Abstract
The options for an extremely low bunch charge regime (20 pC) of the European XFEL project are studied. The parameter study (saturation length and power) is performed for a wide range of the beam normalized emittance, bunch length and energy spread. The study is based both on analytical scaling of the SASE FEL performance and numerical simulations.

INTRODUCTION
The European XFEL project [1] is aiming to generate ultra-short pulses of spatially coherent photon beams with wavelength down to 0.1 nm and a peak brilliance of about \(10^{33}\) photons/s/mm²/mrad²/0.1%BW. After acceleration in the main linac (up to 14 GeV) the electron beam with a normalized emittance of 1.4 mm-mrad enters the hundreds meters long undulator section. Five photon beam lines will deliver X-ray pulses to the experimental stations.

The nominal operating point foresees a 1 nC bunch charge. Low bunch charges operating regime have the advantages that a smaller emittance beam is produced in the gun and short pulse length can be achieved.

In this report we present the results of study a SASE FEL performance for a wide range of the beam normalized emittance, bunch length and energy spread.

In this article with the analytical scaling numerical simulation results are presented. The numerical calculations are made using the GENESIS [2] 3D simulation codes.

PARAMETER CHOICES
At the first stage of SASE FEL design one looks for the dependence of the FEL performance on the electron beam parameters and undulator parameters. These dependence can be found solving the FEL eigenvalue equations analytically. An approximate solution of this equations exist. The solution of this equation was fitted using 3 dimensionless groups of parameters, and 19 fitting coefficients. The scaling function for the gain length (saturation length is proportional to the gain length) is given by

\[ L_{sat} = \frac{\lambda_u}{4\pi\sqrt{3}\rho}(1+\eta) \]  

(1)

where \(\rho\) is the Pierce parameter, \(\lambda_u\) is undulator period length. The parameter \(\eta\) describe the effects caused by the beam energy spread, emittance and diffraction and can be found using Xie fitting formula [3]. The saturation power \(P_{sat}\) is given by the total electron beam power \(P_{beam}\) as [4]:

\[ P_{sat} \approx \frac{1.6}{(1+\eta)^{\frac{1}{3}}} \rho P_{beam} \]  

(2)

We will consider the case when the electron beam energy is 14 GeV, energy spread is 1.4 MeV and the radiation wavelength is 0.1 nm. The SASE1 undulator section design is based on a FODO cell with length of 12.2 m, implying that the focusing quadrupoles placed after each 5 m long undulator segment. The design average beta function 32 m. The total length of the undulator section is 201.3 m. The only difference is that in our considered case the undulator \(K\) value is equal to 2.6, because the radiation wavelength is kept the same and the electron beam energy is less then design one (the electron beam design energy is 17.5 GeV). The design value of undulator \(K\) parameter is 3.3. The undulator \(K\) parameter is given by

\[ K = \frac{eB\lambda_u}{2mn_e} = 0.934 \cdot B[T] \cdot \lambda_u [cm] \]

where \(B\) is the peak magnetic field and \(\lambda_u\) is the undulator period length (for SASE 1 undulator \(\lambda_u = 3.56 cm\)). Keeping the undulator period length constant undulator \(K\) parameter can be changed from 3.3 to 2.6 by increasing the undulator gap.

Using Equations 1 and 2 the FEL performances (saturation length and power) can be found. However, realistic FEL performance is obtained from careful 3D numerical simulations.

NUMERICAL SIMULATIONS
In this section we present the results of our study for the SASE1 performance (saturation length \(L_{sat}\) and saturation power \(P_{sat}\) ) at low charge operation regime. The sensitivity of FEL performances on beam normalized emittance and bunch rms length have been considered.

By means of GENESIS steady-state simulations the behaviour of the saturation length and saturation power for the beam normalized emittance 0.2, 0.3 and 0.4 mm-mrad have been investigated. For each case the bunch rms length varies from 1 to 4 \(\mu m\).
Figure 1 shows the behaviour of saturation power for the beam parameters mentioned above. With the simulation results analytical predictions are also presented.

![Figure 1: Dependence of saturation power on beam rms length at three different values of beam normalized emittance. (points – simulation results, lines – analytical scaling).](image1)

These results shown that for the case when beam normalized emittance is 0.2 mm-mrad, the saturation power will decrease from 27.4 to 3.4 GW, with the increasing of bunch rms length from 1 to 4 μm. For the cases when beam normalized emittance is 0.3 and 0.4 mm-mrad the variation of saturation length is 21.7 - 2.4 GW and 18.2 – 1.8 GW, respectively.

In Figure 1 is shown the variation of saturation length for above considered cases.

![Figure 2: Dependence of saturation length on beam rms length at three values of beam normalized emittance.](image2)

The simulation results show that when the electron beam normalized emittance grows the saturation length increases. Also the saturation length grows with increasing of bunch rms length. For the case when beam normalized emittance is 0.2 mm-mrad and bunch rms length increase from 1 to 4 μm the saturation length growth about 62 %. The variation of saturation length is 70% for the case when the beam normalized emittance is 0.3 mm-mrad, and 76% for the case when emittance is 0.4 mm-mrad.

**TIME DEPENDENCE SIMULATIONS**

One of the most important FEL radiation parameters is the brilliance. It is possible to calculate the brilliance by processing the results of GENESIS time dependent simulations.

In this section we present GENESIS time-dependence simulation results. Time dependence simulations have been performed for the case when the beam normalized emittance is 0.4 mm-mrad. The bunch longitudinal shape is assumed to be Gaussian with rms length \( \sigma = 1.2 \mu m \). Only ± 2\( \sigma \) bunch length around the bunch centre is included into the simulation because all radiation comes from the central part of the bunch (Fig. 3).

![Figure 3: Power profile along the bunch.](image3)

In Fig. 4 is shown the average FEL power variation along the undulator line.

![Figure 4: Average FEL power.](image4)

For this range of parameters the FEL saturates after 114 m at 0.1 nm wavelength. The peak power at saturation is 26 GW. The photon beam size is 66 μm rms and the divergence angle is 2.6 μrad (at saturation point). Note that the quoted values are the rms from both transverse dimensions. The pulse length is about 4.2 fs.
The FEL performance dependence on the energy spread has been investigated in the energy spread deviation range from 1.4 to 4.3 MeV. Fig. 5 presents the power distribution along the bunch at the saturation point for four different values of energy spread.

![Graphs](image)

Figure 5: Power distribution along the bunch at the saturation point for the following values of energy spread: 1.4 (a), 2.3 (b), 3.3 (c) and 4.3 MeV (d).

Saturation length increases by 30 % of its smallest value with the rise of the energy spread. The peak power at saturation point decreases by about 90 %. The reason of decreasing peak brilliance is that with the rise of energy spread the peak power decreases and the photon beam size and the divergence angle increases. For the case when energy spread is 4.3 MeV the pulse length is about 20 % shorter than for 2.3 MeV case. Saturation length and radiation parameters (peak power at saturation, radiation size, divergence and brilliance) dependence on the energy spread is presented in Table 1.

<table>
<thead>
<tr>
<th>Energy Spread [MeV]</th>
<th>( L_{sat} )</th>
<th>P</th>
<th>( \sigma_r )</th>
<th>( \sigma_\theta )</th>
<th>Br.</th>
</tr>
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<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
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</table>

**CONCLUSION**

The numerical results for SASE1 undulator section at low bunch charge regime, based on GENESIS simulations, show the following:

- The sensitivity of FEL saturation length is about 60-70 % for the beam normalized emittance and bunch rms length.
- By means of GENESIS time dependence simulations, saturation length increases by 33 % with the rise of energy spread from 1.4 to 4.3 MeV. For this variation range of energy spread the peak power at saturation will decrease by about 60%. The peak brilliance decreases by about 90 % of its maximum value.

The results presented in this article allows to estimate the FEL performance for European XFEL project at low bunch charge operation regime for wide range of beam normalized emittance, bunch length and energy spread.

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**REFERENCES**