

## PONDEROMOTIVE INSTABILITIES AND MICROPHONICS – A TUTORIAL\*

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### Abstract

Phase and amplitude stabilization of the fields in superconducting cavities in the presence of ponderomotive effects and microphonics was one of the major challenges that had to be surmounted in order to make superconducting rf accelerators practical. This was of particular concern in low-velocity proton and ion accelerators since the beam loading was often negligible, but was usually not relevant in electron accelerators since the beam loading was often high and the gradients low. More recent or future applications of electron linacs – for example JLab upgrade, energy recovering linacs (ERLs) – will operate at increasingly higher gradients with little beam loading, and the issues associated with microphonics and ponderomotive instabilities will again become relevant areas of research. This paper will describe the ponderomotive instabilities and the conditions under which they can occur, and review the methods by which they, and microphonics, can be overcome.

### HISTORICAL BACKGROUND

Ponderomotive instabilities were first observed in normal conducting resonators in the 1960s in the Soviet Union [1-3]. In that work stability conditions were derived using energetic arguments, comparing the rate of transfer of energy from the electromagnetic mode to the mechanical mode and the rate of dissipation of energy of the mechanical mode. The analysis was valid when the decay time of the electromagnetic mode was much less than the period of the mechanical mode ( $\tau\Omega_\mu \ll 1$ ), or when the rate of transfer of energy was very high.

In the late 60s early 70s, as part of the R&D activities at Karlsruhe toward the development of a superconducting proton accelerator, Schulze [4,5] extended the analysis of ponderomotive instabilities in generator-driven systems, with and without phase and amplitude feedback, to arbitrary  $\tau\Omega_\mu$ , which would be appropriate for superconducting structures. His analysis was based on control system methods (Laplace transforms, transfer functions, *etc.*). That work made first mention and demonstrated the effectiveness of using ponderomotive effects to damp mechanical modes.

In the mid 70s, as part of the R&D activities at Caltech toward the development of a heavy-ion superconducting accelerator, Delayen [6,7] analyzed the behavior of resonators operated in self-excited loops, with and without phase and amplitude feedback, in the presence of

ponderomotive effects and microphonics. That analysis was also based on control systems methods and made use of stochastic analysis to quantify the performance of the feedback systems. That work also introduced the I/Q control method as well as microprocessor-based control systems for superconducting cavities.

### THE ADIABATIC THEOREM AND SUPERCONDUCTING CAVITIES

An important theorem of classical mechanics states that for periodic system whose properties change slowly with time (as defined by a slowness parameter  $\varepsilon$ ) the action  $J = \oint p dq$  changes more slowly than a power of  $\varepsilon$ . When applied to harmonics oscillators – where the action is  $U/\omega$ , the ratio of energy and frequency – then  $U/\omega$  changes more slowly than any power of  $\varepsilon = \frac{1}{\omega^2} \frac{d\omega}{dt}$  if the

frequency changes smoothly, *i.e.* it is an adiabatic invariant to all orders [8]. The dimensionless parameter  $\varepsilon$  is the relative change in frequency during one radian. Since in the case of superconducting cavities it would be difficult to change the frequency significantly during one radian, the action  $U/\omega$  can be assumed to be constant and, in particular, any relative change in frequency is equal to any relative change in energy content:

$$\frac{\Delta\omega}{\omega} = \frac{\Delta U}{U}.$$

In the quantum picture, this would mean that the system stays in the same eigenstate and that the number of photons remains constant ( $U = N\hbar\omega$ ).

The energy content in a resonator is given by

$$U = \int_V dv \left[ \frac{\mu_0}{4} H^2(\vec{r}) + \frac{\varepsilon_0}{4} E^2(\vec{r}) \right], \quad (1)$$

and the change in energy content is equal to the work done by the radiation pressure:

$$\Delta U = - \int_S dS \vec{n}(\vec{r}) \cdot \vec{\xi}(\vec{r}) \left[ \frac{\mu_0}{4} H^2(\vec{r}) - \frac{\varepsilon_0}{4} E^2(\vec{r}) \right] \quad (2)$$

where  $\vec{n}(\vec{r})$  and  $\vec{\xi}(\vec{r})$  are the normal vector and the displacement vector, respectively, at location  $\vec{r}$ .

The relative change in frequency is then given by

$$\frac{\Delta\omega}{\omega} = \frac{\int_S dS \vec{n}(\vec{r}) \cdot \vec{\xi}(\vec{r}) \left[ \frac{\mu_0}{4} H^2(\vec{r}) - \frac{\varepsilon_0}{4} E^2(\vec{r}) \right]}{\int_V dv \left[ \frac{\mu_0}{4} H^2(\vec{r}) + \frac{\varepsilon_0}{4} E^2(\vec{r}) \right]}, \quad (3)$$

which, in microwave engineering, is known as Slater's formula [9].

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## PONDEROMOTIVE EFFECTS

Any cavity will have an infinite number of mechanical eigenmodes of vibration represented by a complete infinite set of orthonormal displacement functions  $\phi_\mu(\vec{r})$ . The actual displacements of the cavity wall,  $\xi(\vec{r})$ , and the forces on the wall,  $F(\vec{r})$ , can be expanded into the functions  $\phi_\mu(\vec{r})$ :

$$\begin{aligned}\xi(\vec{r}) &= \sum_\mu q_\mu \phi_\mu(\vec{r}) & q_\mu &= \int_S \xi(\vec{r}) \phi_\mu(\vec{r}) dS \\ F(\vec{r}) &= \sum_\mu F_\mu \phi_\mu(\vec{r}) & F_\mu &= \int_S F(\vec{r}) \phi_\mu(\vec{r}) dS\end{aligned}\quad (4)$$

where  $q_\mu$  is the amplitude of mechanical mode  $\mu$  whose equation of motion is

$$\frac{d}{dt} \frac{\partial L}{\partial \dot{q}_\mu} - \frac{\partial L}{\partial q_\mu} + \frac{\partial \Phi}{\partial \dot{q}_\mu} = F_\mu \quad (5)$$

with  $L = T - U$ , where  $T$ ,  $U$ , and  $\Phi$  are the kinetic energy, the potential energy, and the power dissipation, respectively.

$$U = \frac{1}{2} \sum_\mu c_\mu q_\mu^2, \quad T = \frac{1}{2} \sum_\mu c_\mu \dot{q}_\mu^2, \quad \Phi = \sum_\mu \frac{c_\mu}{\tau_\mu} \dot{q}_\mu^2 \quad (6)$$

where  $c_\mu$  is the elastic constant,  $\Omega_\mu$  is the frequency, and  $\tau_\mu$  is the decay time of mechanical mode  $\mu$ . Equation (5) then becomes

$$\ddot{q}_\mu + \frac{2}{\tau_\mu} \dot{q}_\mu + \Omega_\mu^2 q_\mu = \frac{\Omega_\mu^2}{c_\mu} F_\mu \quad (7)$$

Since the frequency shift  $\Delta\omega_\mu$  caused by mechanical mode  $\mu$  is directly proportional to  $q_\mu$ , and the force  $F_\mu$  due to the radiation pressure is proportional to the square of the field amplitude  $V$ , the equation for  $\Delta\omega_\mu$  is

$$\Delta \ddot{\omega}_\mu + \frac{2}{\tau_\mu} \Delta \dot{\omega}_\mu + \Omega_\mu^2 \Delta \omega_\mu = -k_\mu \Omega_\mu^2 V^2 + n(t) \quad (8)$$

The constant  $k_\mu$  (the Lorentz coefficient for that mode) represents the coupling between the rf field and mechanical mode  $\mu$ , and  $n(t)$  is an additional driving term representing external vibrations or microphonics. The total frequency shift is  $\Delta\omega(t) = \sum_\mu \Delta\omega_\mu(t)$ , and in steady-state  $\Delta\omega_0 = \sum_\mu \Delta\omega_{\mu 0} = -V^2 \sum_\mu k_\mu$ , and  $k = \sum_\mu k_\mu$  is the static

Lorentz coefficient of the cavity.

When analyzing the stability of the system or its performance in cw operation, Eq. (8) can be linearized around steady-state and becomes

$$\delta \ddot{\omega}_\mu + \frac{2}{\tau_\mu} \delta \dot{\omega}_\mu + \Omega_\mu^2 \delta \omega_\mu = -2\Omega_\mu^2 k_\mu V_0^2 \delta v + n(t) \quad (9)$$

where  $\Delta\omega_\mu = \Delta\omega_{\mu 0} + \delta\omega_\mu$  and  $V = V_0(1 + \delta v)$ .

In the frequency domain the response from amplitude fluctuation to frequency fluctuation, also known as the Lorentz transfer function, is then

$$\frac{\delta\omega_\mu(\omega)}{\delta v(\omega)} = \frac{-2\Omega_\mu^2 k_\mu V_0^2}{(\Omega_\mu^2 - \omega^2) + \frac{2}{\tau_\mu} i\omega} = G_\mu(\omega) \quad (10)$$

The total Lorentz transfer function for a cavity is the sum of all the transfer functions for the individual mechanical modes. It can be obtained by operating the cavity in cw mode at some relatively high field (since the response is proportional to  $V_0^2$ ) and introducing a small modulation of the field of amplitude  $\delta v$  and frequency  $\omega$  and measuring the amplitude and the phase of the modulation of the cavity frequency as a function of  $\omega$ . Examples of Lorentz transfer functions are shown in Fig. 1 for a double-spoke cavity and a 6-cell elliptical cavity. For the former, only a few low-frequency mechanical modes exist and the transfer function is simple, while for the latter it is much more complex due to the large number of low-frequency modes.

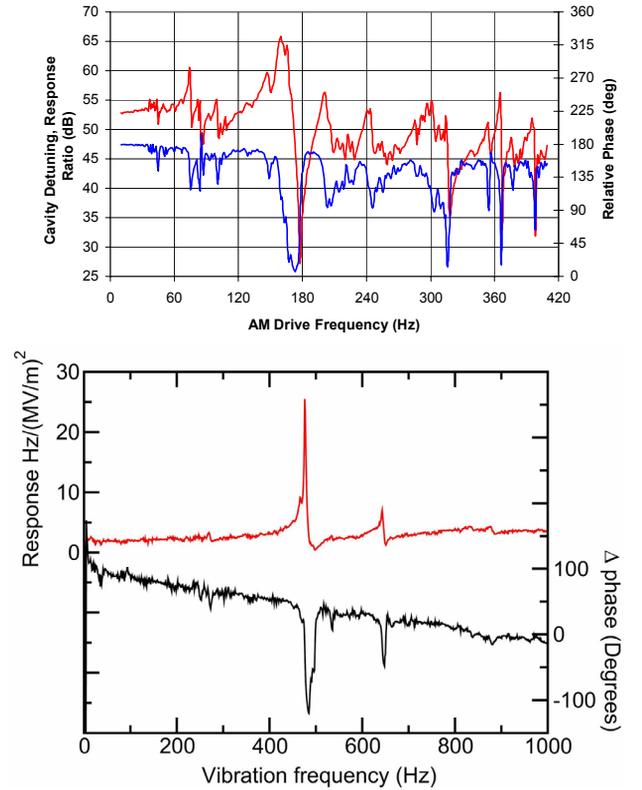


Figure 1: Lorentz transfer function of a  $\beta=0.61$ , 805 MHz 6-cell elliptical cavity (top) [10], and of a double-spoke 352 MHz,  $\beta=0.4$  cavity (bottom) [11].

Equation (8) describes how the cavity frequency is affected by the field amplitude. On the other hand the field amplitude depends on the frequency detuning between the cavity and the rf source. Such a closed feedback system between the electromagnetic mode and the mechanical modes can lead to instabilities. One of them, the monotonic instability is the familiar jump phenomenon illustrated in Figure 2 that can occur on the low-frequency side of the resonance.

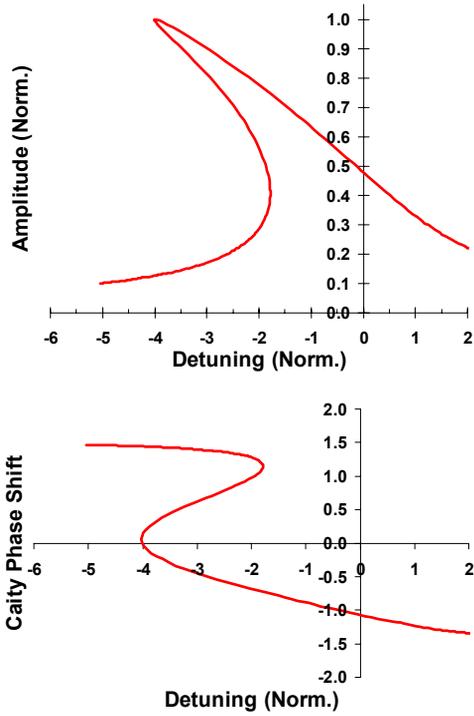


Figure 2: Resonance curve and phase shift across a cavity when driven by a fixed-frequency source in the presence of ponderomotive effects.

There is another type of instability that can occur on the high-frequency side of the resonance, the oscillatory instability, characterized by an exponential growth of the amplitude of mechanical vibrations. These two types of instability are present in generator driven systems in the absence of phase and amplitude feedback. Introduction of feedback can remove those instabilities [4]. Cavities operated in self-excited loops in the absence of feedback are free of ponderomotive instabilities [6]. Stability criteria in the presence of beam loading have also been derived [12].

## MICROPHONICS

Microphonics are the changes in cavity frequency caused by connections to the external world [ $n(t)$  in Eq. (8)], such as vibrations, pressure fluctuations, *etc.* The ponderomotive term  $-k_\mu \Omega_\mu^2 V^2$  cannot be ignored since, in the presence of phase and amplitude feedback, it can affect, positively or negatively, the response of the cavity frequency to external noise; in particular ponderomotive effects can be used to damp the mechanical modes and reduce microphonics [5, 13]. The presence of external noise  $n(t)$  will generate a fluctuation of the cavity frequency  $\delta\omega_{ex}$  through the transfer function  $G_\mu$  of a harmonic oscillator defined in Eq. (10) as shown in Fig. 3.

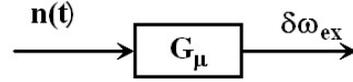


Figure 3: Transfer function representation of microphonics  $\delta\omega_{ex}$  generated by an external disturbance  $n(t)$ .

There can be several different kinds of external driving terms  $n(t)$  for the microphonics. The 2 extreme cases would be a deterministic harmonic drive signal with a well defined frequency and amplitude, and the other a purely stochastic gaussian white noise drive signal. The former could originate from a well defined vibration source such as a pump or a motor; the latter would be due to broadband (relative to the bandwidth of the mechanical mode) ambient noise. The probability density (histogram) of the frequency response  $\delta\omega_{ex}$  to those 2 different driving signals is shown in Fig. 4.

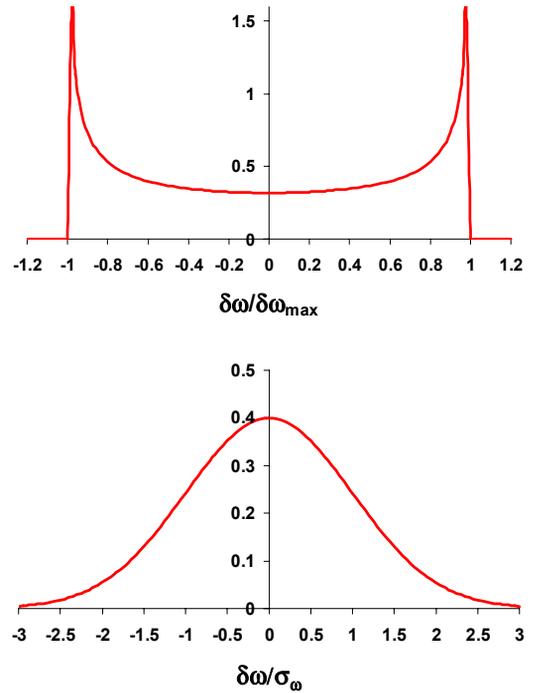


Figure 4: Probability density of cavity frequency fluctuations  $\delta\omega_{ex}$  generated by a sinusoidal (top) or gaussian white noise (bottom) external disturbance  $n(t)$ .

For the sinusoidal drive the probability density of  $\delta\omega_{ex}$  is

$$p(\delta\omega) = \frac{1}{\pi\sqrt{\delta\omega_{\max}^2 - \delta\omega^2}} \quad (11)$$

while for the white noise driving term is it

$$p(\delta\omega) = \frac{1}{\sigma_\omega\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{\delta\omega}{\sigma_\omega}\right)^2\right] \quad (12)$$

where  $\delta\omega$  is the deviation of the instantaneous cavity frequency from its average. Examples of these two types of probability densities have been observed (see Fig. 5),

although the gaussian density is much more common and has been seen over 5 orders of magnitude [14]. The presence of a non-gaussian microphonics probability density is often indicative of the presence of localized source of vibration that needs to be better isolated.

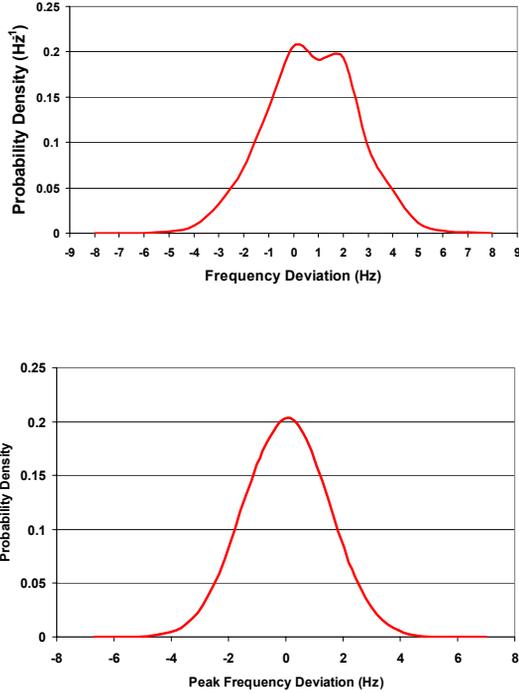


Figure 5: Examples of bimodal (top) and gaussian (bottom) probability densities measured on SNS cavities [10].

From a long time series of  $\delta\omega_{ex}(t)$ , its autocorrelation function can also be calculated. The autocorrelation function of a stationary signal  $x(t)$  is defined as

$$R_x(\tau) = \langle x(t)x(t+\tau) \rangle = \lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T dt x(t)x(t+\tau) \quad (13)$$

In the case of a harmonic oscillator representing a mechanical mode of frequency  $\Omega_\mu$  and decay time  $\tau_\mu$ , driven by a periodic signal of frequency  $\omega_d$ , the normalized autocorrelation function of the output  $\delta\omega_{ex}$  is

$$r_{\delta\omega}(\tau) = \frac{R_{\delta\omega}(\tau)}{R_{\delta\omega}(0)} = \cos(\omega_d \tau) \quad (14)$$

The correlation function is sinusoidal at the frequency of the drive signal and remains finite for arbitrary long time separating 2 measurements.

If instead the harmonic oscillator is driven by a gaussian white noise then the normalized autocorrelation function is

$$r_{\delta\omega}(\tau) = \frac{R_{\delta\omega}(\tau)}{R_{\delta\omega}(0)} = \cos(\Omega_\mu \tau) e^{-|\tau|/\tau_\mu} \quad (15)$$

In this case the correlation function has the frequency and the decay time of the mechanical mode. This implies that values of the microphonics separated by more than  $\tau_\mu$  are uncorrelated and one cannot be used to estimate the other.

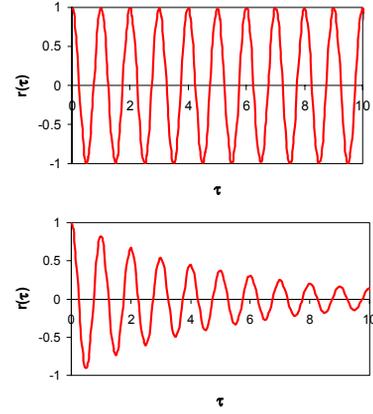


Figure 6: Normalized autocorrelation function of the output of a harmonic oscillator when driven by a sinusoidal input (top) and gaussian white noise (bottom).

Such autocorrelation functions were calculated using Eq.(13) for microphonics measurements made on SNS cavities, and examples are shown in Fig. 7. The picture at the top shows an example where microphonics are caused mainly by 2 sinusoidal driving terms. The picture on the bottom displays the behavior when microphonics are generated by white noise and, as a result, values of the frequency fluctuation separated by more than a decay time of the mechanical mode are uncorrelated. This implies that a feedforward control system to reduce the level of microphonics can use only measurements performed in the past no further than the decay time of the mechanical mode. An autocorrelation function for the microphonics that does not decay to 0, as in top Fig. 7, is indicative on a non-stochastic driving term that could be removed by better isolation or cancelled by feedforward control.

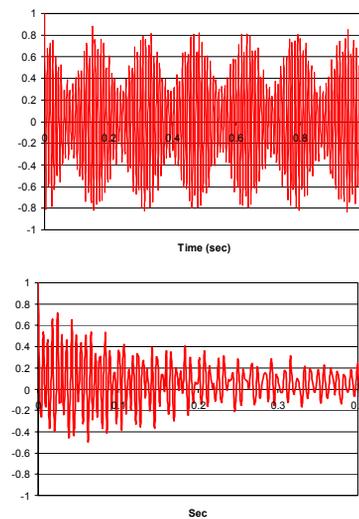


Figure 7: Examples of autocorrelation function of frequency fluctuations measured on SNS cavities. At the top microphonics are generated by 2 single-frequency sources; below they are generated by broad-band noise.

External disturbances  $n(t)$  will generate microphonics  $\delta\omega_{ex}$ , which in turn will cause fluctuations of the phase ( $\delta\phi$ ) and amplitude ( $\delta v$ ) of the fields in the cavity.

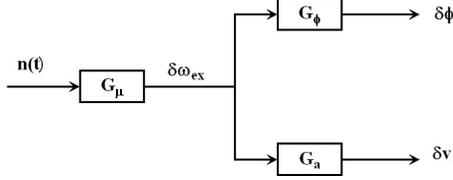


Figure 8: Transfer function representation of the field errors caused by an external disturbance  $n(t)$ .

The transfer functions  $G_a$  and  $G_\phi$  include the properties of the cavity and of the rf control system [6,12]. Assuming that  $n(t)$  is a white noise stationary stochastic process of spectral density  $A^2$ , the mean square values of  $\delta\omega_{ex}$ ,  $\delta v$ , and  $\delta\phi$  are

$$\begin{aligned} \langle \delta\omega_{ex}^2 \rangle &= A^2 \int_{-\infty}^{\infty} d\omega |G_\mu(i\omega)|^2 = A^2 \frac{\pi\tau_\mu}{2\Omega_\mu^2} \\ \langle \delta v^2 \rangle &= \langle \delta\omega_{ex}^2 \rangle \frac{2\Omega_\mu^2}{\pi\tau_\mu} \int_{-\infty}^{\infty} d\omega |G_\mu(i\omega)G_a(i\omega)|^2 \\ \langle \delta\phi^2 \rangle &= \langle \delta\omega_{ex}^2 \rangle \frac{2\Omega_\mu^2}{\pi\tau_\mu} \int_{-\infty}^{\infty} d\omega |G_\mu(i\omega)G_\phi(i\omega)|^2 \end{aligned} \quad (16)$$

Equations (16) establish the relationships between the measured microphonics ( $\delta\omega_{ex}$ ), the properties of the mechanical mode of the cavity ( $G_\mu$ ,  $\Omega_\mu$ , and  $\tau_\mu$ ), the properties of the rf control system ( $G_a$ ,  $G_\phi$ ), and the residual field errors ( $\delta v$ ,  $\delta\phi$ ).

## FINAL COMMENTS

Stabilization of the rf fields in superconducting cavities in the presence of ponderomotive effects and microphonics was one of the outstanding issues that needed to be addressed early on in order to make srf accelerators practical. This was particularly crucial for heavy ion accelerators since the currents were very low and the accelerating structures lacked mechanical rigidity. This challenge was met by a combination of analytical and theoretical work, the development of new superconducting structures, and the adoption of more advanced electronic control system. While these issues are now well understood they are being rediscovered in medium- to high- $\beta$  applications for relatively low beam current.

These applications present new challenges. They tend to be of a much larger scale than the heavy-ion accelerators of decades past: the cavities are larger and they operate at much higher gradient (as a result they have a much larger energy content), and they are in larger number. Thus there is a need for optimization since unnecessary conservatism would be expensive.

While some new medium- to high-beta accelerators would have lower current than previous ones (RIA and the JLab upgrade for example), the currents are still finite

and the presence of the current can modify the stability conditions and needs to be taken into account [12].

In the ERL applications, the low net beam current results from the not-quite-perfect cancellation of 2 large currents. Small fluctuations in any accelerator parameter can lead to large random fluctuation in the beam loading, possibly coupling to the ponderomotive effects and leading to instabilities.

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