charged particle causes generation of an electric field wave-packet proportional to its charge and synchronized with its initial position in the bunch:

\[ E_{\text{total}}(\zeta) = E_o \cdot \text{Im} \left( X \cdot \sum_{i, \text{hadrons}} K(\zeta - \zeta_i)e^{ik(\zeta - \zeta_i)} - \sum_{j, \text{electrons}} K(\zeta - \zeta_j)e^{ik(\zeta - \zeta_j)} \right) \]

Evolution of the RMS value resembles stochastic cooling!

Best cooling rate achievable is \( \sim 1/N_{\text{eff}} \), \( N_{\text{eff}} \) is effective number of hadrons in coherent sample \((\Lambda_k = N_c \lambda)\)

\[ \langle \delta^2 \rangle' = -2\xi \langle \delta^2 \rangle + D \]

\[ \xi = -g \left( \delta, \text{Im}(K(\Delta \zeta_i)e^{ik\Delta \zeta_i}) \right) / \langle \delta^2 \rangle;\quad D = g^2 N_{\text{eff}} / 2; \]

\[ g = G_o \frac{Z^2}{A} \frac{r_p}{\varepsilon_{\text{ln}}} \left\{ 2f(\varphi_2)(1 - \cos \varphi_1) \frac{l_2}{\beta} \right\} \]

\[ \Delta_{\text{CeC}}(\text{max}) = \frac{\Delta}{2\sigma_\gamma} = \frac{2}{N_{\text{eff}}} \left( kD\sigma_\varepsilon \right) \propto \frac{1}{N_{\text{eff}}} \]

Fortunately, the bandwidth of FELs \( \Delta f \sim 10^{13}-10^{15} \text{ Hz} \) is so large that this limitation does not play any practical role in most HE cases.
Possible layout using 20 MeV BNL's R&D ERL for the Proof-of-principle of Coherent Electron Cooling

<table>
<thead>
<tr>
<th>Ions, N per bunch</th>
<th>$1 \times 10^9$</th>
<th>Z, A</th>
<th>79, 197</th>
</tr>
</thead>
<tbody>
<tr>
<td>Energy Au, GeV/n</td>
<td>40</td>
<td>γ</td>
<td>42.63</td>
</tr>
<tr>
<td>RMS bunch length, nsec</td>
<td>3.2</td>
<td>Relative energy spread</td>
<td>0.037%</td>
</tr>
<tr>
<td>Emittance norm, μm</td>
<td>2.5</td>
<td>$\beta_{z}, m^*$</td>
<td>8</td>
</tr>
<tr>
<td>Electrons, energy, MeV</td>
<td>21.79</td>
<td>Peak current, A</td>
<td>60</td>
</tr>
<tr>
<td>Charge per bunch, nC</td>
<td>5 (or 4 x 1.4)</td>
<td>Bunch length, RMS, psec</td>
<td>83</td>
</tr>
<tr>
<td>Emittance norm, μm</td>
<td>5 (4)</td>
<td>Relative energy spread</td>
<td>0.15%</td>
</tr>
<tr>
<td>$\beta_{z}, m$</td>
<td>5</td>
<td>Modulator ,m</td>
<td>4</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$\lambda_{FEL}$, μm</th>
<th>18</th>
<th>$\lambda_{w}$, cm</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_{w}$</td>
<td>0.555</td>
<td>$L_{60}$, m</td>
<td>0.67</td>
</tr>
<tr>
<td>Amplitude gain $\approx 150$, $L_{w}$, m</td>
<td>6.75 (7)</td>
<td>$L_{63D}$, m</td>
<td>1.35</td>
</tr>
<tr>
<td>Kicker, m</td>
<td>3</td>
<td>Cooling time, local, minimum</td>
<td>0.05 minutes</td>
</tr>
<tr>
<td>$N_{turns, \tilde{N}, 5% BW}$</td>
<td>$8 \times 10^3$ to $6 \times 10^4$</td>
<td>Cooling time, beam, min</td>
<td>2.6 minutes</td>
</tr>
</tbody>
</table>
Conclusions

• Coherent electron cooling has potential of cooling high intensity TeV scale hadron beams for significant luminosity increases in hadron colliders from RHIC to LHC

• Electron accelerator of choice for such cooler is energy recovery linac (ERL)

• ERL seems to be capable of providing required beam quality for such coolers

• Majority of the technical limitation and requirements on the beam and magnets stability are well within limit of current technology, even though satisfying all of them in nontrivial fit

• We plan a proof of principle experiment of coherent electron cooling with Au ions in RHIC at ~ 40 GeV/n and existing R&D ERL as part of eRHIC R&D

Supported by the Office on Nuclear Physics, US DoE
Back-ups
Coherent electron cooling, ultra-relativistic case ($\gamma >> 1$)

Economic option

Electron density modulation is amplified in the FEL and made into a train with duration of $N_c \sim L_{\text{gain}}/\lambda_w$ alternating hills (high density) and valleys (low density) with period of FEL wavelength $\lambda$. Maximum gain for the electron density of HG FEL is $\sim 10^3$.

\[ v_{\text{group}} = (c + 2v_\parallel)/3 = c\left(1 - \frac{1 + a_w^2}{3\gamma^2}\right) = c\left(1 - \frac{1}{2\gamma^2}\right) + \frac{c}{3\gamma^2}(1 - 2a_w^2) = v_{\text{hadrons}} + \frac{c}{3\gamma^2}(1 - 2a_w^2) \]

Economic option requires: $2a_w^2 < 1$ !!!

V.N. Litvinenko, 2009 Particle Accelerator Conference, Vancouver, May 8, 2009
Response - 1D FEL after 10 gain lengths

\[ v_g = \frac{c + 2\langle v_z \rangle}{3} = c \left( 1 - \frac{1 + a_w^2}{3\gamma_o^2} \right) \]

Green-function envelope (Abs, Re and Im)

Maximum located at 3.744 slippage units, (i.e. just a bit further than expected 3 and 1/3)
The Green function (with oscillations) had effective RMS length of 1.48 slippage units.
FEL's Green Function

1D - analytical approach

3D - 3D FEL codes RON and Genesis 1.3

\[ G(\tau; z) = \text{Re}\left( \tilde{G}_z(\tau) e^{i\omega_0 \tau} \right) \]

FEL parameters for Genesis 1.3 and RON simulations

FEL gain length: 1 m (power), 2m (amplitude)

---

Main FEL parameters for eRHIC with 250 GeV protons

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Energy, MeV</td>
<td>136.2</td>
</tr>
<tr>
<td>( \gamma )</td>
<td>266.45</td>
</tr>
<tr>
<td>Peak current, A</td>
<td>100</td>
</tr>
<tr>
<td>( \lambda_0, \text{nm} )</td>
<td>700</td>
</tr>
<tr>
<td>Bunchlength, psec</td>
<td>50</td>
</tr>
<tr>
<td>( \lambda_w, \text{cm} )</td>
<td>5</td>
</tr>
<tr>
<td>Emittance, norm</td>
<td>5 mm mrad</td>
</tr>
<tr>
<td>( a_w )</td>
<td>0.994</td>
</tr>
<tr>
<td>Energy spread</td>
<td>0.03%</td>
</tr>
</tbody>
</table>

---

V.N. Litvinenko, 2009 Particle Accelerator Conference, Vancouver, May 8, 2009
Coherent electron Cooling: FEL response

\[ f_{\text{input}}(\vec{r}_\perp, \vec{p}, t) = f_{o \text{ input}}(\vec{r}_\perp, \vec{p}) + \delta f(\vec{r}_\perp, \vec{p}, t) \]

\[ f_{\text{exit}}(\vec{r}_\perp, \vec{p}, t) = f_{o \text{ exit}}(\vec{r}_\perp, \vec{p}) + \int K(\vec{r}_\perp, \vec{p}, \vec{r}_\perp, \vec{p}_1, t - t_1) \cdot \delta f(\vec{r}_1, \vec{p}_1, t_1) \cdot d\vec{r}_\perp d\vec{p}_1 dt_1 \]

1D FEL response

\[ \rho_{\text{exit}}(t; z) = \rho_{o} + \int G(\tau; z) \cdot \delta \rho(t - \tau; 0) \cdot d\tau \]

\[ G(\tau; z) = \text{Re} \left( \tilde{G}(\tau) e^{i\omega_0 \tau} \right) \quad \omega_0 = \frac{2\pi c}{\lambda_0} \]

V.N. Litvinenko, 2009 Particle Accelerator Conference, Vancouver, May 8, 2009
Possible layout for Coherent Electron Cooling proof-of-principle experiment

19.6 m

DX → Kicker 3 m → Wiggler 7m → Modulator 4 m → DX

DX

Possible layout for Coherent Electron Cooling proof-of-principle experiment

19.6 m

DX → Kicker 3 m → Wiggler 7m → Modulator 4 m → DX

DX
**Modulator**

Dimensionless equations of motion

\[
\frac{\partial f_e}{\partial t} + \frac{\partial f_e}{\partial \vec{v}} \cdot \frac{\vec{e} \cdot \vec{E}}{m} + \frac{\partial f_e}{\partial \vec{r}} \cdot \vec{v} = 0; \quad \vec{r}_h(t) \equiv \vec{r}_o + \vec{v}_h t;
\]

\[
(\nabla \cdot \vec{E}) = 4\pi n e \left( \frac{Z}{n_e} \delta(\vec{r} - \vec{r}_h(t)) - \int f_e \, d\vec{v}^3 \right).
\]

\[
\frac{\partial f_e}{\partial \tau} + \frac{\partial f_e}{\partial \vec{v}} \cdot \vec{g} + \frac{\partial f_e}{\partial \vec{r}} \cdot \vec{v} = 0; \quad \vec{g} = \frac{e\vec{E}}{m\omega_p^2 s};
\]

\[
(\nabla_n \cdot \vec{g}) = \frac{Z}{s^3 n_e} \delta(\vec{\rho} - \vec{\rho}_i(t)) - \int f_e \, d\vec{v}^3; \quad \nabla_n \equiv \partial_{\vec{\rho}}.
\]

\[
t = \tau / \omega_p; \quad \vec{v} = \vec{v} \sigma_{v_z}; \quad \vec{r} = \vec{\rho} \sigma_{v_z} / \omega_p; \quad \omega_p^2 = \frac{4\pi e^2 n_e}{m} \quad S = r_{D_z} = \sigma_{v_z} / \omega_p
\]

**Parameters of the problem**

\[
R = \frac{\sigma_{v_{1}}}{\sigma_{v_{z}}}; \quad T = \frac{v_{hx}}{\sigma_{v_{z}}}; \quad L = \frac{v_{hz}}{\sigma_{v_{z}}}; \quad \xi = \frac{Z}{4\pi n_e R^2 s^3};
\]

\[
A = \frac{a}{s}; \quad X = \frac{x_{ho}}{a}; \quad Y = \frac{y_{ho}}{a}.
\]
R&D ERL
Commissioning
start 2009
Velocity map & buncher \((\gamma > 1000)\)

\[
\frac{\delta E}{E}(z,r) = -Z e r \frac{\gamma z}{\left(\gamma^2 z^2 + r^2\right)^{3/2}} \cdot c \Delta t
\]

\[
\delta E \approx -2 Z e a^2 \frac{L_{pol}}{\gamma} \left(\frac{z}{|z|} - \frac{z}{\sqrt{a^2 / \gamma^2 + z^2}}\right)
\]

V.N. Litvinenko, 2009 Particle Accelerator Conference, Vancouver, May 8, 2009
Exact calculations: solving Vlasov equation

\[
f_0(r, \vec{p}_\perp, z, \gamma) = \frac{\theta(r - a) \cdot \theta(z - l)}{a^2/2 \cdot l_z} \cdot \frac{1}{\sqrt{2\pi} \sigma_\gamma} e^{-\frac{(\gamma - \gamma_o)^2}{2\sigma_\gamma^2}} \cdot g(\vec{p}_\perp)
\]

\[
\delta \gamma = \frac{\delta \gamma_i}{\gamma_o} - A \frac{\gamma_o z_i}{\left(r_i^2 + \gamma_o^2 z_i^2\right)^{3/2}}; \quad z = z_i + D \left(\frac{\delta \gamma_i}{\gamma_o} - A \frac{\gamma_o z_i}{\left(r_i^2 + \gamma_o^2 z_i^2\right)^{3/2}}\right);
\]

\[
l_z \rho(z) = \Phi(s) = \frac{1}{\kappa^2 \sqrt{2\pi}} \int_0^{\kappa^2} dy \int_{-L/2}^{L/2} \left[ \exp\left\{-\frac{1}{2\kappa^2} \left(s - u \left(1 - \frac{G}{(y + u^2)^{3/2}}\right)\right)^2\right\} - \exp\left\{-\frac{(s - u)^2}{2}\right\}\right] du;
\]

\[
G = Z \frac{r_e L_{\text{mod}} |D|}{\left(\gamma_o \sigma_{p_i} |D|\right)}; \quad \kappa = \frac{a}{\gamma_o \sigma_{p_i} |D|}; \quad L = \frac{l_z}{\sigma_{p_i} |D|} = \frac{1}{\sigma_{p_i} |D|} \int_0^{\kappa^2} dy \int_{-L/2}^{L/2} \left[ \exp\left\{-\frac{1}{2\kappa^2} \left(s - u \left(1 - \frac{G}{(y + u^2)^{3/2}}\right)\right)^2\right\} - \exp\left\{-\frac{(s - u)^2}{2}\right\}\right] du;
\]

For 7 TeV p in LHC CeC case: simple “gut-feeling” estimate gave 22.9 boost in the induced charge by a buncher, while exact calculations gave 21.7.
Comprehensive studies
Analytical, Numerical and Computer Tools to:

1. find reaction (distortion of the distribution function of electrons) on a presence of moving hadron inside an electron beam

\[
\frac{\partial f_e}{\partial t} + \frac{\partial f_e}{\partial \vec{r}} \cdot \frac{e\vec{E}}{m} + \frac{\partial f_e}{\partial \vec{v}} \cdot \vec{v} = 0; \quad \vec{r}_h(t) \equiv \vec{r}_o + \vec{v}_h t;
\]

\[
(\vec{\nabla} \cdot \vec{E}) = 4\pi e n_e \left( \frac{Z}{n_e} \delta(\vec{r} - \vec{r}_h(t)) - \int f_e d\vec{N}^3 \right).
\]

2a. Find how an arbitrary $\delta f$ is amplified in high-gain FEL

\[
f_{\text{exit}}(\vec{r}_\perp, \vec{p}, t) = f_{\text{exit}}(\vec{r}_\perp, \vec{p}) + \int K(\vec{r}_\perp, \vec{p}, \vec{r}_\perp, \vec{p}_1, t - t_1) \cdot \delta f(\vec{r}_1, \vec{p}_1, t_1) \cdot d\vec{r}_\perp d\vec{p}_1 dt_1
\]

2b. Design cost effective lattice for hadrons + coupling

3. Find how the amplified reaction of the e-beam acts on the hadron (including coupling to transverse motion)
Genesis: 3D FEL

Evolution of the normalized bunching envelope

The Green function (with oscillations) after 10 gain-lengths had also smaller effective RMS length [1] of 0.96 slippage units (i.e. about 38 optical wavelengths, or 27 microns)

©Y.Hao, V.Litvinenko, S.Reiche

Evolution of the bunching and optical power envelopes (vertical scale is logarithmic)
### PoP test using BNL R&D ERL:
**Au ions in RHIC with 40 GeV/n, $L_{\text{cooler}} = 14$ m**

<table>
<thead>
<tr>
<th>$N$ per bunch</th>
<th>$10^9$</th>
<th>$Z, A$</th>
<th>79, 197</th>
</tr>
</thead>
<tbody>
<tr>
<td>Energy Au, GeV/n</td>
<td>40</td>
<td>$\gamma$</td>
<td>42.63</td>
</tr>
<tr>
<td>RMS bunch length, nsec</td>
<td>3.2</td>
<td>Relative energy spread</td>
<td>0.037%</td>
</tr>
<tr>
<td>Emittance norm, $\mu$m</td>
<td>2.5</td>
<td>$\beta_\perp, m^*$</td>
<td>8</td>
</tr>
<tr>
<td>Energy $e^-$, MeV</td>
<td>21.79</td>
<td>Peak current, A</td>
<td>60</td>
</tr>
<tr>
<td>Charge per bunch, nC</td>
<td>5 (or 4 x 1.4)</td>
<td>Bunch length, RMS, psec</td>
<td>83</td>
</tr>
<tr>
<td>Emittance norm, $\mu$m</td>
<td>5 (4)</td>
<td>Relative energy spread</td>
<td>0.15%</td>
</tr>
<tr>
<td>$\beta_\perp, m$</td>
<td>5</td>
<td>$L_1$ (lab frame), m</td>
<td>4</td>
</tr>
<tr>
<td>$\omega_{pe}, CM, Hz$</td>
<td>5.03 $10^9$</td>
<td>Number of plasma oscillations</td>
<td>0.256</td>
</tr>
<tr>
<td>$\lambda_{\perp}, \mu$m</td>
<td>611</td>
<td>$\lambda_{</td>
<td></td>
</tr>
<tr>
<td>$\lambda_{\text{FEL}}, \mu$m</td>
<td>18</td>
<td>$\lambda_w, cm$</td>
<td>5</td>
</tr>
<tr>
<td>$a_w$</td>
<td>0.555</td>
<td>$L_{G0}, m$</td>
<td>0.67</td>
</tr>
<tr>
<td>Amplitude gain =150, $L_w, m$</td>
<td>6.75 (7)</td>
<td>$L_{G3D}, m$</td>
<td>1.35</td>
</tr>
<tr>
<td>$L_2$ (lab frame), m</td>
<td>3</td>
<td>Cooling time, local, minimum</td>
<td>0.05 minutes</td>
</tr>
<tr>
<td>$N_{\text{turns}}, \bar{N}$, 5% BW</td>
<td>$8 \times 10^6$ &gt; $6 \times 10^4$</td>
<td>Cooling time, beam, min</td>
<td>2.6 minutes</td>
</tr>
</tbody>
</table>
**325 GeV polarized protons in RHIC, L\textsubscript{cooler} fits in IR**

<table>
<thead>
<tr>
<th>N per bunch</th>
<th>2 \times 10^{11}</th>
<th>Z, A</th>
<th>1, 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Energy Au, GeV/n</td>
<td>250</td>
<td>γ</td>
<td>266.45</td>
</tr>
<tr>
<td>RMS bunch length, nsec</td>
<td>1</td>
<td>Relative energy spread</td>
<td>0.04%</td>
</tr>
<tr>
<td>Emittance norm, μm</td>
<td>2.5</td>
<td>β\textsubscript{⊥}, m</td>
<td>10</td>
</tr>
<tr>
<td>Energy e\textsuperscript{-}, MeV</td>
<td>136.16</td>
<td>Peak current, A</td>
<td>100</td>
</tr>
<tr>
<td>Charge per bunch, nC</td>
<td>5</td>
<td>Bunch length, nsec</td>
<td>0.2</td>
</tr>
<tr>
<td>Emittance norm, μm</td>
<td>3</td>
<td>Relative energy spread</td>
<td>0.04%</td>
</tr>
<tr>
<td>β\textsubscript{⊥}, m</td>
<td>10</td>
<td>L\textsubscript{1} (lab frame), m</td>
<td>30</td>
</tr>
<tr>
<td>ω\textsubscript{pe}, CM, Hz</td>
<td>4.19 \times 10^{9}</td>
<td>Number of plasma oscillations</td>
<td>0.25</td>
</tr>
<tr>
<td>λ\textsubscript{D\perp}, μm</td>
<td>1004</td>
<td>λ\textsubscript{D</td>
<td></td>
</tr>
<tr>
<td>λ\textsubscript{FEL}, μm</td>
<td>0.5</td>
<td>λ\textsubscript{w}, cm</td>
<td>5</td>
</tr>
<tr>
<td>a\textsubscript{w}</td>
<td>0.648</td>
<td>L\textsubscript{G0}, m</td>
<td>0.87</td>
</tr>
<tr>
<td>Amplitude gain =100, L\textsubscript{w}, m</td>
<td>13 (→ 15)</td>
<td>L\textsubscript{G3D}, m</td>
<td>1.22</td>
</tr>
<tr>
<td>L\textsubscript{2} (lab frame), m</td>
<td>10</td>
<td>Cooling time, local, min</td>
<td>1.96</td>
</tr>
<tr>
<td>N\textsubscript{min turns} or Ŵ in 10% BW</td>
<td>6.7 \times 10^{6} &gt; 5.9 \times 10^{6}</td>
<td>Cooling time, beam, min</td>
<td>49.2</td>
</tr>
</tbody>
</table>

_Not optimized!_
**Au ions in RHIC with 100 GeV/n, \(L_{\text{cooler}} \sim 20\) m**

<table>
<thead>
<tr>
<th>N per bunch</th>
<th>(2 \times 10^9)</th>
<th>Z, A</th>
<th>79, 197</th>
</tr>
</thead>
<tbody>
<tr>
<td>Energy Au, GeV/n</td>
<td>100</td>
<td>(\gamma)</td>
<td>106.58</td>
</tr>
<tr>
<td>RMS bunch length, nsec</td>
<td>1</td>
<td>Relative energy spread</td>
<td>0.1%</td>
</tr>
<tr>
<td>Emittance norm, (\mu m)</td>
<td>2.5</td>
<td>(\beta_\perp, m)</td>
<td>5</td>
</tr>
<tr>
<td>Energy e(^-), MeV</td>
<td>54.5</td>
<td>Peak current, A</td>
<td>50</td>
</tr>
<tr>
<td>Charge per bunch, nC</td>
<td>5</td>
<td>Bunch length, nsec</td>
<td>0.1</td>
</tr>
<tr>
<td>Emittance norm, (\mu m)</td>
<td>3</td>
<td>Relative energy spread</td>
<td>0.1%</td>
</tr>
<tr>
<td>(\beta_\perp, m)</td>
<td>10</td>
<td>(L_1) (lab frame) ,m</td>
<td>8.5</td>
</tr>
<tr>
<td>(\omega_{pe}, CM, Hz)</td>
<td>5.9 (10^9)</td>
<td>Number of plasma oscillations</td>
<td>0.25</td>
</tr>
<tr>
<td>(\lambda_{D\perp}, \mu m)</td>
<td>78</td>
<td>(\lambda_{D</td>
<td></td>
</tr>
<tr>
<td>(\lambda_{FEL}, \mu m)</td>
<td>3</td>
<td>(\lambda_w, cm)</td>
<td>5</td>
</tr>
<tr>
<td>(a_w)</td>
<td>0.603</td>
<td>(L_{G0}, m)</td>
<td>0.5</td>
</tr>
<tr>
<td>Amplitude gain =200, (L_w, m)</td>
<td>8.11 (-&gt; 9)</td>
<td>(L_{G3D}, m)</td>
<td>0.77</td>
</tr>
<tr>
<td>(L_2) (lab frame),m</td>
<td>5</td>
<td>Cooling time, local, minimum</td>
<td>0.08 minutes</td>
</tr>
<tr>
<td>(N_{\text{min turns}}) or (\tilde{N}) in 5% BW</td>
<td>(6 \times 10^5) &gt; (2 \times 10^5)</td>
<td>Cooling time, beam, min</td>
<td>1.93 minutes</td>
</tr>
</tbody>
</table>
## 7 TeV protons in LHC: CeC ~200m
Potential of 4x increase in luminosity

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>N per bunch</td>
<td>$1.4 \times 10^{11}$</td>
<td>$Z, A$</td>
<td>$1, 1$</td>
</tr>
<tr>
<td>Energy Au, GeV/n</td>
<td>7000</td>
<td>$\gamma$</td>
<td>7460</td>
</tr>
<tr>
<td>RMS bunch length, nsec</td>
<td>0.25</td>
<td>Relative energy spread</td>
<td>0.0113%</td>
</tr>
<tr>
<td>Emittance norm, $\mu$m</td>
<td>3.8</td>
<td>$\beta_{\perp}, m$</td>
<td>47</td>
</tr>
<tr>
<td>Energy e-, MeV</td>
<td>3812</td>
<td>Peak current, A</td>
<td>100</td>
</tr>
<tr>
<td>Charge per bunch, nC</td>
<td>5</td>
<td>Bunch length, nsec</td>
<td>0.05</td>
</tr>
<tr>
<td>Emittance norm, $\mu$m</td>
<td>3</td>
<td>Relative energy spread</td>
<td>0.01%</td>
</tr>
<tr>
<td>$\beta_{\perp}, m$</td>
<td>59</td>
<td>$L_1$ (lab frame), m</td>
<td>70</td>
</tr>
<tr>
<td>$\omega_{pe}$, CM, Hz</td>
<td>$2.44 \times 10^9$</td>
<td>Number of plasma oscillations</td>
<td>0.0121</td>
</tr>
<tr>
<td>$\lambda_{D\perp}, mm$</td>
<td>3.7</td>
<td>$\lambda_{D</td>
<td></td>
</tr>
<tr>
<td>$\lambda_{FEL}, \mu m$</td>
<td>0.01</td>
<td>$\lambda_w$, cm</td>
<td>5</td>
</tr>
<tr>
<td>$a_w$</td>
<td>4.61</td>
<td>$L_{Go}$, m</td>
<td>2.7</td>
</tr>
<tr>
<td>Amplitude gain =1000, $L_w$, m</td>
<td>61.8</td>
<td>$L_{G3D}$, m</td>
<td>3.9</td>
</tr>
<tr>
<td>$L_2$ (lab frame), m</td>
<td>35</td>
<td>Cooling time, local, min</td>
<td>3 minutes</td>
</tr>
<tr>
<td>$N_{\text{min turns}}$ or $\bar{N}$ in 10% BW</td>
<td>$2 \times 10^6 &gt;&gt; 2.8 \times 10^5$</td>
<td>Cooling time, beam</td>
<td>23 minutes</td>
</tr>
</tbody>
</table>