# COUPLER KICK STUDIES IN CORNELL'S 7-CELL SUPERCONDUCTING CAVITIES\*

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### Abstract

Cornell is developing a 5 GeV Energy Recovery Linac operating at 100 mA with very small emittances ( $\sim$ 30 pm at 77 pC bunch charge) in the horizontal and vertical directions. We investigate the effect of the fundamental RF power couplers of the main linac SRF cavities on the beam using the ACE3P software package. The cavities in the ERL main linac will be operated at very high loaded quality factors of up to  $6.5 \times 10^7$ , corresponding to a full bandwidth of only 20 Hz. Cavity microphonics will detune the cavities by more than one bandwidth during operation, thereby causing a time dependent change of the coupler kick in addition to its fast oscillation at the RF frequency. We show that a compensation stub geometry located opposite to the input coupler port can be optimized to reduce the overall kick given to the beam and the emittance growth, but for the Cornell input coupler geometry is satisfactory without a compensation stub. Finally, our calculation of the coupler kick as a function of detuning shows the necessity of operating within the designed microphonic limits.

## **INTRODUCTION**

This paper presents work that for the first time calculates the time dependent coupler kick on a bunch in a beam as a function of RF cavity detuning. Understanding this effect is very important to ensure that the beam emittance in Cornell's Energy Recovery Linac (ERL) remains small ( $\sim$ 30 pm-rad at 77 pC bunch charge)[1].

The fact that these simulations are done as a function of detuning is of central importance because ERLs operating detuning is large  $(\Delta f/f_{1/2} \ge 1)$ .

Previous work optimized a superconducting 1.3 GHz 7cell main linac cavity with respect to the beam break-up current through the accelerator[2]. To this design, a fundamental power coupler was added with  $Q_{ext} = 6.5 \times 10^7$ . The effect of the coupler on the beam has previously been explored with Microwave Studio for the case of on resonance excitation.[3] However, due to the required sensitivity, and the need to compute the kick as a function of detuning, we used SLAC's ACE3P software suite for these simulations.[4]

It has also been shown that the coupler kick effects can be mitigated by introducing a compensation stub directly opposite the coupler to reduce asymmetry in the coupler region.[3] Cavity geometries with and without a compensation stub are shown in Figs. 1–2.



Figure 1: View of 7-cell cavity coupler region of a model without a compensation stub. The mesh coloring corresponds to electric surface field intensity.



Figure 2: Model of 7-cell coupler region with a short compensation stub diametrically opposing fundamental power port. Mesh coloring corresponds to electric surface field intensity.

To investigate the minimization of bunch kicks due to the fundamental power coupler, we investigate the maximum normalized coupler kick as a function of stub height by either increasing or decreasing the height of the compensation stub, as illustrated in Fig. 3.



Figure 3: Coupler region of 7-cell cavity geometry. Orange cylinder illustrates method used to tune the compensation stub height, by an additional amount H.

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#### **METHODS**

The dimensions of the optimized 2D model of the cavity were used to generate a 3D geometry, and a coupler and RF probe ports were added to realistically model the final cavity design.

Fields in the cavity were computed with S3P, a frequency domain solver, that launched a TEM wave through the fundamental power coupler and computed the resulting fields in the cavity. The frequency of the TEM wave at the top of the coupler is a tunable parameter, enabling the calculation of the kick as a function of detuning. An example of the fields calculated by S3P at two different frequencies are presented in Fig. 4.



Figure 4: Coupler region of cavity showing fields for two frequencies. Left is on resonance for the 1.3 GHz 7-cell structure and right is obtained by launching a wave through the fundamental power coupler with a frequency 14 kHz higher.

The theoretical basis for calculating of coupler kicks has been dealt with thoroughly in other sources[3]; here we just summarize a few key results for completeness.

Given the fields along the beam axis of the cavity as a function of frequency of the incident wave, the resonant frequency,  $f_0$ , is defined implicitly as the frequency that results in the maximal momentum gain to the beam:

$$P_{z}(f_{0}) = \max \left| \frac{q}{c} \int_{z=0}^{z=L} \mathbf{E}_{z}(f; x=0, y=0, z) e^{2\pi i f \cdot z/c} dz \right|$$
(1)

Since the longitudinal momentum transfer is proportional to the voltage in the cavity, one can use the general form of a resonator with near infinite  $Q_0$  and loaded Q,  $Q_L$ to write:

$$P_z(f) \propto V(f) \propto \frac{1}{\sqrt{\left(\frac{f_0}{f} - \frac{f}{f_0}\right)^2 + \frac{1}{Q_L^2}}}.$$
 (2)

This equation can be used with the curve  $P_z(f)$  to determine both the resonant frequency and the loaded Q of the cavity very accurately.

The normalized coupler kick to the bunch can be calculated by integrating the forces along the beam axis to give

$$\begin{pmatrix} \mathbf{P}_{x} \\ \mathbf{P}_{y} \\ \mathbf{P}_{z} \end{pmatrix} = \frac{q}{c} \begin{pmatrix} \int dz \left(\mathbf{E}_{x} - c\mathbf{B}_{y}\right) e^{2\pi i f \cdot z/c} \\ \int dz \left(\mathbf{E}_{y} + c\mathbf{B}_{x}\right) e^{2\pi i f \cdot z/c} \\ \int dz \mathbf{E}_{z} e^{2\pi i f \cdot z/c} \end{pmatrix}$$
(3)

where the coupler kick is given by

$$\kappa(f) = \sqrt{\frac{P_x^2(f) + P_y^2(f)}{P_z^2(f)}}$$
(4)

where the frequency, f, dependence has been made explicit to demonstrate that the bunch kick is a function of detuning. Also, in our case the cavity and coupler is symmetric about the xz-plane, meaning that  $\mathbf{P}_y$  vanishes, allowing the coupler kick,  $\kappa$  to be simply written as  $\mathbf{P}_x/\mathbf{P}_z$ .

It is important to note that since we are in the frequency domain, the momenta are complex valued. In general one can write

$$P_x = |P_x|e^{i\phi_x} \tag{5}$$

and

$$\tilde{P}_z = |P_z|e^{i\phi_z} \equiv |P_z|e^{i\phi_x}e^{-i\delta} \tag{6}$$

where  $\delta$  is an arbitrary phase between  $\phi_x$  and  $\phi_z$ . This means that for a given frequency, the normalized coupler kick is phase (time) independent since

$$\kappa = \frac{|P_x|e^{i\phi_x}}{|P_z|e^{i\phi_x}e^{-i\delta}} = \frac{|P_x|}{|P_z|}e^{i\delta}.$$
(7)

In contrast to this time independent normalized quantity, the transverse momentum imparted to the beam is phase (time) dependent.

#### RESULTS

Typical fields along the beam axis obtained from S3P are presented in Fig. 5, demonstrating the field accuracy that is possible with the ACE3P code.

The resonant frequency, as defined by frequency that gives largest longitudinal momentum transfer to the beam was found to be different than the frequency that has the smallest magnitude coupler kick. An example of this is . shown in Fig. 6.

Note that the transverse fields in the region between the center of the first cell past the fundamental mode coupler and the opposite end-cell ( $z \in [-0.85, 0]$  m, in Fig. 5) have noise comparable to that of the transverse kick in the coupler region. This noise arises due to the fact that  $\mathbf{E}_z$  in this region is very large, so while the relative error in the eigenvector is small, compared to the total field in the cavity transverse field errors can be large. This is not an issue for coupler kicks though, because in the coupler region  $|\mathbf{E}_z| \sim |\mathbf{E}_{\perp}| \sim c|\mathbf{B}_{\perp}|$ , so the code yields accurate eigenvectors. The momentum can also be determined more accurately by only integrating in the range  $z \geq 0$ , since transverse fields positions past this point should vanish by symmetry.



Figure 5: Real and imaginary parts of the field components along the beam axis of the 7-cell cavity. The longitudinal electric field has been scaled down by a factor of 500 for clarity. Note that  $c \cdot B$  is plotted so that the magnetic field effect can be directly compared with the effect of the electric field. The coupler is located at approximately z = 0.14 m, where a small oscillation shows the field interaction with the bunch.



Figure 6: Magnitude of the longitudinal momentum gain and normalized coupler kick versus frequency. The blue lines mark  $\pm 20$  Hz, which is typical detuning operation for the ERL.

The expected microphonics detuning in main linac seven cells is approximately 20 Hz. Because of this, horizontal lines marking  $\pm 20$  Hz show the expected variation in the coupler kick. Note that operation only a few bandwidths off resonance can increase the coupler kick by orders of magnitude.

The coupler kick as a function of frequency was computed for two geometries: one where the bottom of the symmetrizing stub was rounded with a radius of 3 mm, and one retaining the sharp edge of a simple cylindrical stub. For these geometries, 5 stub heights were simulated, along with the case of no stub at all. The results of these simulations are presented in Figs. 7-8. Note that these kicks are time dependent since the frequency change is a function of time, with modulation frequencies between 10-100 Hz. The rapid shifts are very difficult to correct with steering magnets and instead must be overcome with good design and microphonics control to minimize deleterious effects on the beam.



Figure 7: Real part of the normalized coupler kick as a function of compensation stub height. The real part of  $\kappa$  contributes to the transverse kick of the beam. A stub height of zero corresponds to no compensation stub. The  $\pm 20$  Hz values correspond to expected detuning due to microphonics for an operational 7-cell main linac cavity. Top blue line is the worst value of the coupler kick within  $\pm 20$  Hz of the resonant frequency.



Figure 8: Imaginary part of the normalized coupler kick as a function of compensation stub height. The imaginary component of  $\kappa$  contributes to emittance growth. A stub height of zero corresponds to no compensation stub.

#### CONCLUSIONS

From Figs. 7-8., it is clear that an inclusion of a compensation stub of approximately 10 mm is a good compromise on the transverse kick effect and emittance growth effect of the beam. This yields a ~ 2% decrease in  $\Re \kappa$  while only increasing  $\Im \kappa$  by ~ 10% from the baseline 15 mm case. Fortunately, the effect of these normalized kicks are less than the expected cavity x and y pitch errors, meaning that our values of < 10<sup>-5</sup> are more than sufficient to run successfully in the ERL's low emittance mode[5]. Thus, the compensation stub is unneeded for the Cornell main linac coupler geometry, and production can proceed at a reduced cost on schedule for the first prototype 7-cell cavities to be completed Fall 2011.

Most importantly, it was shown that maintaining detuning below a few bandwidths is essential to keep time dependent coupler kicks to reasonable values, and permit the Cornell ERL to operate in low emittance mode.

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