Abstract

So far, the massive numerical simulation studies of the LHC dynamic aperture were performed using thin lens models of the machine. This approach has the clear advantage of speed, but it has also the disadvantage of requiring re-matching of the optics from the real thick configuration to the thin one. The figure-of-merit for the re-matching is the agreement between the beta-functions for the two models. However, the quadrupole gradients are left as free parameters, thus, the impact of the magnetic multipoles might be affected by this approach. In turns, the dynamic aperture computation could be changed. In this paper the new approach is described and the results for the dynamic aperture are compared with the old approach, including detailed considerations on the CPU-time requirements.

INTRODUCTION

At CERN the two work horses for the design and numerical simulations efforts in the domain of particle accelerators are MAD-X [1, 2, 3] and SixTrack [4] codes. The first code is, to a large extent, used for the optics design due to its powerful matching tools and the very flexible language to describe accelerator lattices. The numerical simulations are mainly performed using the latter. To this aim, a special environment has been developed [5, 6] in order to facilitate the massive numerical simulations performed, e.g., during the LHC design stage.

The standard solution to perform the numerical simulations consists in replacing the thick elements with thin ones in order to obtain a sequence of drifts and kicks (linear or non-linear). With the tools currently available in MAD-X (see next section) even elements that could be left thick and treated in a symplectic manner in a tracking code are reduced to thin lens elements. This is the case for quadrupoles. Even if this speeds up the following tracking, such an approach has certainly a number of limitations. In particular, the strength of the thin lens quadrupoles needs to be matched in order to provide a machine optics that approximates the true one, i.e., the one of the thick lattice. This implies maintaining two versions of each optics configuration: one thick and one thin, which is not effective in terms of resources. Furthermore, in order to reproduce the optical parameters, the thin lens gradients are varied and the end results is a model in which the optical parameters are close to the true ones, but the gradients are different from the nominal ones. This implies that the effect of the magnetic field quality that is used in the numerical simulations might be underestimated.

It is also worth mentioning that, indeed, SixTrack is capable of handling thick quadrupoles in a symplectic way. Therefore, it should not be too difficult to test special LHC lattices with thick quadrupoles. Of course, one should carefully consider the advantage of managing one single optical model for both standard studies and numerical simulation, against the efforts to build two models (thick and thin), but also the impact on numerical simulations, namely computational speed and accuracy.

It should also be mentioned that, in principle, most of the issues considered here are indeed addressed by the Poly-morphic Tracking Code (PTC) [7] that is also embedded in MAD-X. In fact, the issue of symplectic integration of the particle's motion and the best model to describe the accelerator lattices is one of the goals of such a code [8]. Following the results of this study it might be envisaged to develop a tool to transfer the appropriate accelerator model built by PTC within MAD-X, with the most appropriate symplectic description of the lattice, to SixTrack for further use in tracking studies.

CONVERSION TO THIN ELEMENTS

MAD-X contains a module called MAKETHIN which converts a sequence with thick elements into one composed entirely of thin elements as required by the default MAD-X tracking or other tracking codes. Slicing is done by the MAKETHIN command:

\begin{verbatim}
MAKETHIN, SEQUENCE = seqname, STYLE = slicing_style;
\end{verbatim}

The default slicing style is TEAPOT which distributes up to four slices in an optimal way [9]. An alternative is SIMPLE which produces equal strength slices at equidistant positions. SIMPLE is always used if more than four slices are selected. Many equidistant slices will be required to do better then TEAPOT.

By default all elements are converted to one thin element positioned at the centre of the thick element. To get a greater slicing for certain elements, a standard MAD-X SELECT command should be used with

\begin{verbatim}
FLAG = MAKETHIN and CLASS, RANGE or PATTERN selections command, like
SELECT, FLAG = MAKETHIN, CLASS = class_name, RANGE = range_list, SLICE = no_of_slices;
\end{verbatim}

The created thin lens sequence has the following properties:

1. It has the same name as the original.
2. The original sequence is no longer available.
3. The slicer also slices any inserted sequence used in the main sequence. These are also given the same names as the originals.
4. Any component changed into a single thin lens has the same name as the original. If a component is sliced into more than one slice, the individual slices have the same name as the original component and a suffix . . . 1, . . . 2, etc., and a marker will be placed at the centre with the original name of the thick element.

Typical parameters used for the LHC studies are:

- nominal LHC lattice: i) main dipoles and quadrupoles: one slice; ii) insertion quadrupoles: two slices; iii) separation dipoles and low-beta triplets: four slices.
- upgrade LHC lattices [10]: i) main dipoles and quadrupoles: two slices; ii) separation dipoles, insertion quadrupoles: four slices; iii) low-beta triplets: sixteen slices.

In the case of the LHC upgrade optics, the number of slices is in general larger than for the nominal lattice, by a factor of two to four. This is due to the optics, featuring more pushed values of $\beta^*$. In the following, only the nominal LHC machine will be considered. From the previous discussion, in all cases in which the number of slices exceeds four, the standard approach of equal, equidistant slices is applied.

**NEW MODEL MIXING THIN AND THICK QUADRUPOLE ELEMENTS**

In the proposed new sequence to be used for tracking studies, the quadrupoles are modelled as follows: The usual thin multipole slices, created and positioned in MAD-X by the MAKETHIN command using the non-equidistant TEAPOT style, are kept unchanged to contain the field error components. The remaining thick magnet length is filled up with newly created thick quadrupole slices to contain the nominal pure quadrupole field.

Two variants of this approach have been considered. The tracking of thick elements requires a longer CPU-time. Therefore, the two options depend on whether all the quadrupoles are kept as thick elements or not. The first lattice represents the more extreme solution, and possibly the slowest in terms of tracking, in which all quadrupoles are kept thick. The second option is made of thin quadrupoles representing the main quadrupoles in the arcs, and thick quadrupoles in the insertions. The 392 main quadrupoles are modelled by four thin multipoles without any thick slices, thus reducing the total number of thick elements from 1918 to 1134. In the regular LHC arc lattice the optics produced with the four-slice TEAPOT model of these quadrupoles is sufficiently close to the thick model optics (see Fig. 1). Similarly, the dispersion function shown in Fig. 2 is better reproduced by the TEAPOT model than by equidistant slices, unless the number of slices is increased by a factor of about five.

To test the new models two sequences were created using *ad hoc* tools, starting from the thin sequence produced by MAKETHIN. For each insertion quadrupole (containing either two or four thin slices) two thick slice types of different length had to be created to fill up the three or five empty spaces between and around the thin multipoles. The integrated nominal gradient had to be transferred from the thin multipoles to the thick slices.

In a second step, the LHC error macros were modified.

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**Figure 1:** Horizontal and vertical $\beta^*$ for the injection (upper) and nominal collision optics (lower). The TEAPOT approach is clearly superior in terms of reproducibility of optical parameters. The lattice used is the one with thin quadrupoles in the arcs.

**Figure 2:** Horizontal dispersion for the injection optics. Also in this case the TEAPOT approach is clearly superior in terms of reproducibility of optical parameters. The lattice used is the one with thin quadrupoles in the arcs.
As the nominal fields were no longer contained in the same elements as the field errors, the error components could not be defined as relative errors (i.e., expressed w.r.t. the nominal field component) as was done previously. They had to be derived from the strengths of adjacent elements. The new macros detect automatically the presence of thick slices, allowing the use of both thin-only and thin-thick quadrupole models in the same sequence.

It is worth noting that in all these studies the dipoles are kept as thin elements represented by one single kick.

**TRACKING SIMULATIONS**

The ultimate test of the thick models is the computation of the dynamic aperture (DA). The configuration of the machine at injection energy is the one considered for these studies. Initial conditions distributed uniformly over 59 phase space angles, with 30 pairs over 2 $\sigma$ amplitude range have been used to probe the phase space stability. The maximum number of turns is $10^5$ and the momentum off-set is $0.75 \times 10^{-3}$ corresponding to $3/4$ of the bucket height. For each of the three lattice models considered, i.e., nominal model with thin quadrupoles, model with all quadrupoles thick, model with thin main quadrupoles and thick insertion quadrupoles, three configurations in terms of magnetic field errors have been considered. In one case errors are assigned to main dipoles, only; then main quadrupoles’ field errors are also included; the last configuration features magnetic field assigned to all magnets. Finally, sixty realisation of magnetic field errors are considered. Therefore, each configuration is made of $59 \times 6 \times 60 = 21240$ jobs. The results are shown in Fig. 3 in which the DA is plotted as a function of the phase space angle. No large difference is found for the DA of the three lattice models considered. Some minor variations are visible for the error bars that represent the DA variation over the sixty realisations.

The other crucial point is to compare the CPU-time required for these simulations. In Fig. 4 the distribution of the CPU-time for the various jobs making each single study is shown. For this comparison, the configuration with field errors in all magnets has been considered and the two extreme lattice models. The difference is clearly visible with a good factor of four between the cases.

**CONCLUSIONS**

A review of the lattice models used for numerical simulations of the nominal LHC machine has been performed. Mixed models with thin and thick quadrupoles have been built providing a good quality of the linear optical parameters. The use of the different models does not produce any significant difference in the computed DA. However, the CPU-time is four times longer for the model with all quadrupoles treated as thick elements. The situation would become even worse for the case of numerical simulations including also beam-beam effects, not to mention the additional constraint for the slicing algorithm of being compatible with the installation of the required thin-beam-beam lenses in the triplets’ region.

**REFERENCES**