In a laser plasma accelerator, a laser pulse is propagated through a plasma, creating a wake of regions with very strong electric fields of alternating polarity [1]. An electron beam that is injected with the appropriate phase can thus be accelerated to high energy in a distance that is much shorter than with conventional acceleration techniques [2]. The simulation of a laser plasma acceleration stage from first principles using the Particle-In-Cell technique in the laboratory frame is very demanding computationally, as the evolution of micron-scale laser oscillations needs to be followed over millions of time steps as the laser pulse propagates through a meter-long plasma for a 10 GeV stage.

Figure 1: Simulations with the code Warp of scaled laser plasma acceleration stages: (top) in the lab; (bottom) in a Lorentz boosted frame (laser pulse in blue/red; plasma wakefield in pale blue/yellow).

A method was recently demonstrated to speed up full PIC simulations of a certain class of relativistic interactions by performing the calculation in a Lorentz boosted frame [3], taking advantage of the properties of space/time contraction and dilation of special relativity to render space and time scales (that are separated by orders of magnitude in the laboratory frame) commensurate in a Lorentz boosted frame, resulting in far fewer computer operations. As illustrated in Fig. 1, which shows snapshots from simulations of a sample LPA stage, in the laboratory frame the laser pulse is much shorter than the wake, whose wavelength is also much shorter than the acceleration distance \( \lambda_{\text{laser}} < \lambda_{\text{wake}} \ll \lambda_{\text{acceleration}} \). In a Lorentz boosted frame co-propagating with the laser at a speed near the speed of light, the laser is Lorentz expanded (by a factor \( 1 + v_f/c \gamma_f \) where \( \gamma_f = (1 - v_f^2/c^2)^{-1/2} \) and \( v_f \) is the velocity of the frame and \( c \) is the speed of light). The plasma (now moving opposite to the incoming laser at velocity \(-v_f\)) is Lorentz contracted (by a factor \( \gamma_f \)). In a boosted frame moving with the wake \( \gamma_f \approx \gamma_{\text{wake}} \), the laser wavelength, the wake and the acceleration length are now commensurate \( \lambda_{\text{laser}} < \lambda_{\text{wake}} \approx \lambda_{\text{acceleration}} \), leading to far fewer time steps by a factor \( (1 + v_f/c)^2 \gamma_f^2 \), hence computer operations [3, 4].

Recently, control of a violent numerical instability that limited early attempts [5, 6, 7] was obtained via the combination of: (i) the use of a tunable electromagnetic solver and an efficient wideband digital filtering method [8], (ii) observation of the benefits of hyperbolic rotation of space-time on the laser spectrum in boosted frame simulations [9], and (iii) identification of a special time step at which the growth rate of the instability is greatly reduced [8]. A novel numerical method for injecting the laser pulse through a moving planar antenna was also introduced [4]. The combination of these methods enabled the demonstration of a speedup of over a million times for the modeling of a hypothetical 1 TeV stage, and over 10,000 for a 10 GeV stage [9].

REFERENCES