Abstract
We have calibrated the lattice model and measured the beta and dispersion functions in Fermilab’s fast-ramping Booster synchrotron using the Linear Optics from Closed Orbit (LOCO) method. We used the calibrated model to implement ramped coupling, dispersion, and beta-beating corrections throughout the acceleration cycle, reducing horizontal beta beating from its initial magnitude of ~30% to ~10%, and essentially eliminating vertical beta-beating and transverse coupling.

INTRODUCTION
The Fermilab Booster is a fast-ramping synchrotron which accelerates protons from 400 MeV to 8 GeV in 33 milliseconds, supplying proton beam to all of the laboratory’s experimental facilities. In order to meet the demands of Fermilab’s planned high-intensity experimental program, the Booster’s total proton throughput will need to be doubled within the next several years [1]. The Booster’s combined-function magnets are powered by a resonant circuit that produces a fixed 15 Hz acceleration cycle, but currently beam is accelerated only during about half of these cycles. Planned RF upgrades will allow for beam to be accelerated during all 15 cycles, and it is therefore necessary to reduce beam loss per pulse so that tunnel activation does not exceed safe levels.

MODEL CALIBRATION METHOD
Measurement Process
We measured the linear response of the closed orbit to each of 96 dipole correctors in the Booster, at each of 102 beam position monitors around the ring. The orbit response was measured at twenty time points during the 33 ms acceleration cycle, using a dipole current change that increased with momentum so that the angular kick was approximately constant throughout the acceleration ramp. To reduce the effect of jitter in the BPM electronics, each measurement was repeated six times and the three most tightly-clustered measurements were used for the linear fit of orbit position vs. angular kick; this gave very good reproducibility in the measurements.

The Booster uses a radial position feedback system, so the dispersion function has a large effect on the response of the closed orbit to a dipole kick. The orbit response at location \(i\) to dipole \(j\), in terms of the optics functions at the location of the radial position feedback monitor RPOS, is

\[
\frac{\Delta x_i}{\partial \theta_j} = \frac{\sqrt{\beta_i \beta_j}}{2 \sin(\pi \nu_x)} \cos\left(\psi_j - \psi_j - \pi \nu_x\right) - \frac{D_i}{D_{\text{RPOS}}} \frac{\sqrt{\beta_{\text{RPOS}} \beta_j}}{2 \sin(\pi \nu_x)} \cos\left(\psi_{\text{RPOS}} - \psi_j - \pi \nu_x\right)
\]  

We measured the dispersion functions by varying the fixed position of the Booster’s radial position feedback system and measuring the orbit response at each BPM.

LOCO Optimization
We compared the measured response matrix with the response matrix generated by a model of the Booster in Elegant, which was based on the pre-existing MAD file. The Booster lattice has significant alignment errors which aren’t included in the MAD model, so agreement between the model optics and measurements had been poor. To calibrate the model, we added a set of hidden parameters (dipole and BPM scaling and rolls, and thin pseudo-quad and skew pseudo-quad errors) for each dipole and BPM, and used SVD-based minimization to find the values for
these parameters that minimized $F$, the difference between measured and model orbit response,

$$F = \sum_{i,j} \left( \frac{\partial x_i}{\partial \theta_j} \right)_{\text{Meas}} - \left( \frac{\partial x_i}{\partial \theta_j} \right)_{\text{Model}} \right)^2 \frac{1}{\sigma^2_{ij}}$$

where $i$ is the BPM index, $j$ is the dipole corrector index, and $\sigma_{ij}$ is the uncertainty in the linear fit parameter of a particular orbit response measurement.

**COUPLING CORRECTION**

The Booster has significant unintentional transverse coupling, due largely to magnet misalignments. Before correction, the transverse coupling was so strong that the minimum betatron tune separation was as large as 0.08 in some parts of the acceleration cycle.

*Before coupling correction:*

*After coupling correction:*

We used the LOCO optimization method to find the set of thin skew pseudo-quad errors that minimized the difference between the model and the measured orbit response. We then put in ramped changes to the Booster’s skew corrector quadrupoles, equal and opposite in strength to the skew pseudo-quad errors at each time point in the acceleration cycle.

A second iteration of orbit response measurements, as well as measurement of the minimum tune separation, confirmed that transverse coupling was corrected locally as well as globally. Figure 2 shows the measured orbit response to a dipole before and after coupling correction. Figure 3 shows the minimum tune separation before and after correction.

**MEASURED OPTICS**

*Initial Optics*

After correcting for transverse coupling, we repeated the orbit response measurements and model calibration. The betatron tunes were not involved in the LOCO calibration algorithm, so that comparison of model tunes with measured tunes could serve as an independent confirmation of the calibration method’s accuracy. The tunes predicted by our calibrated model agree well with measured tunes throughout the acceleration ramp, as is shown in Figure 4.

*Figure 2: Measured orbit response to variation of a horizontal dipole corrector, before and after coupling was corrected. BPM names are given on the horizontal axis.*

*Figure 3: Betatron tune response to changing quadrupole strengths, at about 4 ms after injection. Minimum tune separation is reduced from 0.08 to nearly zero.*

*Figure 4: Tune evolution through the Booster cycle. Calibrated model predictions are shown with crosses. The contour plots show the combined Continuous Fourier Transform (CFT) spectra of the pinged beam (see ref. [4] for details). The top picture shows the CFT spectra from horizontal BPM data, and the bottom shows the CFT spectra from vertical BPM data.*

The optics measurements revealed significant beta- and dispersion-beating, especially near the beginning of the acceleration cycle (see Figs. 5 and 6). This distortion is largely due to the fringe fields of a vertical four-bump dogleg which guides the beam around the extraction septum. These magnets are DC powered, so the distortion...
decreases as the beam momentum increases. The focusing imperfections that the beam encounters also change during the ramp because the position of the closed orbit moves nearly 20 mm during the ramp.

**Optics Corrections**

Since the magnitude and distribution of focusing errors in the Booster changes during its ramp, we implemented ramped corrections to each quadrupole to reduce beta- and dispersion-beating throughout the acceleration cycle. The quadrupole corrections at each time point were calculated by hand, using OptiM to predict the effects of each magnet change on the optics [5]. We adjusted the dipoles packaged with each corrector quadrupole to compensate for quad steering, adding a kick in each plane determined by the size of the quad correction \( \delta q = \frac{B'B}{Bp} \) and the position of the closed orbit through the quadrupole:

\[
\delta \Theta_x = x_{CO} \cdot \delta q, \quad \text{and} \quad \delta \Theta_y = -y_{CO} \cdot \delta q.
\]

We measured the orbit response to each quadrupole to determine the position of the closed orbit relative to the quad center. To first order, the orbit distortion at location \( i \) due to a change in quad \( j \) is analogous to that due to a dipole kick. (Eq. 3)

\[
\frac{\partial x_i}{\partial k_j} \approx -x_0 \frac{\sqrt{\beta_i \beta_j}}{2 \sin(\pi \nu_x)} \cos\left(\nu_i - \nu_j - \pi \nu_x\right)
\]

Figures 5 and 6 show the beta functions before and after correction, at different times in the Booster’s acceleration cycle. Beta beating was reduced to within about ten percent or less, and the orbit distortion caused by these quadrupole corrections was minimal.

**CONCLUSIONS**

Orbit response matrix optimization has proven to be a useful tool for understanding the Booster’s optics throughout the fast-ramping acceleration cycle. The calibration process resulted in a much more realistic model of the Booster’s lattice, allowing us to accurately predict the effects of corrector magnet changes and therefore successfully manipulate the optics. Thus far we have only demonstrated the efficacy of the beta correction method, and it is not yet used in normal operation. Integrating these ramped optics corrections into normal operations, in conjunction with lattice realignments and closed orbit optimization that are currently being undertaken, should result in reduced beam losses and help with meeting the Booster’s increased total proton throughput goals.

**REFERENCES**


