REAL-TIME FEEDBACK ON BEAM PARAMETERS

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Abstract

Traditionally, tight beam parameter stability requirements were most pronounced for light sources and lepton colliders but have now become increasingly important for present and future hadron accelerator operation, not only for performance but also for reasons of machine protection, as recent improvements have led to significantly increased stored beam energies.

In the latest generation machines, performance depends critically on the stability of the beam. In order to counteract disturbances due to magnetic imperfections, misalignments, ground motion, temperature changes and other dynamic effects, fully automated control of the key beam parameters – orbit, tune, coupling, chromaticity and energy – becomes an increasingly important aspect of accelerator operation.

This contribution presents an overview of beam-based feedback systems, their architecture, performance limitations and design choices involved.

INTRODUCTION

With respect to beam-based feedback systems, the wide range of accelerators can be grouped roughly into synchrotron light sources, lepton and hadron colliders that are distinct in their requirements of number and type of feedbacks deployed.

The requirements on beam stability in synchrotron light sources are determined by the quality and properties of the photon beam seen by experiments. Depending on the time scale of the experiment's data integration and perturbation frequency, movements of the beam centroid may either "smear out" the effective emittance, which has a deteriorating effect on the photon beam quality, or lead to an increase of measurement noise. Due to synchrotron radiation, the beam emittance is usually much smaller in the vertical plane. To preserve and minimise the effective emittance, nearly all light sources deploy fast orbit and energy feedbacks. These minimise transverse beam movements, spurious dispersion by centering the beam in the quadrupoles and maintain a stable vertical orbit inside the sextupoles that would otherwise give rise to emittance coupling. A summary and overview of beam stability requirements and stabilisation in synchrotron light source can be found in [1–4].

The beam stability requirements in lepton and present hadron colliders are driven by luminosity optimisation inside the experimental insertions. They favour, similar to light sources, small emittance and stable beam overlap at the interaction point ([5–7]). In addition to orbit feedbacks, tune feedbacks are often also deployed ([8, 9]) to stabilise the beam during acceleration and to avoid resonances that may cause increased particle loss.

Recent improvements in hadron colliders lead to significantly larger stored beam energies which require an excellent control of particle losses inside a superconducting machine. In case of the LHC, the energy stored in the beam is sufficient to quench all magnets and cause serious damage [10]. Thus, most requirements on key beam parameters in superconducting hadron colliders strongly depend on the capability to control particle losses inside the accelerator. In the case of the LHC, the Cleaning System has the tightest constraints on the orbit and requires a stability better than $25\,\mu\mathrm{m}$ during nominal operation at the location of the collimators [11, 12]. Other requirements range from 0.5-0.2 mm r.m.s. for global stabilisation down to $10\,\mu\mathrm{m}$ for physics analysis improvements in the TOTEM experiment [13].

In contrast to lepton machines that require tune stability in the order of $\delta Q \approx 10^{-2} \ldots 10^{-3}$ to avoid up to 4th order resonances, synchrotron radiation damping is negligible in hadron colliders. In order to provide sufficient beam lifetime, resonances of up to the 12th order have to be avoided [14]. The corresponding tune stability δQ is thus required to be better than 0.001 at the LHC. The chromaticity has to be controlled within $Q' \approx 2 \pm 1$, while the uncorrected chromaticity changes are expected to exceed more than 100 units within a few hundred seconds after the start of the ramp [14].

PARAMETER STABILITY

The wide range of perturbation sources that may affect orbit, tune, coupling, chromaticity and energy can be grouped into:

1. Environmental sources, driven by temperature and pressure changes, ground motion, tides and noise induced by human activity which are mostly propagated through quadrupoles and their girders onto the beam,
2. Machine-inherent sources, such as the decay and snap-back of magnet multipoles, cooling liquid flow, vibration of pumps and ventilation, eddy currents and changes of machine optics (final focus),
3. Machine element failures, which are mainly important for large machines such as the LHC where the single circuit failure out of more than 1300 corrector circuits is non-negligible during regular operation.

Their time scale ranges usually from long-term (month to days) over medium term (days to hours) down to short term (hours to milliseconds). Beam-based feedbacks can contribute and improve beam parameter stability for perturbations on slow to medium time scales but are ultimately limited by thermal drifts, noise and systematics of corrector circuits and beam instrumentation [15]. The sensitivity to thermal drifts in 3^{rd} generation light sources lead to a rigorous stabilisation of not only the orbit but also the temperature of the experimental hall, tunnel, cooling water, and vacuum chamber to a level of about $\pm 0.1\,^{O}\mathrm{C}$ [16]. The quest for temperature stabilisation leads also to 'top-up' operation that maintains a constant beam current and thus constant heat load inside the tunnel [1, 2].

FEEDBACK CONTROL DESIGN

In case of low-order beam parameters – orbit, tune, coupling, chromaticity and energy – the effect of individual corrector circuits is, for most accelerators, sufficiently linear and can be cast into matrices. In the case of the orbit, for example, one can write

$$\Delta\underline{z}(t) = \mathbf{R}\cdot\underline{\delta}(t) \tag{1}$$

with $\Delta\underline{z} = (z_1,\dots,z_m)^T$ holding the readings of m beam position monitors (BPMs) and $\underline{\delta} = (\delta_1,\dots,\delta_n)^T$ holding the strengths of n dipole corrector circuits (CODs) and the matrix elements R_{ij} describing the response of the i-th BPM to the j-th COD circuit.

The external perturbations, corrector circuit strengths $\underline{\delta}(t)$ and thus the beam parameters themselves are usually a function of time. Many feedback designs on beam parameters decouple the control into what is further referred to as *space* and *time domain* which makes the choice of parameter correction strategy and the controller adjusting the temporal behaviour of the corrector circuits more flexible, particularly in the presence of element failures that require quick adjustments of the feedback controller and response matrices.

Space Domain

The parameter control in space domain establishes corrector circuit strengths $\underline{\delta}_{ss} = \lim_{t\to\infty}(\delta_1(t),\dots,\delta_n(t))^T$ that for steady-state perturbations minimises the residual r of

$$r = \left\|\underline{z}_{ref} - \underline{z}_{actual}\right\|_2 = \|\mathbf{R}\cdot\underline{\delta}_{ss}\|_2 < \epsilon \tag{2}$$

with $\mathbf{R}$ the beam response matrix, $\underline{z}_{ref}$ the reference and $\underline{z}_{actual}$ the measured parameter. The two-norm (or r.m.s.) of the parameter vector is defined as $\|\underline{x}\|_2 = \frac{1}{N}\sqrt{\sum_{i=0}^{N} x_i^2}$ with x_i being the individual vector entry and N the total number of entries.

The control in space domain consists essentially of the inversion of the beam response matrices. Singular-Value-Decomposition (SVD) is one of the most popular and widely used inversion algorithms ([1–3]) and a generalisation of the Jacobi matrix eigenvalue decomposition to the general case of non-square matrices [17, 18]. As shown in [17], any matrix $\mathbf{R}\in\mathbb{R}^{m\times n}$ with $m \geq n$ can be decomposed into

$$\mathbf{R} = \mathbf{U}\cdot\lambda\cdot\mathbf{V}^T \tag{3}$$

with $\mathbf{U}\in\mathbb{R}^{m\times n}$ being a dense unitary matrix, $\lambda = diag(\lambda_1,\dots,\lambda_n)$ a diagonal matrix holding the eigenvalues of $\mathbf{R}$, and $\mathbf{V}\in\mathbb{R}^{n\times n}$ an orthogonal matrix containing the eigenvectors of $\mathbf{R}$ in its columns.

Depending on the device layout and lattice parameters, equation 1 may contain singularities that can, using SVD, be identified by eigenvalues close or equal to zero. The SVD can be used to compute a so-called *pseudo-inverse* response matrix $\tilde{\mathbf{R}}^{-1}$ that relates the required steady-state circuit strengths $\underline{\delta}_{ss}$ to the measured orbit error $\Delta\underline{z}$:

$$\underline{\delta}_{ss} = \mathbf{V}\cdot\tilde{\lambda}^{-1}\cdot\mathbf{U}^T\cdot\Delta\underline{z} = \tilde{\mathbf{R}}^{-1}\cdot\Delta\underline{z} \tag{4}$$

Possible singularities are removed by setting the inverse of singular eigenvalues to zero. The number of eigenvalues used for the inversion defines the trade-off between precision and robustness of the correction: A higher number of eigenvalues provides a better convergence but at the same time tends to be more prone to spurious parameter readings and noise.

In case the true accelerator response differs from the one used during the design of the parameter control, the correction error gradient $\underline{\delta}_s s$ may point off the true minimum, as illustrated in Figure 1. It is visible that independent of the

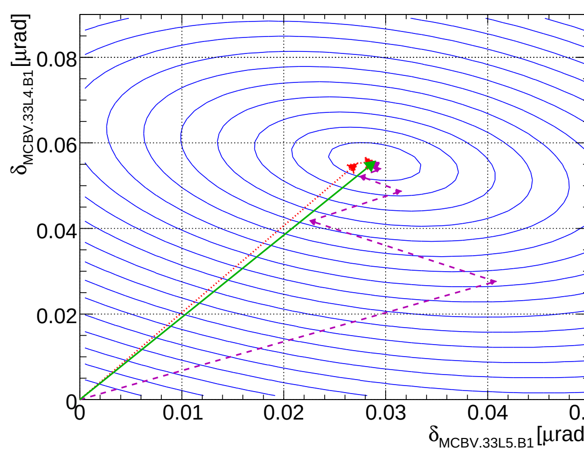

Figure 1: SVD residual contour: The perfect correction (solid green), the correction with optics error (dotted red) and COD calibration errors (dashed violet) are indicated. The contour lines correspond to constant values of the residual orbit error r. The correction is based on a regular FODO lattice (LHC arc) and shows the projection of two selected COD settings.

type of error, the corrections converge to the same steady-state setting. However, depending on the errors the convergence speed can vary between one (perfect) and about seven iterations (20% COD calibration errors).

In most digital approximations, analogue systems are sampled at least 10 times higher than the analogue bandwidth. Since the deteriorating effect due to beam response matrix uncertainties can be mitigated through a higher sampling frequency, it is usually favourable to sample much higher than a factor of 10. In case of synchrotron light sources, typical sampling to bandwidth ratios are about 40 or more [1–3].

Time Domain

A simple loop block diagram consisting of a single-input-single-output (SISO) process $G(s)$ and controller $D(s)$ is shown in Figure 2. The stability and sensitivity

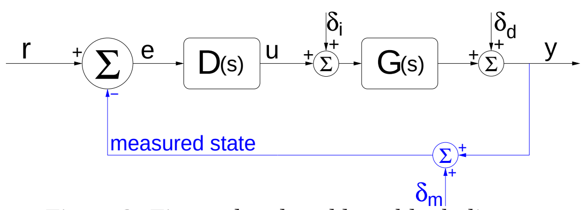

Figure 2: First order closed loop block diagram

to perturbations and noise is defined by the following functions

$$T(s) := \frac{y}{r} = \frac{D(s)G(s)}{1+D(s)G(s)} \quad (5)$$

$$S_d(s) := \frac{y}{\delta_d} = \frac{1}{1+D(s)G(s)} \quad (6)$$

$$S_i(s) := \frac{y}{\delta_i} = \frac{G(s)}{1+D(s)G(s)} \quad (7)$$

$$S_u(s) := \frac{u}{\delta_d} = \frac{D(s)}{1+D(s)G(s)} \quad (8)$$

where $T(s)$ is the complementary (nominal) transfer function, $S_d(s)$ the *nominal sensitivity* defining the loop disturbance rejection, $S_i(s)$ the *input-disturbance sensitivity* and $S_u(s)$ the *control sensitivity*. The state variable is indicated in Figure 2. The sensitivity to measurement noise is equal to the nominal transfer function T_0.

Classic feedback designs rely on the discussion of denominator zeros in equation 5 and 6 while keeping constraints such as required bandwidth, minimisation of overshoot, limits on the maximum possible excitation signal and robustness with respect to model and measurement errors. For ideal processes, this yields adequate controller designs but often falls short in providing a simple comprehensive method for estimating and modifying the loop sensitivity (robustness) in the presence of process uncertainties, non-linearities and noise.

This paper focuses on Youla's affine parameterisation method for optimal controllers, which is based on the analytic process inversion, first introduced in [19]. For an open-loop stable process $G(s)$, the nominal closed-loop transfer function is stable if and only if $Q(s)$ is an arbitrary stable proper transfer function and $D(s)$ parameterised as:

$$D(s) = \frac{Q(s)}{1-Q(s)G(s)} \quad (9)$$

The stability of the closed loop system follows immediately out of the above definition if inserted into equations 5 to 8. The sensitivity functions in the $Q(s)$ form are given as:

$$T(s) = Q(s)G(s) \quad (10)$$

$$S_d(s) = 1-Q(s)G(s) \quad (11)$$

$$S_i(s) = (1-Q(s)G(s))G(s) \quad (12)$$

$$S_u(s) = Q(s) \quad (13)$$

Assuming $G(s)$ is stable, the only requirement for closed loop stability is for $Q(s)$ to be stable. The strength of this method is the explicit controller design with respect to required closed loop performance, as visible in equation 10, and required stability (equations 11 to 13). Equations 10 and 11 are complementary and illustrate the intrinsic limiting trade-off of feedbacks that either have a good disturbance rejection or are robust with respect to noise. The ultimate limit is thus rather defined by the bandwidth and noise performance of the corrector circuits and beam measurements than by the feedback loop design itself. Systematic and thorough analysis of involved beam instrumentation and corrector circuits are thus essential for achieving best beam parameter stabilisation.

The design formalism can be demonstrated using a simple first order system $G_0(s) = \frac{K_0}{\tau\cdot s+1}$ with open-loop gain K_0 and time constant τ. A common controller design ansatz is to write $Q(s)$ as

$$Q(s) = F_Q(s)\cdot G_0^i(s) \quad (14)$$

with $F_Q(s)$ a trade-off function and $G_0^i(s)$ the pseudo-inverse of the process. Since G_0 does not contain any unstable zeros, the pseudo-inverse equals the inverse and is given by $G_0^i(s) := [G_0(s)]^{-1} = \frac{\tau\cdot s+1}{K_0}$. $Q(s)$. In order for $D(s)$ to be biproper, $F_Q(s)$ must have a degree of one and can be written as:

$$F_Q(s) = \frac{1}{\alpha s+1} \quad (15)$$

Inserting equation 14 into Youla's controller parameterisation equation 9 yields the following controller

$$D(s) = \frac{\tau}{K_0\alpha} + \frac{1}{K_0\alpha s} = K_p + K_i\cdot\frac{1}{s} \quad (16)$$

which shows a a simple PI controller structure with proportional gains K_p and integral gain K_i. Inserting equation 14 into equation 10 yields

$$T_0(s) = F_Q(s) \quad (17)$$

that the closed loop response is essentially determined by the choice of trade-off function $F_Q(s)$ and that the closed loop bandwidth is proportional to the parameter $1/\alpha$. This can be used to tune the closed loop between: high disturbance rejection but high sensitivity to measurement noise (small α) and low noise sensitivity but low disturbance rejection (large α) depending on the operational scenario.

The maximum possible closed loop bandwidth is limited by the excitation, as described by equation 13. In case of power converters, for example, the excitation is limited by the maximum available voltage.

The same method can be extended to open-loop unstable and multi-input-multi-output (MIMO) systems [19]. Real life feedbacks may contain significant delays λ (due to e.g. data transmission, data processing etc.) and non-linearities $G_{NL}(s)$, due to e.g. saturation and rate limits of the corrector circuits' power supplies. The modified process can be written, for example as:

$$G(s) = G_0(s) \cdot e^{-\lambda s} G_{NL}(s) \tag{18}$$

Using the same pseudo-inverse $G_0^i(s)$ as for the above example and inserting equation 14 into equation 9 yields a controller parameterisation $D_{NL}(s)$ including a classic Smith-Predictor and anti-windup paths, discussed in more detail in [20, 21]. Inserting equation 14 including the delay and non-linearities into equation 10 yields the following closed loop transfer function:

$$T(s) = F_Q(s) \cdot e^{-\lambda s} G_{NL}(s) \tag{19}$$

Similar to the linear case discussed above, the closed loop is essentially defined by the function $F_Q(s)$ that within limits can be chosen arbitrarily based on the required disturbance rejection and robustness during possibly different operational scenarios (gain-scheduling). Further information and a review on Youla's parameterisation can be found in [21, 22].

DEPENDABILITIES AND CROSS-TALK

In many accelerators, beam-based feedbacks are usually established and designed one by one, and often, little effort is put into the study of cross-dependency and decoupling of these loops. However, for robust and reliable control it is necessary to address possible cross-constraints, cross-talk and coupling between several simultaneous and possible nested loops already in the design stage.

A typical cross-dependency is intrinsic to the stability requirements on orbit and tune: though tight constraints on orbit excursion to micrometre level are beneficial to minimise feed-down effects and beam life-time, it also imposes constraints on other feedbacks such as tune and chromaticity, the measurements of which rely on transverse excitations and momentum modulation. In the case of the LHC, the tight constraints lead to the development of a robust diode-based tune measurement technique that is capable of detecting nanometre scale beam oscillations ([23]) and that enables a tune and coupling phase-locked-loop (PLL) that can operate with transverse excitation levels below $1\,\mu\mathrm{m}$ [24, 25].

Another possible cross-dependency is given by coupling due to the beam response itself. As described in [26], a robust and reliable tune PLL requires also the measurement and control of global coupling. Classic tune PLL designs ([8,9]) often model the PLL as a first order process defined by the phase detector's filter time constant and open loop gain K_0 that depends on the angle of the phase slope at the location of the tune resonance. In the presence of varying chromaticity, the open loop gains K_0 and thus the optimal controller parameter are functions of chromaticity itself. Using linear control design only, this cross-dependence implies either a controller design that is optimal for large chromaticities, which becomes sensitive to noise and unstable for low values of chromaticity, or a controller design that is optimal for small chromaticities but lags behind the real tune for large values of chromaticity [24, 27].

A more complex example for inter-loop coupling can be illustrated by the LHC, which requires a simultaneous control of orbit, tune, coupling, chromaticity and energy. The forseen nested control scheme for chromaticity, tune and coupling is shown in Figure 3. The tune PLL is the innermost loop measuring the global tunes and coupling parameters. The loop is first nested within the loop that measures and controls the chromaticity and is then surrounded by the feedback loop controlling the global tunes and coupling. The decoupling is obtained by choosing gradually reduced bandwidths for the tune PLL ($f_{bw} \approx 8\,\mathrm{Hz}$), chromaticity ($f_{bw} \approx 1\,\mathrm{Hz}$) and tune feedback ($f_{bw} < 1\,\mathrm{Hz}$). This nesting hierarchy is required in particular to eliminate the cross-talk between tune and chromaticity feedback, as the tune feedback would otherwise minimise the momentum-driven modulation as well as tune modulation and thus compromise the chromaticity measurement.

In addition, cross-talk is introduced between the chromaticity and orbit/energy feedback through the dispersion orbit that is driven by the momentum modulation required by the chromaticity feedback. In order to minimise this cross-dependence, the foreseen LHC orbit feedback filters and separates the dispersion orbit from the measured closed orbit prior to performing any orbit correction.

CONCLUSIONS

Beam-based feedbacks can contribute and improve beam parameter stability for perturbations on slow to medium time scales but are ultimately limited by thermal drifts, noise and systematics of involved devices on long time scales from days to months. Systematic and thorough analysis of involved beam instrumentation and corrector circuits are thus essential for achieving best beam parameter stabilisation.

The use of imperfect design beam response does not necessarily affect the precision of the correction but may lead to a reduction of effective feedback bandwidth. This effect can be mitigated by higher sampling frequencies which are usually in the order of 40 times higher than the desired feedback bandwidth.

Youla's affine parameterisation provides a simple yet powerful design tool for optimal adaptive non-linear control. Its strength is the explicit controller representation that enables an unobscured feedback design with respect

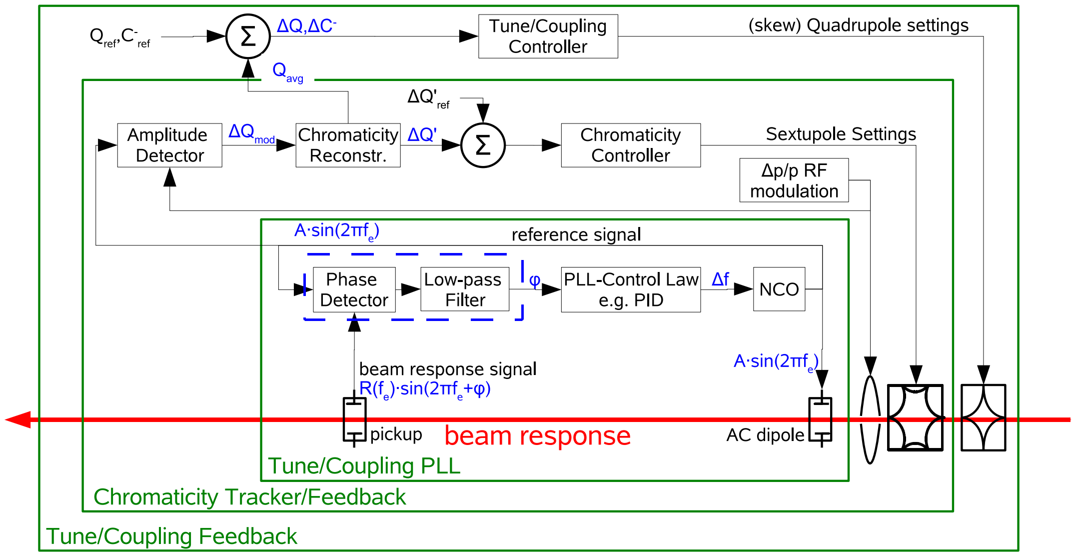

Figure 3: Nested loop scheme required for a coherent control of tune, coupling and chromaticity.

to closed loop robustness (noise insensitivity) and steering precision. It can further be used to design controllers with adaptive gains that are scheduled depending on the requirements of the applicable operational scenario.

It is advisable to address the incorporation of cross-constraints and coupling of multiple simultaneous nested loops early at the design stage as well as design feedbacks as an ensemble.

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