# LORENTZ DETUNING FOR A DOUBLE-QUARTER WAVE CAVITY*

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*Abstract*

The frequency change due to the radiation pressure on the walls of an RF cavity is known as Lorentz detuning. We present benchmarking studies of Lorentz detuning calculations for a Double-Quarter Wave Crab Cavity (DQWCC) using the codes ACE3P and CST. The results are compared with the Lorentz detuning measurements performed during the cold tests of the Proof-of-Principle (PoP) DQWCC at BNL.

## LORENTZ DETUNING

Lorentz detuning is the change of frequency associated with the deformation of an RF cavity due to the pressure exerted by the electromagnetic field (known as radiation pressure) on the cavity walls.

The radiation pressure $P_{rad}$ exerted by an electromagnetic wave on the walls of a cavity is given by:

$$P_{rad} = \frac{1}{4}\left(\mu_0 {H_{pk}}^2 - \varepsilon_0 {E_{pk}}^2\right), \qquad (1)$$

where $E_{pk}$ and $H_{pk}$ are the peak values of the electric and magnetic field, respectively [1]. These are the field values automatically provided by simulation codes such CST and ACE3P. The expression of $P_{rad}$ presented here is therefore the time-average radiation pressure, not the instant value.

According to Eq. (1), there will be inwards pressure in the region with dominant electric energy (negative sign – as the attractive force between two capacitive plates) and outwards pressure in the region with dominant magnetic energy (positive sign).

The Slater's perturbation theorem gives the frequency change experienced by a cavity which suffers a small change in its volume $V \to V + dV$:

$$\Delta f \propto (\varepsilon_0 E^2 - \mu_0 H^2)\, dV \qquad (2)$$

Some conclusions can be extracted from the combination of Eqs. (1) and (2): a) the detuning is proportional to the amplitude of the electromagnetic field, thus the Lorentz detuning is especially important at high field operation; b) frequency always decreases, and c) it is impossible to design a cavity with net frequency change equal to zero.

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## LORENTZ DETUNING CALCULATION

The Lorentz detuning coefficient can be calculated after a sequence of coupled simulations using electromagnetic and structural solvers. The general calculation workflow consists of three main stages:

1) Run eigenmode solver to obtain resonant frequency of the cavity and electromagnetic field distribution in the RF volume (vacuum volume).

2) Run structural solver to calculate the displacement of cavity walls (shell volume) due to the radiation pressure.

3) Run eigenmode solver to solve resonant frequency for deformed cavity.

Several simulation codes can be used to calculate the Lorentz detuning coefficient. This paper focuses on the calculation performances of two simulation codes, ACE3P [2] and CST [3].

### *Lorentz Detuning Calculation with ACE3P*

The parallel finite-element code suite ACE3P was developed by SLAC. Omega3P is the eigenmode solver of ACE3P and TEM3P is its structural solver. Simulated volumes must be meshed in Cubit and later processed by acdtool, so that Omega3P and TEM3P can work with them. Simulation results are postprocessed with acdtool and visualized with Paraview.

Omega3P provides the resonant frequency and electromagnetic field distribution for RF volume. TEM3P takes the electromagnetic field distribution as an input. Then it calculates the corresponding radiation pressure on the inner surface of the shell model for a given accelerating gradient set by the user. From the radiation pressure and for the given boundary conditions on the shell model, TEM3P computes the corresponding displacement of the cavity walls. As a result, it generates a file with the deformed vacuum volume. This deformed vacuum volume is like the original vacuum volume, but the node coordinates have been updated according to the displacement due to pressure radiation. The deformed vacuum volume file can be used as input for Omega3P to calculate the shifted resonant frequency, and thus obtain the Lorentz detuning.

### *Lorentz Detuning Calculation with CST*

A similar routine is used with the code package CST Suite. The eigenmode solver from the CST MWS package provides the resonant frequency and electromagnetic field distribution for the RF volume. It is equipped with a postprocessing tool that calculates the Lorentz force generated by the electromagnetic field on the cavity walls.




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This Lorentz force can be then imported into the structural solver from the CST MPhysics package as a load (from CST version 2014 onwards). The code allows setting the Lorentz force load for a given stored energy in the cavity. The structural solver will then calculate the corresponding displacement of the cavity walls for the given shell model and boundary conditions. The resulting displacement can be imported by the sensitivity analysis tool of the eigenmode solver. The sensitivity analysis tool takes the displacement generated by the Lorentz force and calculates the frequency shift by the perturbation method, using the information of the nominal value and the first derivative. Results presented here are from simulations run in CST version 2015.

## BENCHMARK: PoP DQWCC

The cavity used for this benchmark study is the PoP DQWCC. The PoP DQWCC is an SRF crab cavity designed to operate at 2 K in CW. The crabbing mode is the fundamental mode at 400 MHz. The cavity was designed as a first prototype to validate the cryogenic performances for the nominal deflecting voltage per cavity of 3.34 MV required for the crab crossing system of LHC [4].

The field distribution for the crabbing mode of the PoP DQWCC is shown in Fig. 1.

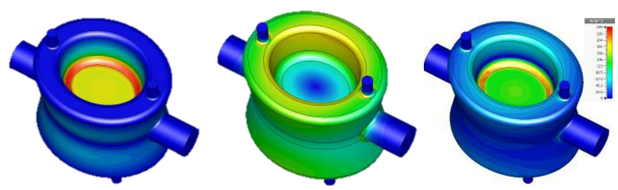

Figure 1: Electric (left) and magnetic (center) field distributions for the fundamental mode of the PoP DQWCC and corresponding Lorentz force density (right) [CST simulation].

During the cryogenic RF tests the PoP DQWCC was reinforced and supported by a dedicated stiffener. The stiffener held the two capacitive plates of the cavity, the most sensitive region to deformation where the highest electric field is located. The stiffener consisted of two titanium frames made of 2" wide and 0.5" thick titanium bars and four titanium plates that fixed the cavity central plates. Fours niobium bars were electron beam welded to the cavity central plates and then the titanium plates were bolted to the niobium bars. Fig. 2 shows the PoP DQWCC and its stiffener.

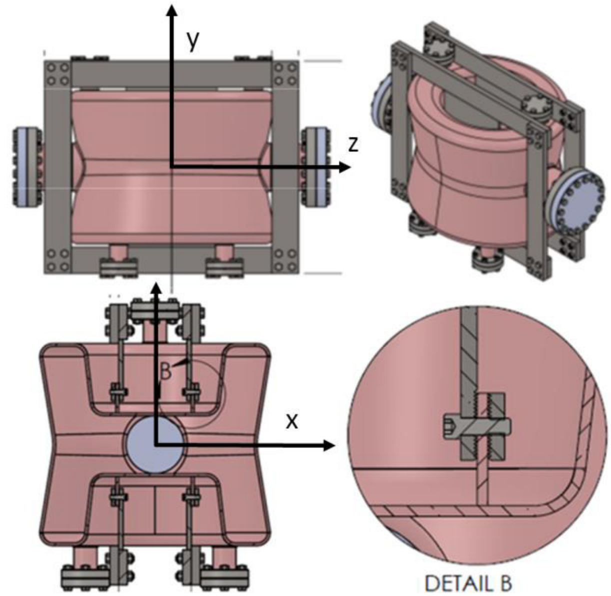


Figure 2: Geometry of the PoP DQWCC with its titanium stiffener.

The Lorentz detuning coefficient measured for the PoP DQWCC during the cryogenic RF tests conducted at BNL was -206 Hz/(MV)$^2$. Typically the Lorentz detuning coefficient $K$ is defined as the frequency shift due to radiation pressure, $\Delta f$, over the accelerating voltage, $V_{acc}$. Crab cavities are a type of deflecting cavities; thus the Lorentz detuning coefficient discussed in this paper is calculated with respect to the deflecting voltage $V_t$: $K_t = \Delta f/(V_t)^2$.

The cavity was made of 4 mm-thick niobium sheets. Ultrasound measurements of cavity thickness were performed after all chemical surface treatments. The thickness at the cavity central plates was 3.73 mm. The Lorentz force density is especially large in the central plate region, as shown in Fig. 1. Therefore, a cavity model with 3.73 mm-thick walls was used for the simulations presented in this paper.

## SIMULATIONS

The Lorentz detuning coefficient was calculated for three different cavity models:

- Free cavity without stiffener (CST)
- Cavity with simplified stiffener (ACE3P and CST)
- Cavity with realistic stiffener (CST)

### *Cavity Model with Simplified Stiffener*

As a first approach, a null displacement was set as a boundary condition for the interfaces of stiffener and central plates. This assumption corresponds to the case of an infinitely rigid, ideal stiffener which completely prevents the cavity central plates to move. Fig. 3 shows the shell volume with the zero-displacement faces. Ports are free according to the test setup described in Ref. [2].

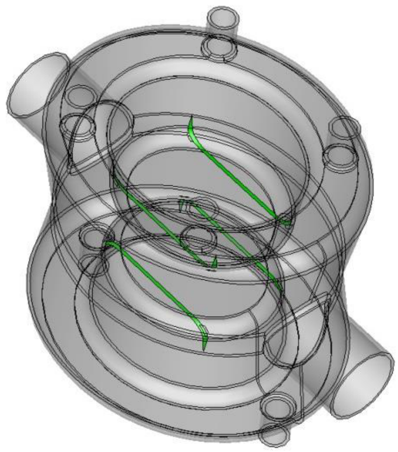

Figure 3: Shell model with an ideal stiffener: zero-displacement was applied to the interfaces where the stiffener was welded to the cavity central plates (highlighted in green).

The PoP DQWCC was RF tested in a saturated helium bath at 2 K. The mechanical properties for RRR niobium and titanium grade-II at 2 K used in the simulations are listed in Table 1 [5].

Table 1: Mechanical Properties for RRR Niobium (Nb) and Titanium Grade-II (Ti)

| **Property at 2 K** | **Nb** | **Ti** | |
|---|---|---|---|
| Young's modulus, $E$ | 110.9 | 124 | [GPa] |
| Poisson's ratio, $\nu$ | 0.393 | 0.37 | |
| Density, $\rho$ | 8610 | 8553 | [kg/m$^3$] |

The mesh of vacuum and shell volumes for the ACE3P simulations was generated by Cubit. The cavity was split into two halves along the vertical plane. Only one half of the simulated model was used to reduce the time and computing resources. About 1.1 million elements were used to mesh half of the vacuum volume for Omega3P simulations and about 509 thousand elements were employed to mesh half of the shell volume for TEM3P simulations.

All ACE3P simulations were run on the NERSC supercomputer Hopper. It took about 1.26 CPU hours and 0.83 CPU hours to get the mesh files for the vacuum and shell volumes, respectively, using 24 CPUs. The memory required was about 1,211 MB for the vacuum volume and 1,211 MB for the shell volume. The Omega3P simulations consumed about 3 GB and 185 CPU hours each using 480 CPUs. The TEM3P simulation required about 2 GB and 185 CPU hours using 480 CPUs. Post-processing calculations used about 2 GB and 1.23 CPU hours from 24 CPUs. The Lorentz detuning coefficient provided by ACE3P was about -85 Hz/(MV)$^2$.

For the CST simulations, about 344 thousand tetrahedral elements were used to mesh the vacuum volume whereas about 1 million elements were used for the shell volume. The eigenmode simulations required about 11 GB RAM; the structural simulation – 5 GB. It took about 1 h to complete each eigenmode simulation and about 4 h for the structural one, running on a PC with 4 CPUs.

A maximum displacement of 0.88 μm was found right in the middle of the central plates (the cavity is fully symmetric with respect to the xy and xz planes) for the nominal deflecting voltage. The Lorentz detuning coefficient was -73 Hz/(MV)$^2$.

The Lorentz detuning coefficient calculated from ACE3P and CST for the simplified model of the stiffener showed a good agreement. However, the calculated values presented a large discrepancy with the Lorentz detuning coefficient measured during the cold tests.

### *Cavity Model with Realistic Stiffener*

The Lorentz detuning coefficient for the free cavity (cavity with no stiffener) is pretty large, about -891 Hz/(MV)$^2$ according to CST simulations. This large Lorentz detuning coefficient suggested that a small deformation of the stiffener due to the effect of the radiation pressure on the cavity walls would have a non-negligible impact on the Lorentz detuning.

Therefore, another model with a more realistic stiffener was prepared. Fig. 4 shows the cavity model. CST was used to calculate the Lorentz detuning coefficient for the cavity model with more realistic stiffener. The model was still a simplification of the real one: no bolts, no joints, all merged in a single piece.

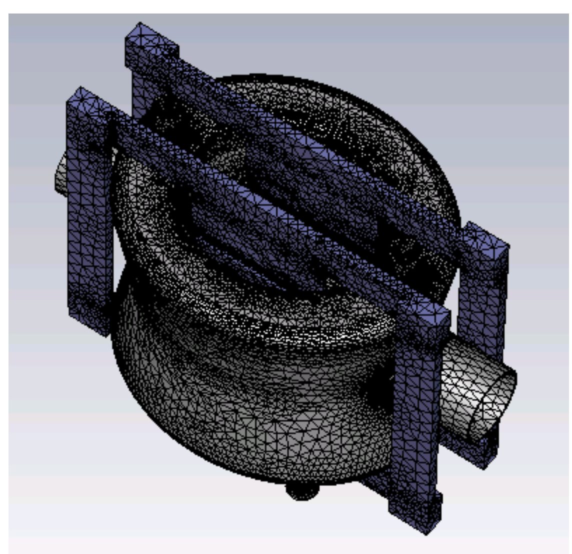

Figure 4: Mesh generated for structural simulation in CST MPhysics solver.




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The shell volume and the more realistic stiffener model were meshed with about 726 thousand tetrahedral elements. This structural simulation took about 1h30' using 12 CPUs and required about 4.8 GB.

The maximum displacement was larger than for the ideal stiffener, 1.73 µm, and the calculated Lorentz detuning coefficient, -218 Hz/(MV)$^2$, was in very good agreement with the measured value. Fig. 5 shows the cavity wall displacement calculated by CST. Table 2 summarizes the Lorentz detuning coefficient evaluated with different simulation codes for different models.

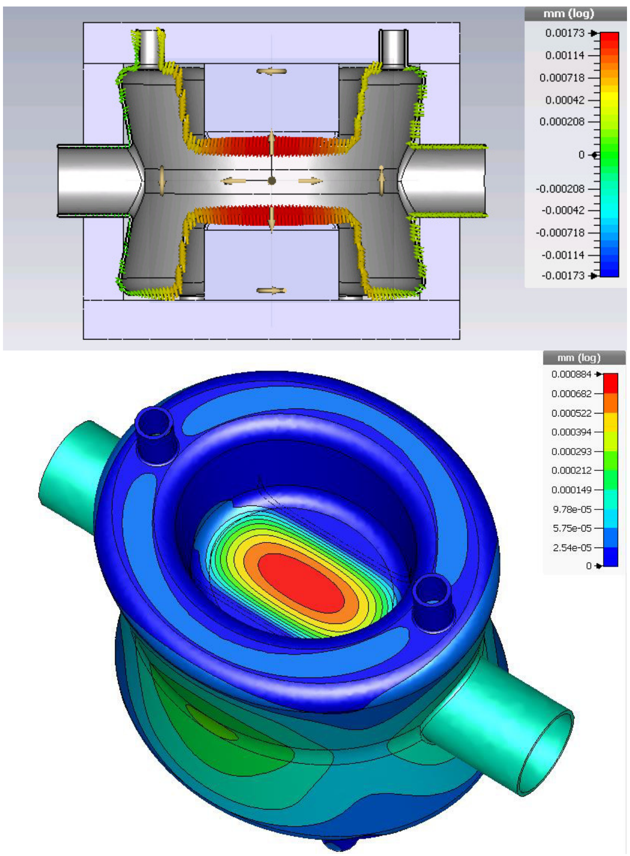


Figure 5: Cavity wall deformation due to radiation pressure at nominal deflecting voltage of 3.34 MV (CST simulation).

Table 2: Calculated and Measured Lorentz Detuning Coefficient for PoP DQWCC

| **Lorentz detuning coefficient $\Delta f/V_t^2$** | **[Hz/(MV)$^2$]** |
|---|---|
| *Ideal stiffener* | |
| ACE3P | -85 |
| CST | -73 |
| *No stiffener* | |
| CST | -891 |
| *Realistic stiffener* | |
| CST | -218 |
| *Measured* | -206 |

## CONCLUSIONS

The benchmark study found a good agreement between ACE3P and CST simulation codes for the calculation of the Lorentz detuning coefficient.

Simulation codes showed that the PoP DQWCC stiffening frame plays a non-negligible role to limit the cavity deformation due to radiation pressure and to limit the subsequent frequency shift associated to this deformation.

## ACKNOWLEDGMENTS

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