Atomic Physics in Traps

QED – Fundamental Constants – CPT Invariance

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Quantum mechanics, Relativity, and P.A.M. Dirac

- Quantum mechanics
- Special Relativity
- Dirac theory

- electron magnetic moment
- energy levels of H-like ions
- negative energy states
- existence of anti-matter

- g-factor
- Lamb shift
- few-el. ions
- CPT tests
Quantum Electrodynamics (QED)

QED = Dirac theory + quantized radiation field

basic processes in QED:

- self energy
- vacuum polarization
- vertex correction

QED coupling parameter: finestructure constant $\alpha = \frac{e^2}{2\varepsilon_0hc} \approx \frac{1}{137} \approx 0.007$

Ref.:
bound-state QED: quantum physics in strong fields

basic processes in bound-state QED:

self energy  vacuum polarization  vertex correction

bound-state QED coupling parameter for U^{91+}:  \( Z\alpha \approx 0.67 \)

Ref.:
Magnetic moment (g-factor) of the electron

\[ \mu = g \cdot \frac{e}{2m} J \]

m: magnetic moment
g: g-factor
e: charge
m: mass
J: angular momentum
\[ g = 2 + \frac{\alpha}{\pi} \]
\[ g_{\text{free}} = 2 \left( 1 + C_1 \frac{\alpha}{\pi} + C_2 \left( \frac{\alpha}{\pi} \right)^2 + C_3 \left( \frac{\alpha}{\pi} \right)^3 + C_4 \left( \frac{\alpha}{\pi} \right)^4 + C_5 \left( \frac{\alpha}{\pi} \right)^5 + \ldots \right) \]

1st order in \( \alpha \): Schwinger term
\[ C_1 = \frac{1}{2} \]

The theory of quantum electrodynamics is, I would say, the jewel of physics - our proudest possession.

Ref.:
J. Schwinger, Phys. Rev. 73, 416 (1948); Hanneke et al., PRL 100, 120801 (2008)

R. Feynman
Free electron: QED contributions of 2\textsuperscript{nd} and 3\textsuperscript{rd} order

\[ g_{\text{free}} = 2 \left( 1 + C_1 \alpha / \pi + C_2 (\alpha / \pi)^2 + C_3 (\alpha / \pi)^3 + C_4 (\alpha / \pi)^4 + C_5 (\alpha / \pi)^5 + \ldots \right) \]

2\textsuperscript{nd} order in \( \alpha \):
\[ C_2 = -0.328\,478\,966 \]
7 graphs

3\textsuperscript{rd} order in \( \alpha \):
\[ C_3 = 1.1765 \]
72 graphs

not shown:
4\textsuperscript{th} order in \( \alpha \):
\[ C_4 = -1.9108 \]
891 graphs

Ref.:
Free electron: QED contributions of 5th order

\[ g_{\text{free}} = 2 \left( 1 + C_1 \alpha/\pi + C_2 (\alpha/\pi)^2 + C_3 (\alpha/\pi)^3 + C_4 (\alpha/\pi)^4 + C_5 (\alpha/\pi)^5 + \ldots \right) \]

Harvard g-2 measurement 2008:

\[ g_{\text{free}} = 2 \left( 1.00115965218073(28) \right) \rightarrow \text{determination of } \alpha \]

5th order in \( \alpha \):

\[ C_5 = 9.16 \]

12672 graphs


„I am digging at the roots of physics to see whether there is some treasure there."

Toichiro Kinoshita
Determinations of the finestructure constant $\alpha$
Bound-electron g-factor:
Feynman graphs 1st order in $\alpha/\pi$

\[
g_{\text{bound}}/g_{\text{free}} \approx 1 - (Z\alpha)^2/3 + \alpha(Z\alpha)^2/4\pi + ...\]

Dirac theory  
bound-state QED

Ref.:  
Bound-electron g-factor

![Graph showing the bound-electron g-factor vs. nuclear charge number Z.](image)

- **C)**: Z = 6
- **O**: Z = 8
- **Si**: Z = 14
- **Ca**: Z = 20
- **Pb**: Z = 82
- **U**: Z = 92

H-like ions

free electron g = 2.002 319 304 361 460
g-Factor of the electron bound in a hydrogen-like ion

Larmor precession frequency of the bound electron:

\[ \omega_L^e = \frac{g_J}{2} \frac{e}{m_e} B \]

\[ g_J = 2 \frac{\omega_L^e}{\omega_c^{ion}} \frac{m_e}{M_{ion}} \frac{Q^{ion}}{e} \]

Ion cyclotron frequency:

\[ \omega_c^{ion} = \frac{Q}{M_{ion}} B \]

→ 'experimental g-factor'
→ comparison with theory

→ comparison with theory

COOL13, Mürren, CH, 14 June 2013, Wolfgang Quint
A single highly charged ion stored in a Penning trap

radial confinement

endcap

ring

dendcap

potential

$B_0$
magnetic

physical

electric

combined ion motion

$U_0$

axial confinement

ion

$\omega_z$

AXIAL MOTION

$\omega_e$

MAGNETRON DRIFT

(MODIFIED) CYCLOTRON MOTION
Highly charged ion g-factor apparatus

SUPERCONDUCTING MAGNET WITH ROOM TEMPERATURE BORE

CRYOSTAT

CRYO ELECTRONICS @ 4 K

SUPERCONDUCTING SOLENOIDS

PENNING TRAP @ 4K

MICROWAVE INLET

PRECISION TRAP

‘DOUBLE TRAP’

MINI EBIS

TARGET

FEP
Electronic detection of a single trapped ion: Resistive cooling and active feedback cooling

\[ dE_t/\text{dt} = P_{\text{cool}} = -I^2R \]

\[ v_z = 680 \text{ kHz} \]

feedback electrode
ring electrode
compensation electrode
end cap

resistive cooling
Resistive cooling of trapped $^{12}\text{C}^{5+}$ ions

- final temperature: $T = 4$ Kelvin

Resistive cooling of trapped $^{12}\text{C}^{5+}$ ions
High-resolution cyclotron frequency measurement of a single highly charged silicon ion

$^{28}\text{Si}^{13+}$

$w_{\text{FWHM}} = 53(1) \text{ mHz}$

$(v_+ - 26\,857\,368 \text{ Hz}) / \text{ Hz}$
Bound electron magnetic moment measurement on hydrogen-like silicon $^{28}\text{Si}^{13+}$


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GSI Helmholtzzentrum für Schwerionenforschung GmbH, Planckstraße 1, 64291 Darmstadt, Germany

(Received 6 May 2011; published 7 July 2011)

We determined the experimental value of the $g$ factor of the electron bound in hydrogenlike $^{28}\text{Si}^{13+}$ by using a single ion confined in a cylindrical Penning trap. From the ratio of the ion’s cyclotron frequency and the induced spin flip frequency, we obtain $g = 1.995348958(7)(3)(8)$. It is in excellent agreement with the state-of-the-art theoretical value of 1.995348958 0(17), which includes QED contributions up to the two-loop level of the order of $(Z\alpha)^2$ and $(Z\alpha)^3$ and represents a stringent test of bound-state quantum electrodynamics calculations.
Comparison of theory and experiment: g-Factor of the bound electron in H-like carbon $^{12}\text{C}^{5+}$, oxygen $^{16}\text{O}^{7+}$ and silicon $^{28}\text{Si}^{13+}$

<table>
<thead>
<tr>
<th>Element</th>
<th>g$_J$ (Theory)</th>
<th>g$_J$ (Measurement)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{12}\text{C}^{5+}$</td>
<td>2.001 041 590 18 (3)</td>
<td>2.001 041 596 4 (10)(44)</td>
</tr>
<tr>
<td>$^{16}\text{O}^{7+}$</td>
<td>2.000 047 020 32 (11)</td>
<td>2.000 047 025 4 (15)(44)</td>
</tr>
<tr>
<td>$^{28}\text{Si}^{13+}$</td>
<td>1.995 348 958 0 (17)</td>
<td>1.995 348 958 7 (5)(3)(8)</td>
</tr>
</tbody>
</table>

Lit.:
- V. Shabaev et al., PRL 88, 091801 (2002)
- V. Yerokhin et al., PRL 89, 143001 (2002)
- K. Pachucki, V. Yerokhin et al., PRA 72, 022108 (2005)
- S. Sturm et al., PRL 107, 023002 (2011)
Bound-electron g-factor

CONTRIBUTION TO G-FACTOR

NUCLEAR CHARGE Z

Ref.: D. Glazov

Dirac
1-loop QED free electron
1-loop QED-BS
2-loop QED-BS

Häffner 2000
Verdu 2004
Sturm 2011
Sturm 2013

Köhler 2013, preliminary

1-loop QED-BS
2-loop QED-BS

Dirac
1-loop QED
2-loop QED

Verdu 2004
Determination of electron mass

Larmor precession frequency of the bound electron:

$$\omega_L^e = \frac{g_J}{2} \frac{e}{m_e} B$$

Ion cyclotron frequency:

$$\omega_c^{ion} = \frac{Q}{M_{ion}} B$$

$$\frac{m_e}{M_{ion}} = \left(\frac{g_J}{2}\right) \cdot \frac{\omega_c^{ion}}{\omega_L^e} \cdot \frac{e}{Q}$$

→ determination of electron mass

theory as input parameter

our measurement
Determination of the electron mass from g-factor measurements on H-like carbon $^{12}\text{C}^{5+}$ and oxygen $^{16}\text{O}^{7+}$

$^{12}\text{C}^{5+}$ g-factor measurement

\[ m_e^{(12}\text{C}^{5+}) = 0.000\ 548\ 579\ 909\ 32 (29) \text{ u} \]

$^{16}\text{O}^{7+}$ g-factor measurement

\[ m_e^{(16}\text{O}^{7+}) = 0.000\ 548\ 579\ 909\ 60 (41) \text{ u} \]

Van Dyck et al.,
comparison of cycl. frequencies $\nu_e/\nu(\text{C}^{6+})$

\[ m_e^{(\text{UW})} = 0.000\ 548\ 579\ 911\ 10 (120) \text{ u} \]

Outlook:
1) Improved measurement on carbon $\text{C}^{5+}$,
work in progress by F. Köhler and S. Sturm
2) Measurements on lighter ions, e.g. $^4\text{He}^{1+}$
Bound electron magnetic moment measurement on lithium-like silicon $^{28}\text{Si}^{11+}$

$g_{\text{exp}}(^{28}\text{Si}^{11+}) = 2.000\, 889\, 889\, 9(21)$

$g_{\text{theo}}(^{28}\text{Si}^{11+}) = 2.000\, 889\, 909\, (51)$

Theoretical calculations by D.A. Glazov, A.V. Volotka, V.M. Shabaev

Ref.: A. Wagner et al. PRL 110, 033003 (2013)

Precision test of
• electron-electron interaction
• screened QED contributions
Dirac sea: contribution of negative energy states to bound electron magnetic moment in Li-like HCI

integration over negative energy states for internal electron lines

Ref.: D. Glazov
HITRAP at the ESR storage ring / GSI

**UNILAC**

experiments with particles at rest or at low energies

![Diagram](image)

**ESR**

400 MeV/u

**SIS**

U^{91+}

U^{73+}

strip target

U^{91+}

electron cooling and deceleration down to 4 MeV/u

**EXPERIMENTS WITH HIGHLY CHARGED IONS AND ANTIPROTONS AT EXTREMELY LOW ENERGIES:**

- g-factor measurements of the bound electron
- laser spectroscopy
- mass measurements
- reaction microscope, atomic collisions
- surface studies
- x-ray spectroscopy
Determination of the proton g-factor

\[ \omega_c = \frac{e}{m_p} B \]

Cyclotron frequency

\[ \omega_c = \sqrt{\omega_+^2 + \omega_-^2 + \omega_z^2} \]

\[ \omega_+ \approx 2\pi \cdot 29 \text{ MHz} \]
\[ \omega_\pm \approx 2\pi \cdot 690 \text{ kHz} \]
\[ \omega_- \approx 2\pi \cdot 8.5 \text{ kHz} \]

\[ \omega_L = g \frac{e}{2m_p} B \]

Larmor frequency

\[ g = 2 \frac{\omega_L}{\omega_c} \]

\[ \hbar \omega_L \]

\[ \omega'_z (\uparrow) - \omega'_z (\downarrow) = \Delta \omega_z \]
A single trapped proton and the continuous Stern-Gerlach effect

Axial frequency shift due to spin flip:

\[ \Delta \nu_z \approx \frac{1}{2\pi^2} \frac{\mu_z B_2}{m \nu_z} \]

Proton measurement is 10,000 times harder compared to electron g-2 measurement.

\[ B_2 = 0.3 \text{ T/mm}^2 \]
\[ \Delta \nu_z = 190 \text{ mHz} \]
First Larmor resonance curve of a single proton in the Penning trap

✓ Axial temperature reduced
✓ Larmor resonance narrower

\[ \frac{\Delta v_L}{v_L} = 1.2 \cdot 10^{-6} \]

\[ g = \frac{2 v_L}{v_c} \]

Next steps:
• Reduce axial frequency fluctuations further

Selected for a Viewpoint in Physics

Observation of Spin Flips with a Single Trapped Proton

S. Ulmer,1,2,3 C. C. Rodegheri,1,2 K. Blaum,1,3 H. Kracke,2,4 A. Mooser,2,4 W. Quint,3,5 and J. Walz2,4

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5GSI—Helmholtzzentrum für Schwerionenforschung, D-64291 Darmstadt, Germany

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Radio-frequency induced spin transitions of one individual proton are observed. The spin quantum jumps are detected via the continuous Stern-Gerlach effect, which is used in an experiment with a single proton stored in a cryogenic Penning trap. This is an important milestone towards a direct high-precision measurement of the magnetic moment of the proton and a new test of the matter-antimatter symmetry in the baryon sector.
Proton g-factor measurement with and **without** active feedback cooling

\[ g_p = 5.585696 \pm 0.000050 \]

Ref.: C. Rodegheri et al., NJP 2012

Reduction of axial temperature by application of active electronic feedback
Baryon-Antibaryon Symmetry Experiment – The BASE Collaboration at AD / CERN
Acknowledgements

✓ Group at the institute of physics - Mainz

✓ Group of Klaus Blaum at MPIK Heidelberg

✓ Atomic Physics Division at GSI Darmstadt

Thank you for your attention!
Electronic detection of a single ion by resonance circuit

\[ Q = 5600 \]
\[ \nu = 680 \text{ kHz} \]
\[ R_p = 36 \Omega \]
\[ e_n = 1.3 \text{ nV/}\sqrt{\text{Hz}} \]

Particle acts as a perfect short

Line width
\[ \delta \nu_z \propto N_p \]

Ref.:
A. Mooser
Continuous Stern-Gerlach effect: Determination of spin direction

CLASSICAL STERN-GERLACH
SEPARATION IN POSITION SPACE

\[ \Delta z = \frac{\mu L^2}{2KE} B_1 \]

CONTINUOUS STERN-GERLACH
SEPARATION IN FREQUENCY SPACE

\[ \Delta \omega_z = \frac{\mu}{m\omega_z} B_2 \]
Quantum jumps of a single HCl in a Penning trap

![Graph showing frequency difference and ion mass relation.](image)

- Frequency difference [Hz] vs. ion mass M [u]
- Ions: $^{12}\text{C}^{5+}$, $^{16}\text{O}^{7+}$, $^{28}\text{Si}^{13+}$
- $B_z = 10 \text{ mT/mm}^2$

Inset: Time [min] vs. axial frequency [Hz] offset with $\Delta \omega$ indicator.