ENHANCEMENT OF UNDULATOR FIELD IN BULK HTSC STAGGERED ARRAY UNDULATOR WITH HYBRID CONFIGURATION*

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Abstract
To realize a short-period strong-magnetic field undulator, we have developed a novel undulator using bulk high-temperature superconductor magnets in a staggered array structure. To enhance the undulator field of the undulator, we studied numerically introducing hybrid staggered array configuration in the undulator. As a result, we found that the undulator field in the hybrid type was stronger than the normal type at the same critical current density of the bulk high-temperature superconductor magnet. We also found that the critical current density and the amount of the solenoid field change required to generate the certain undulator field were less in the hybrid type than in the normal type. This implied that the hybrid type can operate at higher temperature and the lower cost than the normal type.

INTRODUCTION
The combination of short-period strong-magnetic field undulators and middle-energy storage rings is the promising way to obtain high-brightness undulator radiation in hard x-ray region [1, 2]. To realize a short-period strong-magnetic-field undulator, there have been several studies. In-vacuum undulators mainly have been used in large numbers of synchrotron facilities. Cryogenic permanent magnet undulator in which the magnetizations of the permanent magnets are enhanced by cooling the magnets down to 100 – 150 K has been developed in SPring-8 [3] and operated in the real facilities. The undulator using low-temperature superconductor wires in vacuum is proposed [4].

Recently, the magnetic properties of bulk high-temperature superconductor (bulk HTSC) have been largely studied. The trapped field of over 17 T in 26 mm diameter bulk HTSC was reported [5]. Use of the bulk HTSC to undulators is proposed by Spring-8 [6, 7] and out institute [8]. At our institute, we have developed the undulator using bulk HTSCs in the staggered array structure. The detailed principle of bulk HTSC staggered array undulator (Bulk HTSC SAU) is described in the next session. Here, we describe the problem. In Bulk HTSC SAU, the bulk HTSCs which have the magnetization direction of the same with the solenoid field (z-direction) can generate the undulator field (y-direction). This should be the advantage because they can be magnetized by the one external solenoid and the amplitude of the undulator field is controlled by the solenoid. At the same time, this should be the disadvantage because multi-times stronger magnetization and solenoid field than the undulator field is required. Therefore we improve the configuration of Bulk HTSC SAU to effectively use the strong magnetization of the bulk HTSCs as the undulator field.

In this paper, we will explain the enhancement of undulator field in the undulator in which the hybrid staggered configuration is introduced to Bulk HTSC SAU (Hybrid Bulk HTSC SAU). First, the basic concept of these undulators will be explained. Second, the calculation model and the method in RADIA which is the numerical code for undulator field developed by European Synchrotron Radiation Facility [9] will be described. Third, the result of the calculation will be shown and the effect of the enhancement of the undulator field and reduction of the required solenoid field are discussed.

Figure 1: Conceptual design of Bulk HTSC SAU and Hybrid Bulk HTSC SAU

CONCEPT
Figure 1 shows the conceptual design of Bulk HTSC SAU (left side) and Hybrid Bulk HTSC SAU (right side). In Bulk HTSC SAU, the bulk HTSCs and the copper pieces to support them are stacked in the solenoid. After cooling down the undulator below the critical temperature of the bulk HTSC, solenoid field is changed from the initial value to the operation value. From Faraday's electromagnetic induction law, the superconducting currents induced in the bulk HTSCs to negate the changes of the solenoid field in each bulk HTSC. Therefore, bulk HTSCs are magnetized with the inverse direction to the solenoid field. By these longitudinal (z-direction) magnetizations, the transverse (y-direction) undulator field is produced. In Hybrid Bulk HTSC SAU, there are the soft magnetic pieces instead of the copper pieces and they are magnetized with the same direction to the

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solenoid field. Because the bulk HTSCs and the soft magnetic pieces make the closed magnetic circuits, the undulator field is expected to be enhanced. This idea is based on the hybrid staggered array undulator [10] in which the undulator field is enhanced from the original staggered array undulator [11].

**CALCULATION METHOD**

To calculate the magnetic field in the structure consists of the bulk HTSCs and the soft magnetic pieces, we used RADIA code which can treat the saturation of the soft magnetic piece, and made the calculation with the model of the bulk HTSC developed in our previous work [12] based on Bean’s critical state model [13] of type II superconductor.

**Calculation Structure**

Figure 2 shows the cross-sectional structure used of Bulk HTSC used in the magnetic field calculation by RADIA code. We modelled Bulk HTSC by the current loop with the current $I = J_c d_y D_z$. Where $J_c$ is the critical current density, $D_z$ is the thickness of Bulk HTSC in $z$-direction, and $d_y$ is the thickness of loop current. Note that, although Bulk HTSC had a dimension of $D_y$ in $y$-direction, we assumed that the current only flow in the region with the distance of $d_y$ from its edge and there were no variation of $d_y$ in $z$-direction in single Bulk HTSC. The current of the loop was determined by $I = J_c d_y D_z$. We assumed the Bean’s critical state model in which the current density was the same with the critical current density $J_c$ anywhere in Bulk HTSC and $J_c$ was constant over Bulk HTSC and there was no individual difference. Here, we defined $\lambda_d = 2 d_y / D_y$ which indicate the relative depth of penetration of current and external magnetic field from its edge.

**Figure 2:** RADIA structure of Bulk HTSC

Figure 3 shows the three-dimensional structure and the coordinate system used in RADIA calculation. The green loops indicate the Bulk HTSC and the gray blocks indicate the copper (Bulk HTSC SAU case) or soft magnetic pieces (Hybrid Bulk HTSC SAU case). We used Vanadium permendur as the soft magnetic piece. The parameters used in the calculation are listed in Table 1.

<table>
<thead>
<tr>
<th>Table 1: Used Parameters</th>
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<tbody>
<tr>
<td>Period $\lambda_0$</td>
</tr>
<tr>
<td>Gap $g$</td>
</tr>
<tr>
<td>Periodic number</td>
</tr>
<tr>
<td>$D_z$</td>
</tr>
<tr>
<td>$D_x$</td>
</tr>
<tr>
<td>Dimension ($x, y, z$)</td>
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<tr>
<td>Penetration ratio $\lambda_d$</td>
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<tr>
<td>Penetration depth $d_y$</td>
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<tr>
<td>Range of critical current density $J_c$</td>
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**Calculation Method**

In the previous work, the amount of the solenoid field change $\Delta B_z$ was given and the penetration depths of each bulk HTSC were calculated iteratively. This was to obtain the distribution of the undulator field along $z$-axis. In this study, only the amplitude of the undulator field $B_y$ at the centre of the undulator should be required to check the enhancement of the undulator field with the Hybrid Bulk HTSC SAU. Therefore, we calculated as following to reduce the time of RADIA calculation which included the nonlinear ferromagnetic pieces. First, we assumed that the penetration ratios were the same in all bulk HTSCs. The penetration depth $d_y$ was decided from given parameters $D_y$ and $\lambda_d$. Second, we assumed that the undulator field was the same with the $y$-component of the calculated magnetic field at the centre of the undulator $B_y(0, 0, 0)$. Third, the applied solenoid field was the same with the $z$-component of the calculated magnetic field at the centre of the bulk HTSC near the centre of undulator $B_z(0, (D_y + g)/2, D_d/2)$ because the solenoid field and the magnetic field generated by Bulk HTSCs should be balanced.

**RESULT AND DISCUSSION**

Figure 4 shows the undulator field dependency on the critical current density in each undulator type. The undulator field in Hybrid Bulk HTSC SAU is stronger than Bulk HTSC SAU. Especially, because the undulator field is almost doubled and enough strong in the range between 2 and 4 kA/mm$^2$. Hybrid Bulk HTSC SAU seems effective. In another view point, the undulator field of 1.2 T is obtained at 3.5 kA/mm$^2$ in Hybrid Bulk HTSC SAU, while it is obtained at 6.5 kA/mm$^2$ in Bulk HTSCs.
SAU. This enables us to use Hybrid Bulk HTSC SAU at higher temperature.

![Graph showing the undulator field dependency on the critical current density in each undulator type]

Figure 4: The undulator field dependency on the critical current density in each undulator type.

Figure 5 shows the dependency of the required amount of the solenoid field change on the undulator field. This shows that the required amount of the solenoid field change to generate the same amplitude of the undulator field, it is related to the required solenoid field, is weaker in Hybrid Bulk HTSC SAU. Especially, Hybrid Bulk HTSC SAU at \( J_c = 3.5 \text{ kA/mm}^2 \) has a field of 2.2 T is just required, while the solenoid field of 4.0 T is required in normal Bulk HTSC SAU at \( J_c = 6.5 \text{ kA/mm}^2 \).

![Graph showing the dependency of the required amount of the solenoid field change on the undulator field in each undulator type.]

Figure 5: The dependency of the required amount of the solenoid field change on the undulator field in each undulator type.

CONCLUSION

To enhance the undulator field of Bulk HTSC SAU, we studied numerically introducing hybrid staggered array configuration in the undulator. To calculate the magnetic field in the structure consisted of the bulk HTSCs and the soft magnetic pieces, we made the calculation by RADIA code with the model of the bulk HTSC and the method to treat it. As a result, we found that the undulator field in the hybrid type was stronger than the normal type at the same critical current density of the bulk high-temperature superconductor magnet. We also found that the critical current density and the amount of the solenoid field change required to generate the certain undulator field were less in the hybrid type than in the normal type. This implied that the hybrid type can operate at higher temperature and the lower cost than the normal type.

REFERENCES