STOCHASTIC INJECTION SCENARIOS AND PERFORMANCE FOR NUSTORM

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Abstract
At Fermilab, we are developing NuSTORM (Neutrinos from STORed Muons), a neutrino beam from muon decay in a long straight section of a storage ring. The baseline design for NuSTORM uses what was called “stochastic injection”. In that method, high-energy protons on a nuclear target produce pions that are directed by a chicane into a straight section of the storage ring. Pions that decay within that straight section can provide lower-energy muons that are within the circulating acceptance of the storage ring. This decay acceptance enables injection for multiple storage ring turns without kickers, and muon accumulation can be reasonably high. The design of a muon storage ring with pion injection is described and simulations of acceptance are discussed.

INTRODUCTION

Particle motion in accelerator physics is dominated by Hamiltonian dynamics within electromagnetic fields, which implies symplectic motion with conservation of phase space (Liouville’s theorem). Great improvements are possible when “non-Liouvillean” processes such as beam cooling (by radiation damping, stochastic cooling and ionization or frictional cooling) and charge exchange injection are enabled. In H charge exchange injection, stripping foil traversal transforms an H ion into a proton (H⁺) of about the same energy, enabling multturn accumulation of protons in the same initial phase space within a storage ring.

The present concept is quite similar. An unstable particle inserted into a storage ring decays into a more stable one of different momentum. If the decayed momentum matches the momentum of the stored particles in the ring, the particle is added to the stored beam, increasing the stored beam phase-space density. The “non-Liouvillean” random process of particle decay enables multturn stacking of particles within the same phase space.

The name of “stochastic injection” was introduced when this concept was first presented in 1980. [1, 2] The naming was influenced by the contemporary use of “stochastic” to describe other important non-Liouvillean beam handling techniques. (i.e., “stochastic cooling”)

The current paper is developed to accompany the proposal. [3, 4]

OVERVIEW OF SCENARIO

The basic scenario concept is presented in Fig. 1. A high-intensity proton source places beam on a target, producing a large spectrum of secondary π’s. Forward π’s are focused by a collection lens and inserted through a chicane into the straight section of the storage ring. π decay within the straight section produces μ’s which are within the circulating acceptance of the storage ring, where they are stored for the muon lifetime. μ-decay within the straight sections will produce ν beams of known flux and flavor. (μ → e⁺νµ +νe) or (μ → e⁺νe +νµ)

For this implementation, we choose a 3.8 GeV/c storage ring to obtain the desired spectrum of ~2—3 GeV ν’s. This means that we must capture π’s at a higher momentum (~5 GeV/c). The primary proton beam is the Fermilab Main Injector beam at 60 to 120 GeV.

Table 1: Parameters of the NuSTORM Muon Ring

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Circumference</td>
<td>480 m</td>
</tr>
<tr>
<td>Injected beam momentum P_i</td>
<td>3.8 GeV/c</td>
</tr>
<tr>
<td>Stored beam momentum P_n</td>
<td>5.0 GeV/c</td>
</tr>
<tr>
<td>Straight section length</td>
<td>180 m</td>
</tr>
<tr>
<td>Straight section (β_max,β_min)</td>
<td>30.2, 23.3 m</td>
</tr>
<tr>
<td>Arc Length</td>
<td>50 m</td>
</tr>
<tr>
<td>Ring Tunes (ν_x,ν_y)</td>
<td>9.72, 7.87</td>
</tr>
<tr>
<td>Maximum B-field</td>
<td>4.3T</td>
</tr>
<tr>
<td>Arc Cell type</td>
<td>DBA</td>
</tr>
</tbody>
</table>

Figure 1: Schematic view of NuSTORM μ storage ring. π → μ scenario is shown. π → μ scenario is obtained by reversing magnet polarities and retuning the injection chicane; with ~same intensity. Arc and straight section lengths will vary when optimized.
sections, where decay in the straight sections provide the collimated ν beams. One of the straight sections is used for injection, with the pion beam inserted at the beginning, in a chicane integrated into the dispersion suppressor of the ring arc (see Fig. 2). The injection straight section is a FODO cell lattice with 20 cells and weak focusing. The return straight has 20 cells with stronger focusing. The arcs consist of three double bend achromat cells plus another achromat matched into the injection straight. The maximum dispersion establishes the separation needed for injected and stored beams.

The injection beam combination section (BCS) design requires particular care, since it requires accommodating injection π and circulating μ transport elements. It is designed in three steps. First, periodic lattice solutions for both 5.0 GeV/π and 3.8 GeV/c μ beams are generated within the same straight FODO cells. Second, the μ beam solution is matched back to the storage ring arc, with an enlarged dispersion at the point of π-μ separation. Third, with the common magnets fixed, the π beam is matched back from the straight through the injection chicane toward the production target. The resulting transports are then checked for acceptance and magnet design constraints.

**Decay Beam Characteristics**

The injection straight is ~180 m long, which, with a pion decay length of 8.4 μm, implies ~50% of injected π’s decay in the straight.

\( \pi \rightarrow \mu + \nu \) decay is an isotropic decay in the π rest frame, producing a μ at a random angle \( \theta \) with a momentum \( p_\mu = (m_\pi^2 - m_\mu^2) / 2m_\pi = 29.8 \text{ MeV/c} \) and energy \( E_\mu = (m_\pi^2 + m_\mu^2) / 2m_\pi = 109.8 \text{ MeV} \). The energy and momentum of the μ in the lab frame are:

\[
P_\mu = P_\pi \sin \theta, \quad E_\mu = E_\pi(1-(m_\pi^2-m_\mu^2)(1-\cos \theta)/(2m_\pi^2)),
\]

where we have used the relativistic approximation \( p_\pi = E_\pi \) or \( \beta_\pi = 1 \) to simplify the energy expression. The more complete expression is:

\[
E_\mu = E_\pi \left[ 1 - \frac{(m_\pi^2 - m_\mu^2)}{2m_\pi^2}(1-\cos \theta) \right] - \frac{(1-\beta_\pi^2)(m_\pi^2 - m_\mu^2)}{2m_\pi^2} \cos \theta.
\]

With \( (m_\pi^2 - m_\mu^2) / 2m_\pi^2 = 0.213 \), we find that the maximum μ energy is \(-E_\pi\) and the minimum is \(-0.575 E_\pi\), with the μ’s distributed evenly in energy throughout that range.

As the π’s travel in the straight section and decay, they produce muons with energy randomly between \(-0.575E_\pi\) and \(-1.0E_\pi\), with a (random-direction) transverse momentum kick of \( p_\pi \sin \theta \), which is relatively small. If the muon energy is within the ring storage acceptance, the muons are added to the stored intensity.

**SIMULATION OF STOCHASTIC INJECTION FOR NUStORM**

We have initiated simulation of stochastic injection at NUStORM parameters, using the G4Beamline beam transport and decay simulation. An initial pion beam is generated at a production target and tracked through the initial transport through the injection and into the decay straight. In ref. [5] a graphite target within a magnetic horn is used, obtaining \(-0.056 \pi^+ / \text{per 60 GeV proton on target within 5±0.5 GeV/c} \) and a 2000μm-rad transport acceptance. These π’s are tracked through the BCS into the storage ring with decays within the acceptance of the storage ring obtaining the broad energy distribution of μ’s shown in Fig. 5. We obtain \(-0.012 \mu^- / \text{POT in that distribution. Transverse phase-space distributions are shown in Fig. 6. (π^-/μ^- intensities in the π^-/μ^- scenario are similar.)} \)

The present ring momentum acceptance for μ’s is approximated by the 3.8±10% band shown in Fig. 5, and with the expected ring acceptance we should obtain ~0.003 μ/POT stored within the ring for ν production.

Design and tracking for the storage ring has been initiated, and results are consistent with this estimate, but have not yet been fully implemented and optimized. More accurate and complete results will be obtained in the near future.
The $\mu$ lifetime in the ring is given in turns by:

$$N_{\text{turns}} = \frac{P_\mu e \tau_\mu}{m_\mu c_{\text{ring}}} \approx 60 \text{ turns}.$$  

Since the magnet transports are fixed, $\pi$'s can be injected over an extended pulse to obtain multturn injection into the ring. For example a full Main Injector circumference pulse would correspond to ~7 turns of injection. The multturn $\mu$ injection could occur with no phase-space dilution. (In the present NuSTORM scenario, shorter pulses can be used because of beam sharing requirements. Pure $\mu$-decay neutrinos would not be obtained from the injection straight until injection is concluded.)

**Variations and Optimization**

The optimum choice of $\pi$ production energy and relative $\mu$ collection energy has not yet been determined and will be studied, and will depend on the actual acceptances of the ring and transports.

We have presented a particular example with production and acceptances as shown in Fig. 5. It may be more desirable to collect $\mu$'s from the lower energy end, where the transverse decay kick is minimal. Broader (or narrower) acceptances would also shift the optima.

We are currently considering other versions of the lattice in order to obtain a performance and cost optimum version for the proposal. A lattice with arc FODO cells is being developed. The present version is unsymmetric with different straights; a more symmetric version will be considered. An FFAG ring with stochastic injection is also being considered. [6]

**OTHER APPLICATIONS OF THE CONCEPT**

The present example is adapted to production of ~1—2 GeV neutrinos, but the concept is readily modified to obtain neutrinos from ~100 MeV to TeV with modification of the injection beam and storage ring parameters. It is desireable that the $\mu$'s be somewhat relativistic to minimize the decay emittance dilution.

The same method could be used to accumulate ions by inserting unstable radioactive ions into a storage ring, with decay to a more stable state by beta-decay. ($X_1 \rightarrow X_2 + e^{-} + \nu$) The ions would be separable at the ends of the straight by their different charge states, and the more stable $X_2$ ion could be accumulated over multiple turns. Non-relativistic motion would be practical; the transverse kick in $\beta$-decay is relatively small.

**ACKNOWLEDGMENTS**

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**REFERENCES**


