Abstract
In this paper we investigate, and compare, the properties of two narrow bandwidth FEL schemes: Self-Seeding [1] and High Gain Harmonic Generation [2, 3]. These schemes have been thoroughly studied analytically and numerically in the past. The aim of this work is to compare the performances of these schemes with respect to several non-ideal properties of the electron beam such as shot to shot energy fluctuations and nonlinear energy chirp. The work has been carried out with the aid of the time dependent FEL codes GENESIS (3D) [4] and PERSEO (1D) [5].

NARROW BANDWIDTH FEL SCHEMES
One goal of several FEL facilities operating in the soft x-ray spectral range, is the production of narrow bandwidth FEL radiation. Several schemes have been proposed to obtain bandwidths narrower than that achievable with SASE.

A self seeded FEL is composed of two undulators separated by a monochromator and a magnetic chicane. The FEL process in the first undulator is started by shot noise and is interrupted well before saturation. While the broad band SASE radiation is sent through a monochromator the electron beam passes through magnetic chicane which destroys the microbunching introduced by the SASE and compensates the delay introduced by the monochromator. The monochromatic radiation and the demodulated electron beam are then sent through the second undulator for a seeded FEL process.

An HGHG scheme is composed of two undulators separated by a magnetic chicane. The first undulator is seeded by an external coherent source. The FEL interaction in the first undulator introduces an energy modulation in the electron beam. The dispersive section transforms the energy modulation in a density modulation on higher harmonics of the seed wavelength. The second undulator is tuned to one of these harmonics. The bunching factor generated by the dispersive section triggers the FEL process in the second undulator.

STUDIED CASE
Electron beam and undulator parameters
The comparison between HGHG and Self Seeding has been carried out using the electron beam parameters of the 1nC, 1.5 GeV working point of the SPARX FEL, shown in Tab 1.

Table 1: Electron Beam Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Energy</td>
<td>1.5 GeV</td>
</tr>
<tr>
<td>Peak Current</td>
<td>1.5 kA</td>
</tr>
<tr>
<td>Uncorrelated Energy Spread</td>
<td>$10^{-4}$</td>
</tr>
<tr>
<td>Normalized emittance</td>
<td>1 mm*mrad</td>
</tr>
<tr>
<td>RMS bunch length (gaussian)</td>
<td>60 μm</td>
</tr>
</tbody>
</table>

For the self seeded scheme both the undulators have a 2.8 cm period, while in the HGHG case the period is 4.2cm for the modulator and 2.8cm for the radiator.
The self seeded scheme operates at 6 nm while for the HGHG we assume a 30 nm seed with 5th harmonic conversion.

Results in the ideal case: self seeding
For the self-seeding option we assume a monochromator with a bandwidth of $3 \times 10^{-5}$ and 20% transmissivity. The $R_{50}$ element of the dispersive section must be larger than about 50μm to completely demodulate the FEL induced bunching at the exit of the first undulator.

Since the emission of SASE radiation is a stochastic process, the radiation after the monochromator suffers from intrinsic statistical fluctuations. To fully describe the FEL process the results of simulations have to be averaged over several independent runs.
The length of the first undulator is chosen so that the average power after the monochromator is at least two orders of magnitude larger than the equivalent shot-noise power. A choice of 410 periods for the first undulator gives an average peak power after the monochromator of 60 kW, well above the shot noise level (roughly 100W). The results of 1-D time dependent simulations are shown in figures 1, 2, and 3.

Figure 1: Pulse energy, averaged over 20 shots, as a function of the position along the undulator.

Figure 2: RMS Spectrum at saturation (averaged over 20 shots). FWHM bandwidth is $4.2 \times 10^{-5}$.

Figure 3: RMS Intensity fluctuations along the second undulator.

The saturation energy is 1.4 mJ, distributed over a bandwidth of $4.2 \times 10^{-5}$. The intensity fluctuations at the second undulator entrance are close to 100% but decrease as the FEL amplifier approaches the non linear regime until reaching a value of 10% at saturation.

Results in the ideal case: HGHG

The seed source has a central wavelength of 30 nm. To tune the modulator to such wavelength, the undulator period is 4.2 cm and the undulator parameter is $K=4.41$. We assume a seed power of 100kW, which provides an energy modulation amplitude of $4\sigma_p$ after 130 undulator periods. The optimum value for the dispersive section strength is $R_{56}=10^{-5}$, which gives a fifth harmonic bunching $b_s=0.1$. The seed pulse has the same length as the electron beam. The saturation energy is roughly 1mJ over a bandwidth of $6.8 \times 10^{-5}$.

Figure 4: Pulse energy along the radiator.

Figure 5: Spectrum at saturation FWHM bandwidth is $6.8 \times 10^{-5}$.

In this ideal example the performances of the two schemes are almost equivalent, with a slightly larger spectral width in the HGHG case which on the other side is not affected by intrinsic shot to shot fluctuations and reaches saturation in a shorter undulator. In the next section we analyse one of the effects associated to a real electron beam, i.e. a non-linear energy chirp in the longitudinal phase space distribution and shot to shot energy fluctuations.

EFFECT OF NON-LINEAR ENERGY CHIRP

A non-linear energy chirp in the electron beam is responsible for spectral broadening. In both schemes spectral broadening is due to the imaginary part of the gain varying with energy along the beam and resulting in a non-linear phase chirp in the radiation pulse. Following the one dimensional model of the FEL (neglecting the effect of slippage) we obtain the following expression for the phase of the electric field:

$$\psi(s) = -2 \cdot k_w \cdot p(s) \cdot z + \Re\{\Lambda(p(s))\} \frac{z}{L_g} \quad (1)$$

where $z$ is the position along the undulator, $p$ is the energy offset, $\Lambda$ is the root of the usual FEL cubic and $L_g$ is the 1-D FEL gain length. Deriving eq 1 with respect to $s$ and linearizing $\Re\{\Lambda(p)\}$ we can express the local frequency offset as:

$$d\lambda(s)/\lambda = 2N_s \lambda \frac{dp}{ds} - \lambda \frac{dp}{ds} N_s \frac{d\lambda}{ds} = \frac{4}{3} N_s \lambda \frac{dp}{ds} \quad (2)$$
In a quadratically chirped beam \( \frac{dp}{ds} \) varies with position, resulting in a frequency modulated radiation pulse with a broader bandwidth.

HGHG suffers from an additional broadening effect due to the quadratically chirped beam passing through a dispersive section: compression in the dispersive section shortens the beam by an amount that is proportional to the derivative of energy with respect to \( s \). Since the number of periods of the energy modulation is constant the frequency is offset by an amount equal to:

\[
d\lambda(s) / \lambda = R_{56} \cdot \frac{dp}{ds}
\]  

(3)

It is worth noting that the \( 2N_a \lambda \frac{dp}{ds} \) term in eq. 2 contains the \( R_{56} \) of the undulator \( (2N_a \lambda) \), and accounts for the same physical effect happening in the dispersive section (local frequency offset due to compression), while the remaining term is due to the FEL gain.

In the self seeded scheme the frequency chirp is introduced in the second undulator while in the HGHG contribution from both the undulators have to be taken into account and the local frequency offset is:

\[
d\lambda(s) / \lambda = \left( \frac{4}{3} \lambda \cdot (nN_{mod} + N_{rad}) + R_{56} \right) \cdot \frac{dp}{ds}
\]  

(4)

Inserting the parameters of section 2 in equations 2 and 4 we obtain:

\[
\frac{d\lambda}{\lambda}_{HGHG} = 5 \cdot \frac{d\lambda}{\lambda}_{S.S.}
\]  

(5)

Figure 6 shows the FWHM bandwidth as a function of the amplitude of the quadratic energy chirp for the schemes described in section 2 (the spectra are calculated with PERSEO FEL code). In the example considered the spectral broadening is almost 5 times bigger for the HGHG scheme, a result that is consistent with the analytical estimate.

Figure 6: FWHM bandwidth at saturation as a function of the quadratic chirp amplitude. The results for self seeding are averaged over 20 independent runs.

The consequence of this sensitivity is that particular care has to be given to the “flatness” parameter and to e-beam phase space optimization for seeded operation [6].

**EFFECT OF BEAM ENERGY FLUCTUATIONS**

The effect of beam shot to shot energy fluctuations has been investigated with time dependent simulations with PERSEO. Figure 8 shows the pulse energy at saturation (at a fixed point along the undulator) as a function of the beam detuning.

The performances of the two schemes with respect to beam energy fluctuations are comparable. This is due to the fact that the only difference between the two schemes is the trigger of the FEL instability and the FEL amplifier in the saturation regime is largely insensitive to the fluctuations of the “starting value” (see figure 3).

Figure 8: Pulse energy at the saturation point as a function of energy detuning. The results for the self seeding are averaged over 20 independent runs.

**START TO END SIMULATIONS**

The performances of the two schemes have been investigated with start to end simulations in the SPARX FEL context. The 1.5GeV, 1nC working point (see figures 9 and 10) has been taken into account.

Figure 9: Energy profile of the SPARX 1.5GeV, 1nC beam.

Figure 10: Current profile of the SPARX 1.5GeV, 1nC beam.
For the seed source the following parameters have been chosen:
- Fractional Bandwidth: $5 \times 10^{-4}$
- Duration: 100 fs
- Peak Power: 10 kW
which are typical of a state of the art HHG in gas source at 30 nm, after monochromatization.

Figure 11: HGHG: pulse energy along the radiator (left) and spectrum at saturation. Pulse energy at saturation is 400 $\mu$J and the FWHM bandwidth is $5.8 \times 10^{-4}$.

Figure 12: HGHG: radiation power (left) and field phase (right) along the beam.

Figure 13: Self Seeding: pulse energy along the radiator (left) and spectrum at saturation. Pulse energy at saturation is 1 mJ and the FWHM bandwidth is $4.1 \times 10^{-5}$.

Figure 14: Self Seeding: radiation power (left) and field phase (right) along the beam.

The undulator parameters are the same as described in section 2 with the only difference that the modulator for the HGHG scheme is made of 3 sections of 55 periods each and the first undulator for the self seeded scheme is made of 7 sections of 75 periods. With this setup we obtain an 8% harmonic bunching factor in the HGHG scheme and 60 kW of average power after the monochromator (assuming a 20% efficiency and $3 \times 10^{-5}$ bandwidth).

The HHG seed pulse is significantly shorter than the electron beam, while the monochromatized SASE pulse for the self seeded scheme has approximately the same length, resulting in a higher pulse energy in the self seeding case.

The longitudinal phase space is very non-linear (see figure 9), resulting in a significant spectral broadening in both cases (see the strong quadratic dependence of the field phase in figures 12 and 14). As predicted in section 4, the broadening effect is much bigger in the HGHG scheme, resulting in a FWHM bandwidth 10 times bigger than the self seeding scheme.

**CONCLUSIONS**

Start to end simulations have been carried out using a realistic electron beam phase space optimized for SPARX SASE operation and state of the art seed parameters. In this conditions the performances of the self seeded scheme exceed those of HGHG by a factor 30 in terms of spectral brilliance due to the spectral broadening associated to non linear longitudinal phase space and to the short duration of the seed pulse. On the other side the beam was optimized for SASE operation, i.e. maximizing the peak current and with no special attention to the longitudinal energy chirp. Also in consideration of the much shorter radiator length required in the HGHG case, the chirp may be reduced with a lower e-beam compression factor and a reduced peak current.

In conclusion the performances of the two schemes have been analyzed and compared taking into account the main non ideal effects that affect narrow bandwidth FEL operation. Both schemes are equally sensitive to beam energy fluctuations and the HGHG scheme is more sensitive to non linear terms in the beam energy profile and requires specific optimization of the electron beam phase space distribution.

**REFERENCES**