DESIGN AND CONTROL OF ULTRA LOW EMITTANCE LIGHT SOURCES*

Johan Bengtsson# 
BNL, Upton, NY 11973, U.S.A.

Abstract
In the quest for brightness, the horizontal emittance remains one of the main performance parameters for modern synchrotron light sources. A control theory approach that takes the nonlinear dynamics aspects into account, using a few simple (linear) optics guidelines, at an early stage generates robust designs. Modern analytic-and computational techniques enable the optics designer to avoid the fallacy of the traditional approach guided by the Theoretical Minimum Emittance (TME) cell: the "chromaticity wall". In particular, by using an interleaved computational approach with the nonlinear dynamics analyst/model. We also outline how to implement the correction algorithms for a realistic model so that they can be re-used as part of an on-line model/control server for commissioning- and operations of the real system.

TRADE-OFFS: GLOBAL OPTIMIZATION

The (natural) horizontal emittance \( \varepsilon_x \) originates from the equilibrium:

\[
\text{damping} \leftrightarrow \text{diffusion}
\]

of three different processes: radiation damping, quantum fluctuations, and IntraBeam Scattering (IBS). One can show that (fundamental limit is IBS):

\[
\varepsilon_x \sim \frac{1}{R^2 \cdot P}
\]

where \( R \) is the bend radius, and \( P \) the radiated power.

The design of a synchrotron light source is essentially a matter of balancing the conflicting entities schematized in Fig. 1 (optimized for Insertion Device (ID) beam lines) [1].

However, the approach is misguided (reductionalist), since it only considers the linear optics, i.e., ignores how to control the resulting (linear) chromaticity, and hence does not lead to realistic/robust designs. In particular, it creates an artificial "chromaticity wall" [4]. It also leads to lattices with dispersion at the cavity; which potentially increases the effective transverse beam size due to synchro-betatron coupling (by i.e. operating with finite (linear) chromaticity).

To capture the control aspects of the nonlinear dynamics from the start of the NSLS-II, we have provided the following (linear) optics guidelines [5]:

- max chromaticity per cell,
- min peak dispersion,
- max values for the beta functions.

For an intuitive (systems) approach, see e.g. the MAX-IV conceptual design [6].

WHAT’S KNOWN

The first dedicated third generation light sources were commissioned in the early 80s, i.e., they have been optimized for over 20 years. Basically:

- The horizontal emittance (isomagnetic lattice) is given by

\[
\varepsilon_x \text{[nm·rad]} = 7.84 \times 10^{3} \cdot \left( \frac{E\text{[GeV]}}{J_x N_b^3} \right) F
\]

where \( N_b \) is the number of dipoles, \( J_x + J_z = 3 \), and \( F \geq 1 \). No dipole gradients \( \Rightarrow J_x \sim 1 \).

- Generalized Chasman-Green lattices: DBA, TBA, QBA, 7-BA [6].

- Effective emittance \( \Rightarrow \) chromatic cells.

- Increasing \( N_b \) reduces \( \varepsilon_x \) but also reduces the peak dispersion, which makes the chromatic correction less effective \( \Rightarrow \) "chromaticity wall".

- Damping wigglers (DWs): damping rings and conversion of HEP accelerators [7-8].

- Mini-Gap Undulators (MGUs), Three-Pole-Wigglers (TPWs) inside the DBA [9].

WHAT’S NEW

The NSLS-II design is conservative, i.e., it is based on well known techniques, but the approach is also novel because it combines these in a unique way:

- Use of damping wigglers to reduce horizontal emittance and as high flux X-ray sources \( \Rightarrow \) achromatic cells and weak dipoles.

- Medium energy ring (3 GeV) with \( \sim 30 \) DBA cells.

- Vertical orbit stability requirements.

- Generalized higher order achromat.
CHALLENGES

Given the design goals and approach, challenges related to non-linear dynamics issues are:

- Medium energy: control of Touschek lifetime and momentum aperture.
- 30 DBA cells: control of tune footprint.
- Control of impact of DWs and IDs -> include leading order nonlinear effects from DWs in the Dynamic Aperture (DA) optimizations.
- Optics requirements for IDs and top-up injection are contradictory: introduce alternating straights with high- and low horizontal beta functions => reduced symmetry (30 => 15).
- DBA: momentum dependence of optics functions => sufficient number of chromatic sextupole families.

There are also technical challenges:

- Weak dipoles: introduce TPWs (adjacent to the dipoles) => control of peak beta functions and horizontal dispersion.
- Vertical orbit stability: sub micron => pushing the state-of-the-art [10-11].

LATTICE PARAMETERS

The main lattice parameters are summarized in Tab. 1, where values specific to the NSLS-II are in bold type.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Energy ($E_0$)</td>
<td>3 GeV</td>
</tr>
<tr>
<td>Circumference ($C$)</td>
<td>791.5 m</td>
</tr>
<tr>
<td>Beam Current ($I_b$)</td>
<td>500 mA</td>
</tr>
<tr>
<td>Bending Radius ($R$)</td>
<td>25.0 m</td>
</tr>
<tr>
<td>Dipole Energy Loss ($U_0$)</td>
<td>286.5 keV</td>
</tr>
<tr>
<td>Emittance: ($\epsilon_x, \epsilon_y$)</td>
<td>(2.1, 0.01)/0.6,0.01</td>
</tr>
<tr>
<td>Momentum Compaction</td>
<td>0.00037</td>
</tr>
<tr>
<td>RMS Energy Spread: bare/w. 8 DWs</td>
<td>0.05/0.1%</td>
</tr>
<tr>
<td>Working Point ($v_x, v_y$)</td>
<td>(32.4, 16.3)</td>
</tr>
<tr>
<td>Chromaticity ($\xi_x, \xi_y$)</td>
<td>(-100, -42)</td>
</tr>
<tr>
<td>Peak Dispersion ($\eta_x$)</td>
<td>0.45 m</td>
</tr>
<tr>
<td>Beta Function ($\beta_x, \beta_y$): long/short straight</td>
<td>(18,3)/(3,3) m</td>
</tr>
</tbody>
</table>

ROBUST DESIGN AND CONTROL

Typically, the approach has been to first design the linear optics, and then attempt to control (fix) the DA, aka perturbative point-of-view. In other words, a “top-up” (reductionist) rather than “top-down” (systems) approach, see e.g. refs [12-14]. Clearly, a prerequisite for a robust design and effective commissioning is a realistic model, see Fig. 2.

Challenge: for a streamlined approach, how to re-use the design model for model based (on-line) control?

- See section MODEL BASED CONTROL.

For a systematic approach one may view the design process as “Closed-Loop” Control, see Fig. 3 applied to:

- lattice design,
- control of DA,
- guidelines for engineering tolerances, ring magnets, and insertion devices,
- correction algorithms,
- aka TQM (Total Quality Management) in industry.

Similarly, a Use Case approach is a rational method to capture and refine the often elusive requirements for:

- model based control [15].

By treating the control system as an abstraction and analyzing how abstract “actors” (e.g. individuals, groups, other sub-systems, etc.) interact with the system, one avoids the typical gridlock between different stake holders (“What’s the requirement?” vs. “What’s the best you can do?”). Instead, by focusing the effort on “what” rather than “how”, the process provides for a sequence of successive refinements that generates a set of quantitative, measurable requirements: aka a spiral approach.
MODELING CONSIDERATIONS

Of course, these methods assume that a realistic model has been provided. In particular, the following aspects must be addressed:

- A confinement problem governed by the Lorentz force: $\dot{p} / dt = q(E + \dot{v} \times B)$.
- The single particle dynamics is described by the relativistic Hamiltonian for a charged particle in an external electro-magnetic field (aka volume preserving flow) $\Rightarrow$ Symplectic integrators.
- The residual beam size is in dynamic equilibrium between “cooling” from radiation damping (described by classical radiation), and “heating” due to diffusion from quantum fluctuations (i.e. recoil form the emitted photons) $\Rightarrow$ Modified symplectic integrator.
- Need to model a realistic lattice: the optics, diffusion field errors, and related correction algorithms.
- Must be able to compute- and optimize the global properties of a realistic lattice: the optics, driving terms, tune footprint, etc.
- No theory of stability (for the general nonlinear case) $\Rightarrow$ Perturbation theory. Hence, “analytic” results must be validated by numerical simulations.
- Control nonlinear effects by the: lattice symmetry, driving terms/resonances (Lie generators), and tune footprint; obtained either from Taylor maps, Lie series, and map normal form (analytically) or frequency maps (numerically).

**Challenge:** How to combine the numerical methods for modeling of a realistic lattice with the analytical techniques for analysis of its properties?

- Introduce a polymorphic number class for transparent floating point- and TPSA (Truncated Power Series Algebra) [16] calculations with object-oriented programming [17] $\Rightarrow$ a Lagrangian object, aka PTC (Polymorphic Tracking Code) [18].

**Challenge:** How to re-use the beam dynamics model and related correction algorithms developed during the design phase as an on-line model for the commissioning?

- Implement a well designed software library that can be re-used by for instance the Controls Group.

MODEL

The Hamiltonian is (equations of motion for a medium size ring)

$$H = (1 + h_{ref}(s))\left\{ -\frac{q}{p_0} A_x(s) + \frac{1}{2(1 + \delta)} \left[ p_x - \frac{q}{p_0} A_x(s) \right]^2 \\
+ \left[ p_y - \frac{q}{p_0} A_y(s) \right]^2 \right\} - \delta + O(p_{x,y}) $$

with the multipole expansion

$$
\frac{q}{p_0} A_x(s) = -\text{Re}\left\{ \sum_{n=1}^{\infty} \left( b_n(s) + i a_n(s) \right)(x + iy)^n \right\}
$$

The map is obtained by splitting the Hamiltonian into two integrable parts ($f(\hat{x})$: $g(\hat{x}) = f(\hat{x}), g(\hat{x})$)

$$H = H_{\text{drift}} + H_{\text{kick}}$$

which leads to

$$M(\Delta s) = \exp\left( -\int_{0}^{\Delta s} H ds \right) = e^{-H\Delta s}:
\\= e^{-H_{\text{drift}}\Delta s/2} e^{-H_{\text{kick}}\Delta s/2} e^{-H_{\text{drift}}\Delta s/2} + O(\Delta s^3)
\\= M_{\text{drift}}(\Delta s) M_{\text{kick}}(\Delta s) M_{\text{drift}}(\Delta s/2) + O(\Delta s^3)

aka a 2nd order symplectic integrator. In particular, it can be generalized to 4th order. For insertion devices, the vector potential can be obtained from the magnetic field (numeric model or measurements) by

$$A_x(s) = -\int_{s} B_y(z) dz, \quad A_y(s) = \int_{s} B_x(z) dz, \quad A_z(s) = 0$$

The corresponding kick $M_{\text{kick}}$ map is provided for instance by RADIA [19].

For an analytic model (to leading order)

$$\frac{q}{p_0} A_x(s) = \frac{B_y}{B_\rho}(k_z(z)) \cos(k_z x) \cos(k_y y) \sin(k_z s), \quad \frac{q}{p_0} A_y(s) = \frac{k_y B_y}{k_z B_\rho}(k_z(z)) \sin(k_z x) \sin(k_y y) \sin(k_z s),$$

with $k_z^2 = k_y^2 - k_x^2 = (2\pi/\lambda_0)^2$.

Assuming that the corresponding Taylor map has been obtained (to an arbitrary order) from the beam dynamics model, the map can be factored (Lie series)

$$M = A^{-1} e^{-f^{(4)}} e^{-f^{(3)}} R A$$

The (Lie) generators (driving terms) provide a means to control the DA [20]. They can also be measured from turn-by-turn data and Fourier analysis [21] $\Rightarrow$ “closing-the-loop” between model and the real lattice [22]. The map can also be (recursively) transformed into normal form [23]

$$M = 1 \exp[-\partial^2(\hat{\mathcal{J}})] \exp[\mathcal{K}(\hat{\mathcal{J}})] \exp[\partial \mathcal{K}(\hat{\mathcal{J}})] \mathcal{R} A$$

from which we obtain the global properties of the lattice, e.g. the tune shift

$$\nu = -\frac{1}{2\pi} \partial \mathcal{K}(\hat{\mathcal{J}})$$

The information flow for the corresponding computer model is shown in Fig. 5.
The requirements are:
- on-momentum Dynamic Aperture (DA): 11 mm (robust top-up injection),
- off-momentum DA: 2.5% (Touschek life time),
- Tune footprint for the bare lattice (w/ DWs): ~0.05 (to accommodate engineering tolerances, IDs, etc.).

The last requirement is based on a (conservative) estimate of the tune footprint for stable beam in existing medium energy light sources, i.e., about ~0.1.

Note, due to the high number of DBA cells (30), as compared to existing medium energy synchrotron light sources, the control of the amplitude dependent tune shift per cell needs to be about 3 times better for a similar nonlinear performance. Hence, tight engineering tolerances are required.

The DA is essentially determined by the tune footprint and the sextupolar resonances (to 2nd order in the sextupole strength, aka 4th order resonances).

The analytic model for the amplitude dependent tune shift and residual nonlinear chromaticity need to include terms to 6th order in the sextupole strength:

$$\tilde{v} = \tilde{v}(\mathbf{J}) + \mathcal{O}(\mathbf{J}, \delta)^4$$

The resulting tune footprint and frequency maps after optimization are summarized in Figs 6 and 7. In particular, the introduction of DWs requires [24]:
- optics correction (local/global control of symmetry and working point),
- and sextupole re-optimization (due to the residual local optics perturbations from the DWs).

**MODEL-BASED CONTROL**

Assuming that a realistic model of the system has been provided, and that a robust design has been delivered, which will be implemented, the question arises:
- How to control the real system?

Since a model-based approach is required, ideally, the model and controls algorithms developed during the design work would be re-used on-line. In particular, by pursuing a client/server approach [25].

**Challenge:**
- How to migrate from high level application prototypes developed for beam studies into thin (simple) clients suitable for day-to-day operations.

By providing a software architecture that provides both [26-27]:
- a flexible environment for rapid prototyping with a scripting language,
- and a model server with thin clients, see Fig 8.

High level controls applications are ideally implemented and tested before commissioning, but one problem is that there is typically quite some lag time until hardware, etc. become available. It is thus desirable to have a simulator for the entire accelerator. A transparent approach is summarized in Fig. 9 [27-28].
CONCLUSIONS

- By using modern methods, a self-consistent, realistic computer model has been implemented, i.e., where the same model is used for numerical simulations and by analytic techniques.

- The model has been used to guide the NSLS-II design. In particular, it has provided an effective framework to control the dynamic aperture, to provide guidelines for engineering tolerances, and magnet- and insertion device design. In other words, “closing-the-loop” between conceptual design and the performance of the final hardware.

- The model is also being used by the Controls Group, as a simulator for the accelerator, by interfacing with the control system, for e.g. testing of high level applications & controls algorithms. Furthermore, it also provides a transparent implementation of a model server with thin clients for the commissioning of the accelerator.

REFERENCES