# EFFECTS OF ELLIPTICALLY DEFORMED CELL SHAPE IN THE CORNELL ERL CAVITY

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#### Abstract

The Cornell ERL cavity is optimized to minimize the dipole mode beam breakup (BBU) parameters to achieve the required high beam current (100mA). Deformations due to errors in fabrication and tuning of the accelerating mode can result in a cavity shape different from the ideal. Elliptically deformed cells can cause dipole mode frequency spread and splitting of the mode polarizations leading to an x-y mode coupling. To investigate these effects, we use a mesh distortion technique to generate an elliptically deformed cell cavity model as a base for studying random imperfections. Simulation results of 50 randomly elliptically deformed cell cavities from the eigensolver Omega3P covering the first three dipole bands will be presented. The results will be used as input to the beam tracking code BMAD to calculate the impact of such imperfections on the dipole mode BBU parameters.

### **INTRODUCTION**

Cornell has proposed a 5 GeV superconducting (SC) energy recovery linac (ERL) light source design, which can provide a high average current of 100mA with low emittance of 30pm-rad at 77pC bunch charge [1]. Because the achieved high current is limited by the dipole modes excited by the beam in the linac cavities, the Cornell 7-cell SC cavity, working at 1.3GHz, is carefully optimized to minimize the dipole mode BBU parameters measured by  $\xi \equiv (R/Q)$  sqrt(Q)/f to suppress beam instabilities [2].

However, due to errors in cavity fabrication and tuning of the accelerating mode, the actual dipole mode properties differ from those of cavities with the nominal dimensions. Figure 1 shows an example of actual dipole mode measurements for the first two dipole bands in the eight cavities of the TTF Module 5. In general, the dipole mode frequencies spread and are shifted to lower values, the mode splitting is larger than expected, and the Qext's have a larger scatter. Therefore, it is essential to determine the threshold current including the effects of varying dipole mode parameters in the Cornell ERL SC cavities.

All SC cavities are fabricated and prepared following the same procedures, and thus the nature of the imperfections is similar. Cavity imperfection models can be generated for five types of deformations (Figure 2) using a mesh distortion method [3]. The first three types of deformations had been investigated using 2D finite element codes CLANS/CLANS2 [2]. Cavity shapes with tiny bumps/dents can only cause field enhancement which affect the fundamental mode performance without much impact on the dipole mode properties [4].

Elliptically deformed cell imperfections can cause the dipole mode frequencies spread and splitting of the dipole mode polarizations. The mode frequencies spread can help suppress the wakefield accumulation in the linac, but the mode polarizations varying will lead to an x-y mode coupling. In this paper, we will investigate the effects of elliptically deformed cell shapes on the dipole modes in the Cornell ERL cavity.



Figure 1: Comparison between ideal cavity results from Omega3P (black dots) and measurements from 8 cavities in the TTF Module 5 (in color).



(1) cell length error (2) Cell radius error (3) Deformed cell surface



(4) Elliptically deformed cell (5) Cell with bump

Figure 2: Meshes for imperfect models with ideal cell in red and deformed cell in blue.

First, we implement a code to automatically and randomly generate a set of elliptically deformed cell cavities within a certain error. Then, the dipole mode parameters covering the first three passbands in the imperfection cavities are calculated using the eigensolver Omega3P [5]. The simulation results are used as input to the beam tracking code BMAD [6] to calculate the BBU threshold current in these elliptically deformed cell cavities.

## ELLIPTICALLY DEFORMED CELL CAVITY MODEL

A mesh distortion technique is used to build an elliptically deformed cell shape as shown in Figure 2 (4). The perturbation of a cell radius can be described by

 $R(\Theta)=Asin(2\Theta)+Bcos(2\Theta)$  (1) The maximum of *R*, *Rmax*, occurs at the elliptically deformed cell polarization angle  $\Theta_{polar}$ . Pairs of (*A*, *B*) can be derived from random choices of (*Rmax*,  $\Theta_{polar}$ ), and thus randomly elliptically deformed cell cavities can be generated by distorting the ideal cavity mesh following the Equation (1).

The geometry model for the Cornell 7-cell ERL cavity is shown in Figure 3. To suppress the BBU, the dangerous dipole higher-order modes (HOM) should be effectively damped by the RF loads in the cavity beam pipes.



Figure 3: Geometry model for the Cornell 7-cell 1.3 GHz ERL cavity with HOM absorber rings in yellow and pink.

The small elliptically deformed cell shape will not change the cell volume, and the fundamental mode field flatness will remain the same as that of the ideal cavity. As a result, it is not necessary to introduce cell length corrections for keeping the fundamental mode field flatness in elliptically deformed cell cavities if the maximum perturbation of the cell radius is less than 1mm.

### DIPOLE MODE PARAMETERS IN ELLIPTICALLY DEFORMED CELL CAVITIES

Using the mesh deformation code, 50 elliptically deformed cell cavities are automatically and randomly generated including 10 cavities with fixed Rmax of 1mm, 20 cavities with fixed Rmax of 0.5mm, and 20 cavities with random Rmax less than 0.5mm. All these 50 cavities have a random orientation for each cell. One of these cavities is shown in Figure 4.

For each passband, the four combinations of electric and magnetic boundary conditions at the ends of the cavity are used to model a chain of identical cavities. In this paper, only the simulation results with electric boundary conditions are discussed. The first three passband dipole mode RF parameters in the 20 imperfect cavities with random Rmax (< 0.5mm) and  $\Theta_{polar}$  (<180<sup>0</sup>) are shown in Figure 5.



Figure 4: Elliptically deformed Cornell ERL cavity.



(c)  $R/Q.sqrt(Q)/f(\Omega/cm^2.MHz)$  (d)  $\Theta$ polar (degree)

Figure 5: The dipole mode parameters in Cornell elliptically deformed cavities with closed electric boundary conditions assuming HOM absorber  $\epsilon r=30-i10$ .

It can be seen from Figure 5 (a) and (b) that small elliptically deformed cell shapes will not change the dipole mode R/Q's and Qload's. Therefore, the dipole mode BBU parameters determined by R/Q, Qload, and f are similar to those in the ideal cavity except for the modes at the end of the second passband. However, the dipole mode polarizations spread in the elliptically deformed cell cavities will lead to the mode x-y coupling. The BBU threshold current should be considered

including the effects of HOM frequencies spread and the mode splitting.

### BBU INSTABILITY IN ELLIPTICALLY DEFORMED CELL CAVITIES

The HOM simulation results in randomly elliptically deformed ERL cavities are used as input to the beam tracking code BMAD for studying the Beam Break-Up (BBU) threshold current [6]. This limiting current is due to transverse beam instability caused by the cavity HOMs in ERL operation, and can be amplified by coherent effects between the cavities having nearly identical HOM frequencies.

The higher-order dipole modes computed via Omega3P were ranked with a figure of merit  $\xi$ , mentioned earlier. This figure of merit is computed for each mode, given the R/Q, loaded quality factor and frequency, and the threshold current is inversely related to the maximal value of  $\xi$  over all HOMs.



Figure 6: Beam break-up results for the 0.5 mm elliptical variations. Mean BBU current is 0.39 A, and 90% of ERLs had current in excess of 0.34 A.



Figure 7: Beam break-up results for the 1.0 mm elliptical variations. Mean BBU current is 0.41 A, and 90% of ERLs had current in excess of 0.36 A.

The ten modes from each cavity with the largest values of  $\xi$  were used as accelerating cavity HOM elements in Cornell ERL lattice file version 8.4. The cavities were then assigned to random locations in the ERL, and 100 such ERLs were simulated. A particle tracking program BMAD was used to compute the threshold current through the cavity before beam break-up occurs. Histograms comparing the 0.5mm variation with the 1.0 mm variation are presented in Figs. 6 and 7. The increased threshold current in the larger elliptically deformed cell cavities is likely due to the larger relative cavity-to-cavity frequency spread of shapes with larger errors.

Since there were not enough cavities to entirely populate the ERL with unique cavities, an artificial relative cavity-to-cavity frequency spread of 0.001 was introduced in the particle tracking program. This has the effect of simulating actual operating conditions in which no two cavities have the exact same HOM properties.

The results show that the X-Y coupling of the dipole modes is not the limiting factor for the threshold current, and is consistent with other simulations, that did not include elliptical defects [7].

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