Low and Intermediate β Cavity Design

A Tutorial

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2003 SRF Workshop

A Few Obvious Statements

Low and medium β

- β<1
- Particle velocity will change

The lower the velocity of the particle or cavity β

- The faster the velocity of the particle will change
- The narrower the velocity range of a particular cavity
- The smaller the number of cavities of that β
- The more important it is that the particle achieve design velocity

Be conservative at lower β

Be more aggressive at higher β

A Few More Statements

Two main types of structure geometries

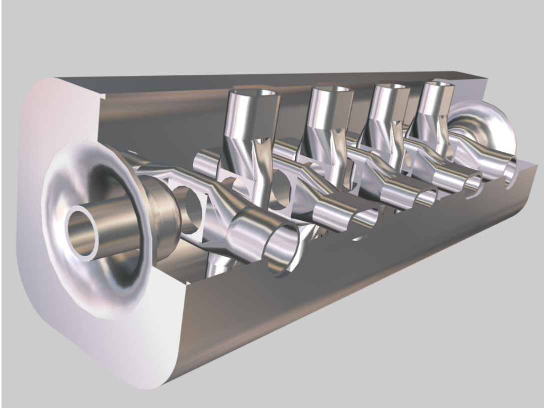
- **TEM class (QW, HW, Spoke)**
- **TM class (elliptical)**

Design issues of medium β elliptical cavities are similar to those of β=1

- **Most of the talk will be on TEM-class cavities**
- **For TM-class cavities see:**
  - **Design criteria for elliptical cavities**
    Pagani, Barni, Bosotti, Pierini, Ciovati, SRF 2001.
  - **Challenges and the future of reduced beta srf cavity design**
    Sang-ho Kim, LINAC 2002.

A Word on Design Tools

TEM-class cavities are essentially 3D geometries



3D electromagnetic software is available

MAFIA, Microwave Studio, HFSS, etc.

3D software is usually very good at calculating frequencies

Not quite as good at calculating surface fields

Use caution, vary mesh size

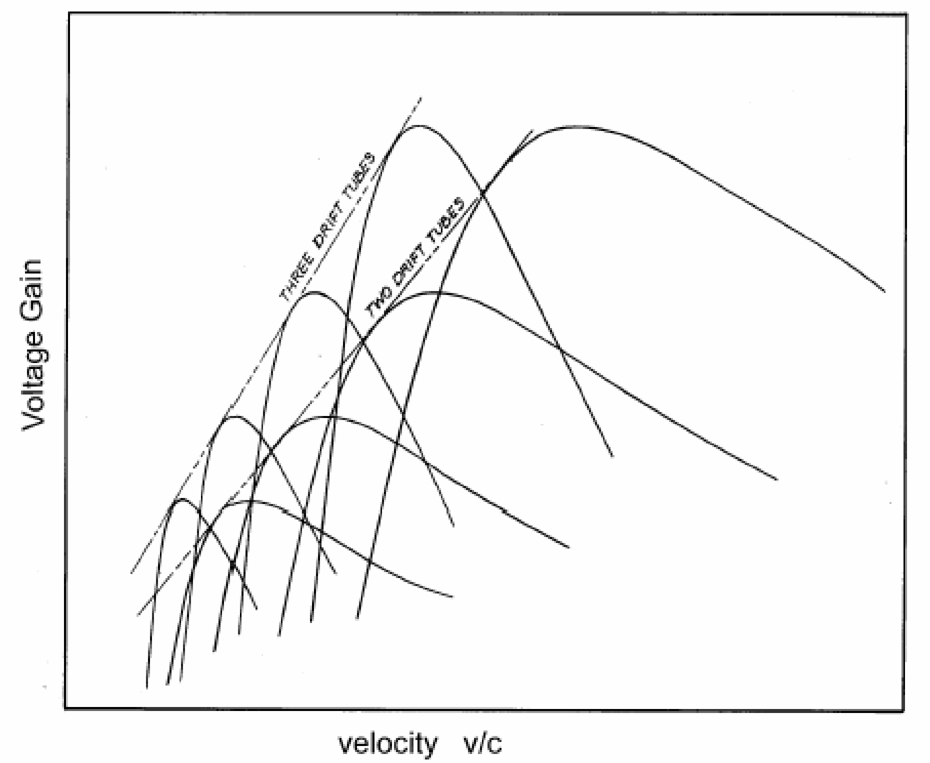
Remember Electromagnetism 101

Design Tradeoffs

Number of cells

Voltage gain

Velocity acceptance



Frequency

Size

Voltage gain

Rf losses

Energy content, microphonics, rf control

Acceptance, beam quality and losses

Energy Gain
Transit Time Factor - Velocity Acceptance

$$\Delta W = q \int_{-\infty}^{+\infty} E(z) \cos(\omega t + \phi)\, dz$$

Assumption: constant velocity

$$\Delta W = q \cos\phi \; \Delta W_0 \; T(\beta) \qquad \Delta W_0 = \Theta \int_{-\infty}^{+\infty} |E(z)|\, dz$$

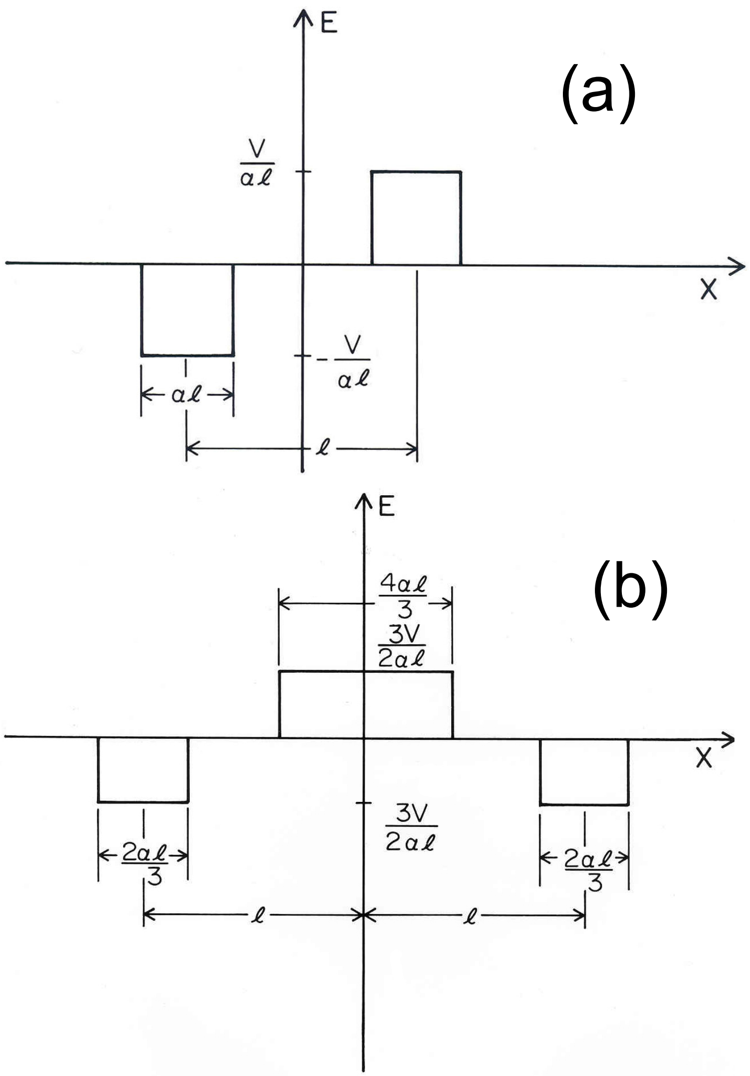
$$\Theta = \frac{\text{Max} \int_{-\infty}^{+\infty} E(z) \cos\left(\frac{\omega z}{\beta c}\right) dz}{\int_{-\infty}^{+\infty} |E(z)|\, dz}$$

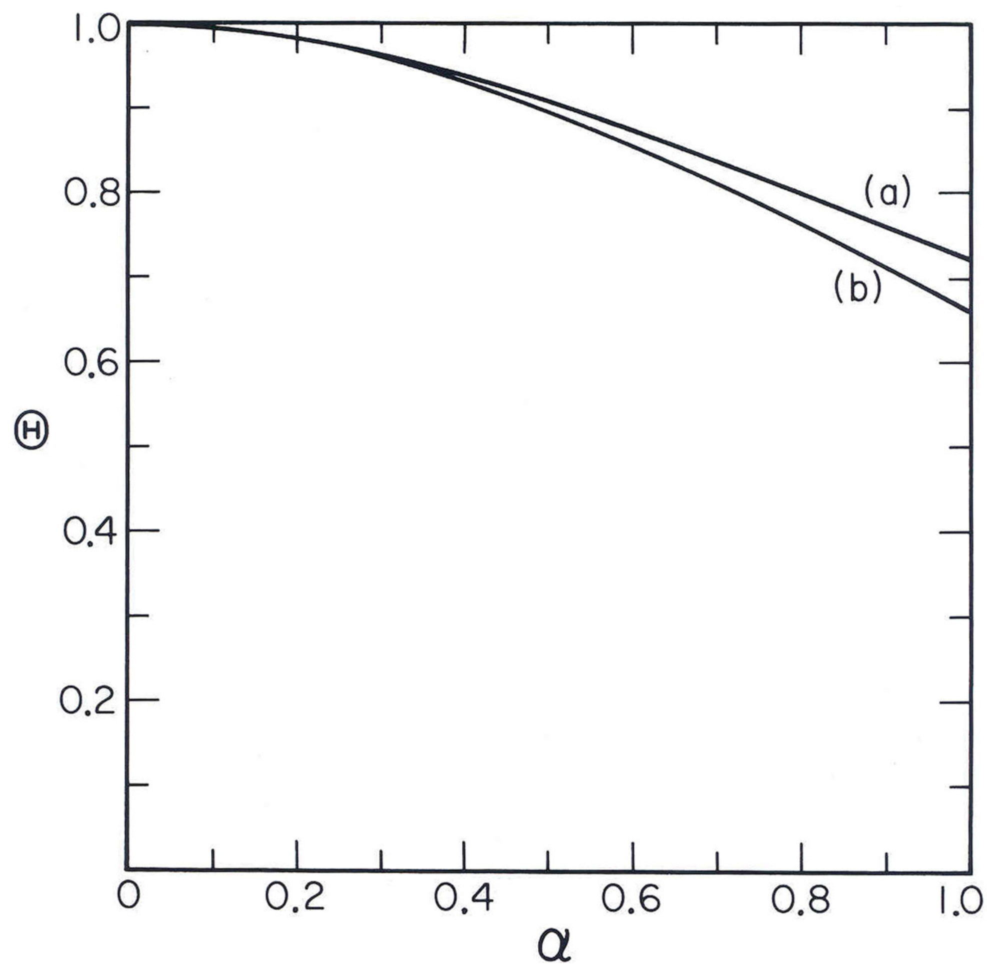
Transit Time Factor

$$T(\beta) = \frac{\int_{-\infty}^{+\infty} E(z) \cos\left(\frac{\omega z}{\beta c}\right) dz}{\text{Max} \int_{-\infty}^{+\infty} E(z) \cos\left(\frac{\omega z}{\beta c}\right) dz}$$

Velocity Acceptance

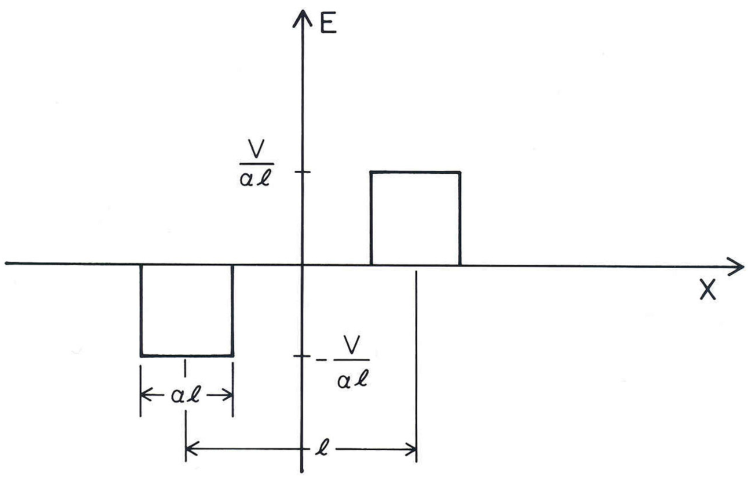
Transit Time Factor



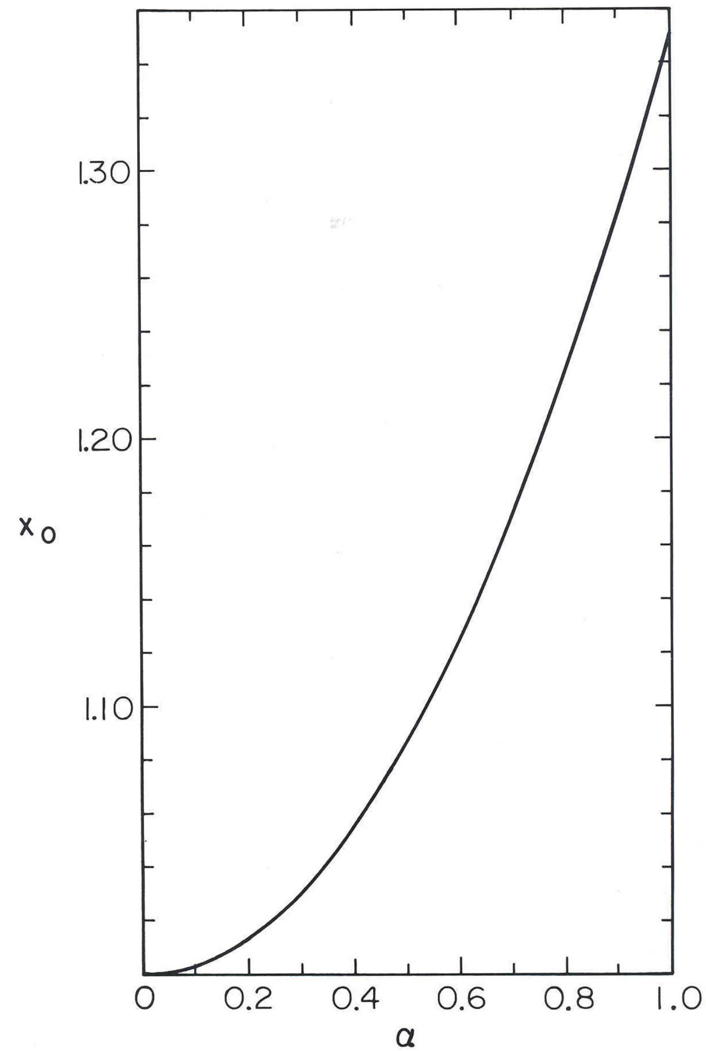


Velocity Acceptance for 2-Gap Structures

$$T(\beta)=\frac{\beta}{\beta_0}\frac{\sin\left(\frac{\pi\alpha}{2x_0}\frac{\beta_0}{\beta}\right)\sin\left(\frac{\pi}{2x_0}\frac{\beta_0}{\beta}\right)}{\sin\left(\frac{\pi\alpha}{2x_0}\right)\sin\left(\frac{\pi}{2x_0}\right)}$$

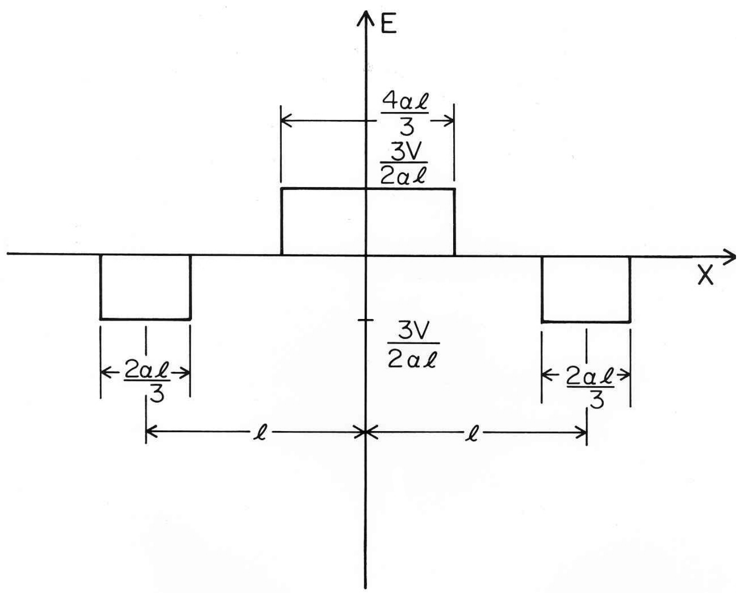


$$x_0=\frac{\beta_0\lambda}{2l}$$

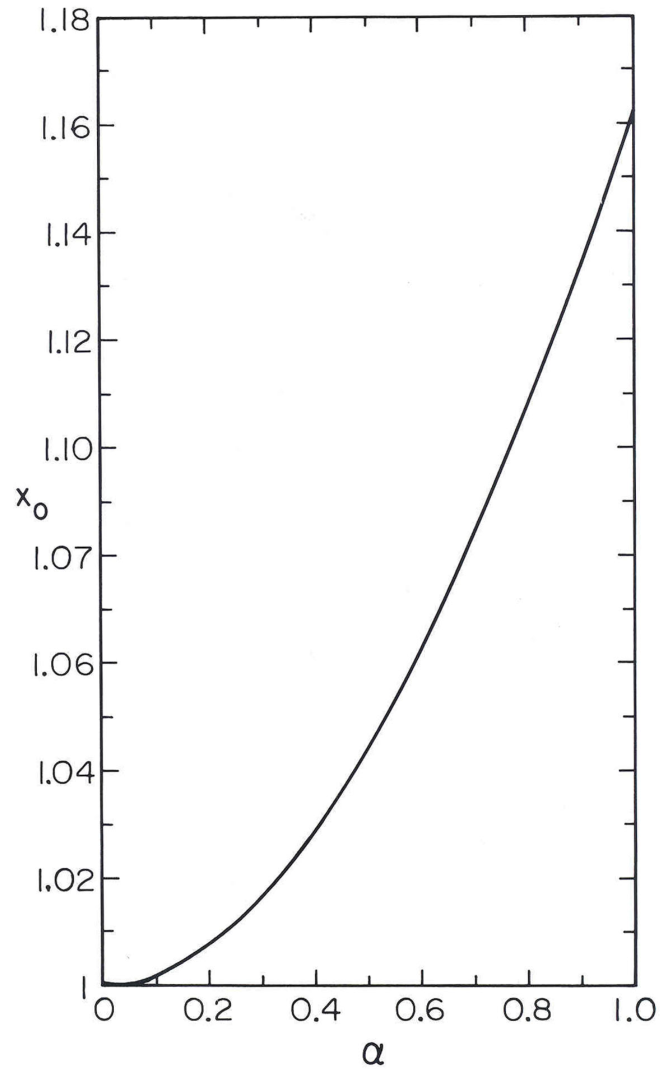


Velocity Acceptance for 3-Gap Structures

$$T(\beta)=\frac{\beta}{\beta_0}\frac{\sin\left(\frac{\pi\alpha}{3x_0}\frac{\beta_0}{\beta}\right)\left[\cos\left(\frac{\pi\alpha}{3x_0}\frac{\beta_0}{\beta}\right)-\cos\left(\frac{\pi}{x_0}\frac{\beta_0}{\beta}\right)\right]}{\sin\left(\frac{\pi\alpha}{3x_0}\right)\left[\cos\left(\frac{\pi\alpha}{3x_0}\right)-\cos\left(\frac{\pi}{x_0}\right)\right]}$$



$$x_0=\frac{\beta_0\lambda}{2l}$$

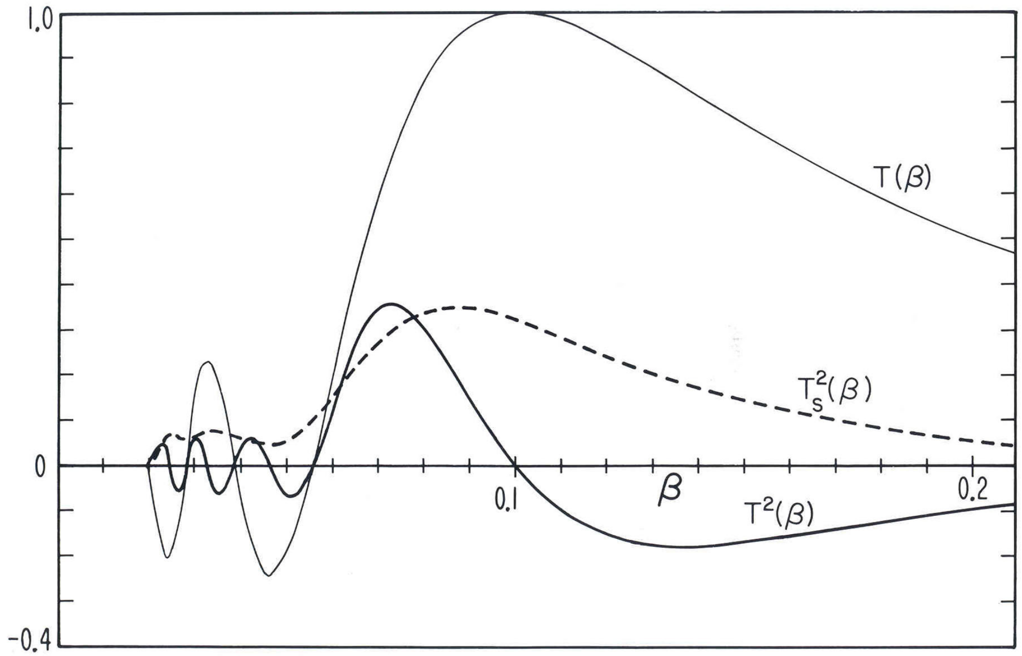


Higher-Order Effects

$$\Delta W = q\cos\phi\,\Delta W_0\,T(\beta) + \frac{(q\Delta W_0)^2}{W}\left[T^{(2)}(\beta) + \sin 2\phi\,T_s^{(2)}(\beta)\right]$$

$$T^{(2)}(k) = -\frac{k}{4}T(k)\frac{d}{dk}T(k) \qquad k = \omega/\beta c$$

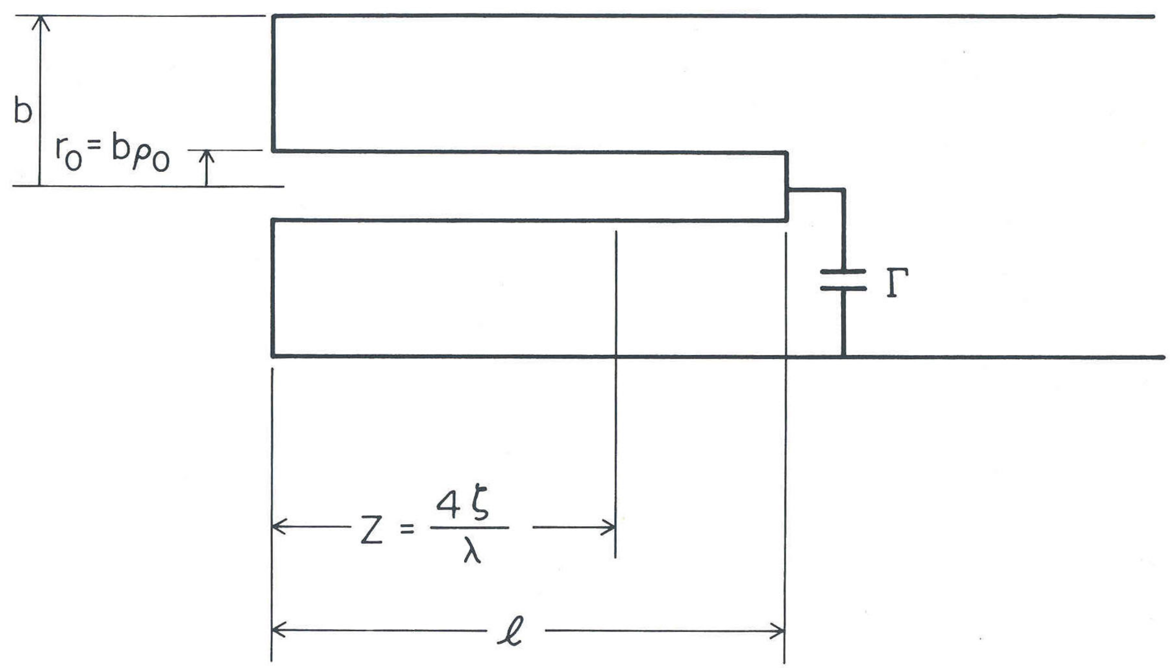
$$T_s^{(2)}(k) = -\frac{k}{4\pi}\int_0^\infty \frac{T(k+k')T(k-k') - T(k)T(k)}{k'^2}dk'$$



A Simple Model: Loaded Quarter-wavelength Resonant Line

If characteristic length <<λ (β<0.5), separate the problem in two parts:

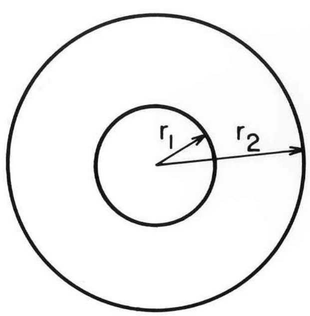
- **Electrostatic model of high voltage region**
- **Transmission line**



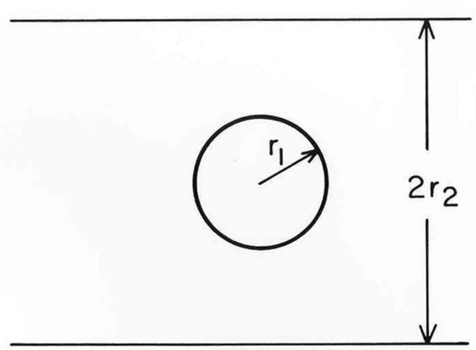
Basic Electrostatics

a: concentric spheres
b: sphere in cylinder
c: sphere between 2 planes
d: coaxial cylinders
e: cylinder between 2 planes

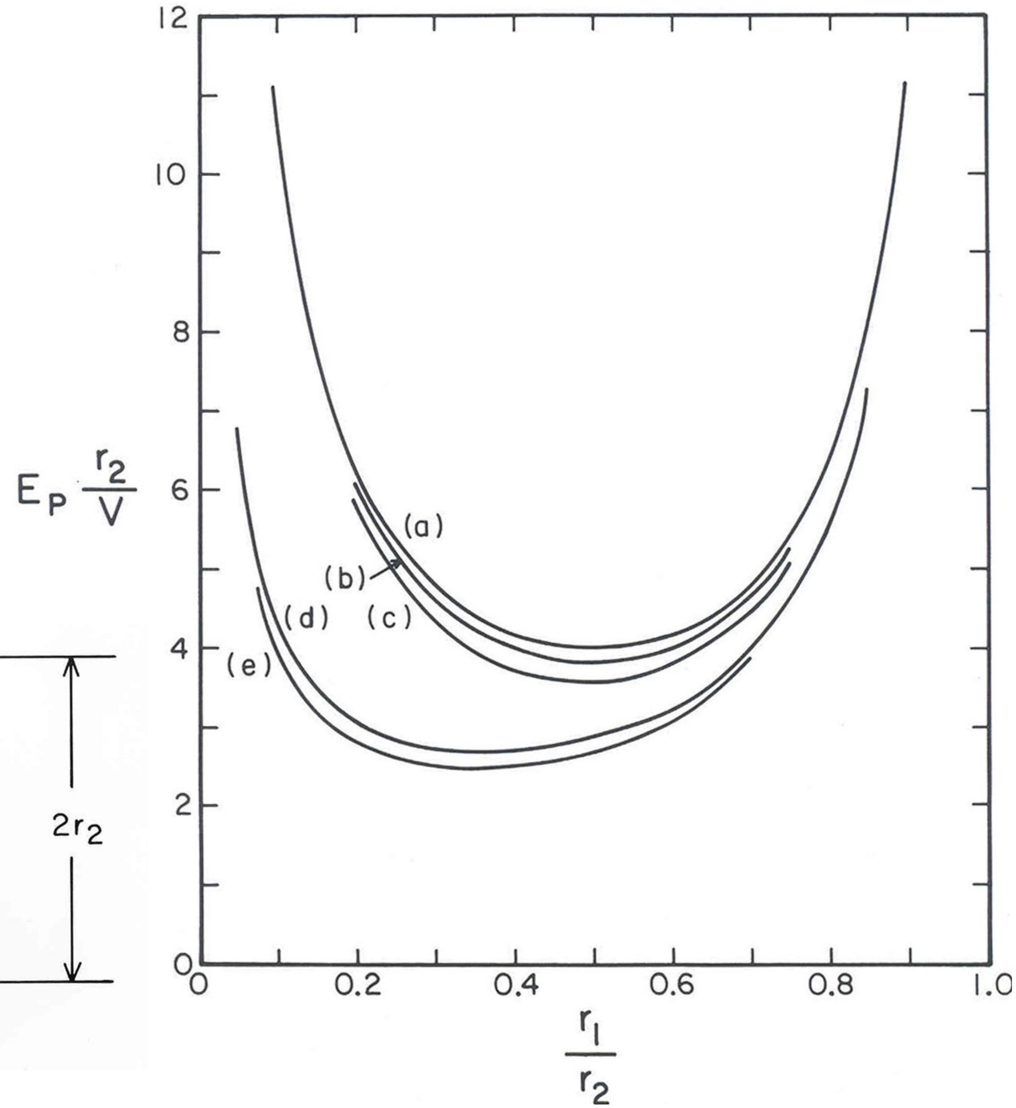
V_p : Voltage on center conductor
Outer conductor at ground
E_p: Peak field on center conductor



(a),(b),(d)



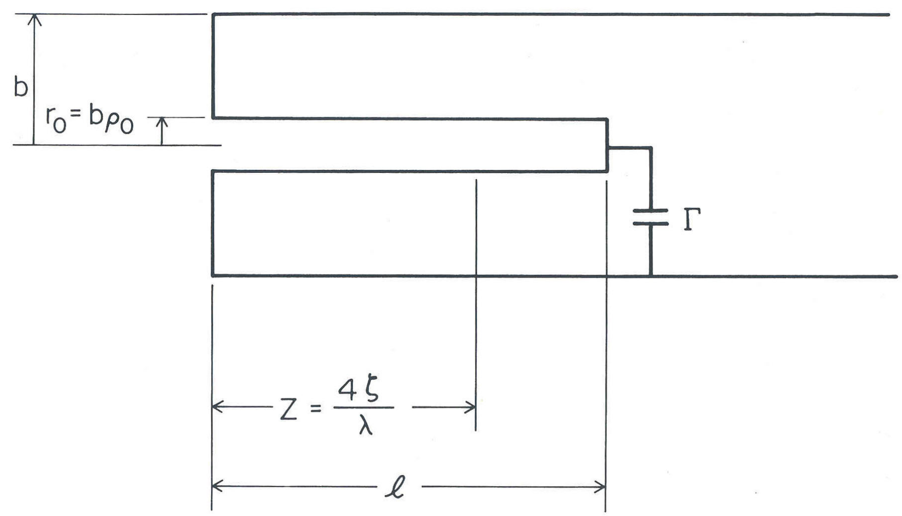
(c),(e)



A Simple Model: Loaded Quarter-wavelength Resonant Line

Capacitance per unit length

$$C = \frac{2\pi\varepsilon_0}{\ln\left(\frac{b}{r_0}\right)} = \frac{2\pi\varepsilon_0}{\ln\left(\frac{1}{\rho_0}\right)}$$



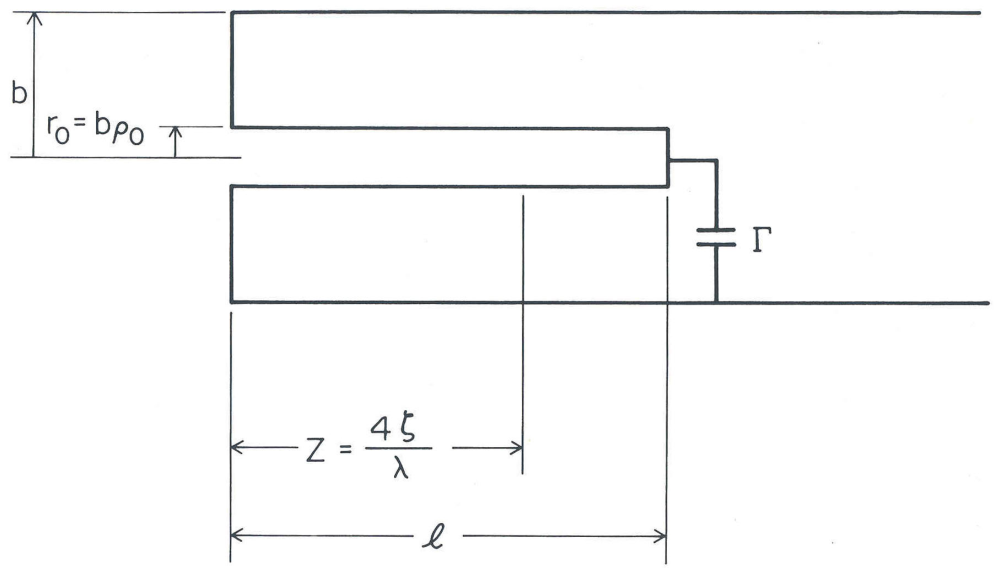
Inductance per unit length

$$L = \frac{\mu_0}{2\pi}\ln\left(\frac{b}{r_0}\right) = \frac{\mu_0}{2\pi}\ln\left(\frac{1}{\rho_0}\right)$$

A Simple Model: Loaded Quarter-wavelength Resonant Line

Center conductor voltage

$$V(z) = V_0 \sin\left(\frac{2\pi}{\lambda} z\right)$$



Center conductor current

$$I(z) = I_0 \cos\left(\frac{2\pi}{\lambda} z\right)$$

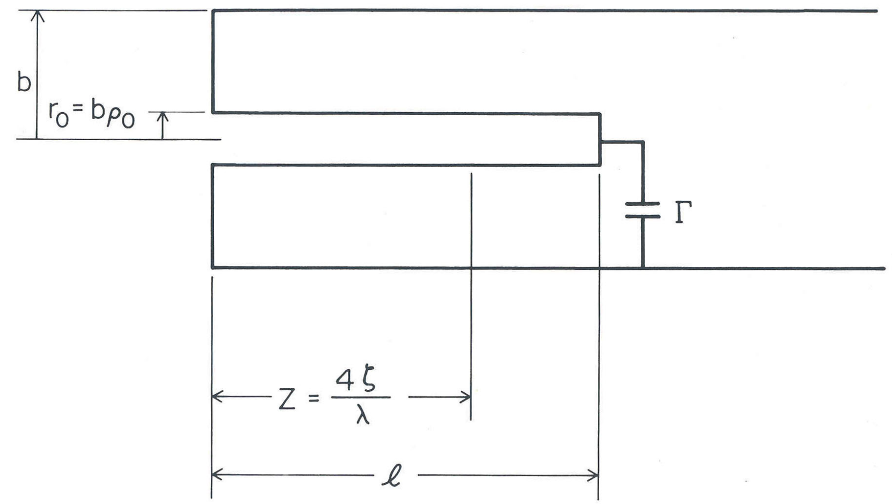
Line impedance

$$Z_0 = \frac{V_0}{I_0} = \frac{\eta}{2\pi} \ln\left(\frac{1}{\rho_0}\right) , \qquad \eta = \sqrt{\frac{\mu_0}{\varepsilon_0}} \simeq 377\Omega$$

A Simple Model: Loaded Quarter-wavelength Resonant Line

Loading capacitance

$$\Gamma(z) = \lambda\varepsilon \frac{\operatorname{cotan}\left(\frac{2\pi}{\lambda} z\right)}{\ln\left(b/r_0\right)} = \lambda\varepsilon \frac{\operatorname{cotan}\left(\frac{\pi}{2}\zeta\right)}{\ln\left(b/\rho_0\right)}$$



Peak magnetic field

$$\frac{V_p}{b} = \begin{Bmatrix} \eta & H \\ c & B \\ 300 & B \end{Bmatrix} \rho_0 \ln\left(\frac{1}{\rho_0}\right) \sin\left(\frac{\pi}{2}\zeta\right) \qquad \begin{Bmatrix} \text{m, A/m} \\ \text{m, T} \\ \text{cm, G} \end{Bmatrix}$$

V_p: Voltage across loading capacitance

$B \simeq 9$ mT at 1 MV/m

A Simple Model: Loaded Quarter-wavelength Resonant Line

Power dissipation (ignore losses in the shorting plate)

$$P = V_p^2 \frac{8}{\pi} \frac{R_s}{\eta^2} \frac{\lambda}{b} \frac{1+1/\rho_0}{\ln^2 \rho_0} \frac{\zeta + \frac{1}{\pi}\sin \pi\zeta}{\sin^2 \frac{\pi}{2}\zeta}$$

$$P \propto \frac{R_s}{\eta^2} E^2 \beta \lambda^2$$

Energy content

$$U = V_p^2 \frac{\pi\varepsilon_0}{8} \lambda \frac{1}{\ln(1/\rho_0)} \frac{\zeta + \frac{1}{\pi}\sin \pi\zeta}{\sin^2 \frac{\pi}{2}\zeta}$$

$$U \propto \varepsilon_0 E^2 \beta^2 \lambda^3$$

A Simple Model: Loaded Quarter-wavelength Resonant Line

Geometrical factor

$$G = QR_s = 2\pi\,\eta\,\frac{b}{\lambda}\,\frac{\ln(1/\rho_0)}{1+1/\rho_0}$$

$$G \propto \eta\,\beta$$

Shunt impedance $\left(4V_p^2/P\right)$

$$R_{sh} = \frac{\eta^2}{R_s}\,\frac{32}{\pi}\,\frac{b}{\lambda}\,\frac{\ln^2\rho_0}{1+1/\rho_0}\,\frac{\sin^2\frac{\pi}{2}\zeta}{\zeta+\frac{1}{\pi}\sin\pi\zeta}$$

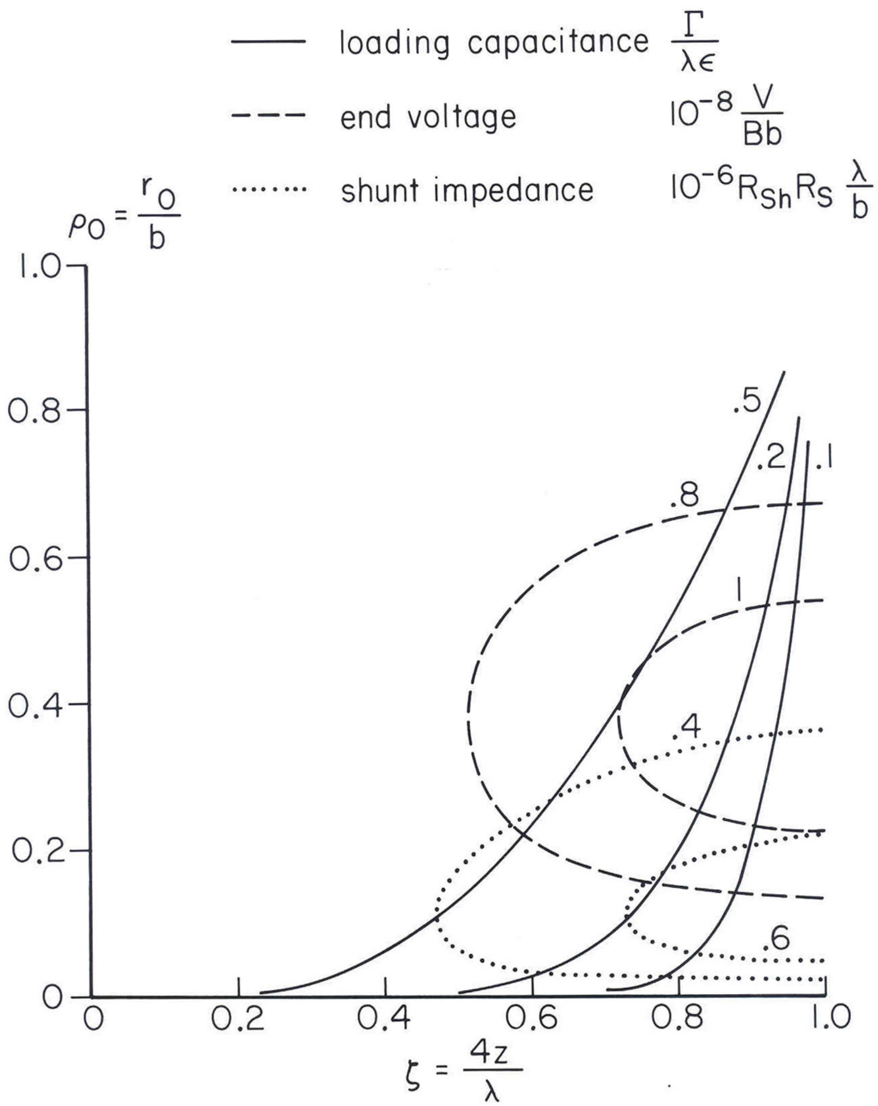
$$R_{sh}\,R_s \propto \eta^2\,\beta$$

R/Q

$$\frac{R_{sh}}{Q} = \frac{16}{\pi^2}\,\eta\ln(1/\rho_0)\,\frac{\sin^2\frac{\pi}{2}\zeta}{\zeta+\frac{1}{\pi}\sin\pi\zeta}$$

$$\frac{R_{sh}}{Q} \propto \eta$$

A Simple Model: Loaded Quarter-wavelength Resonant Line



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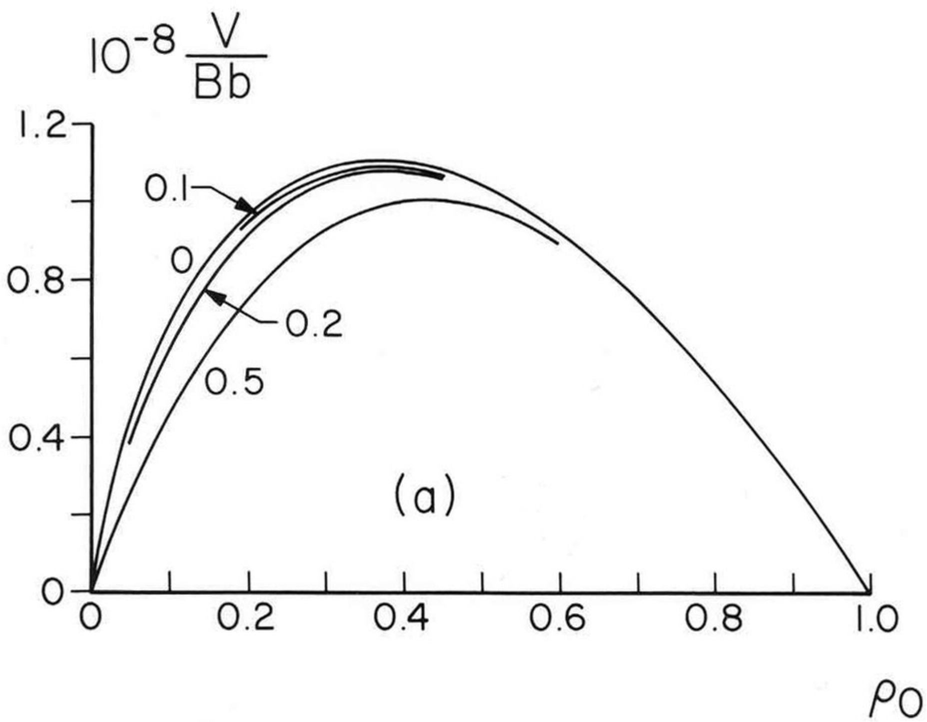
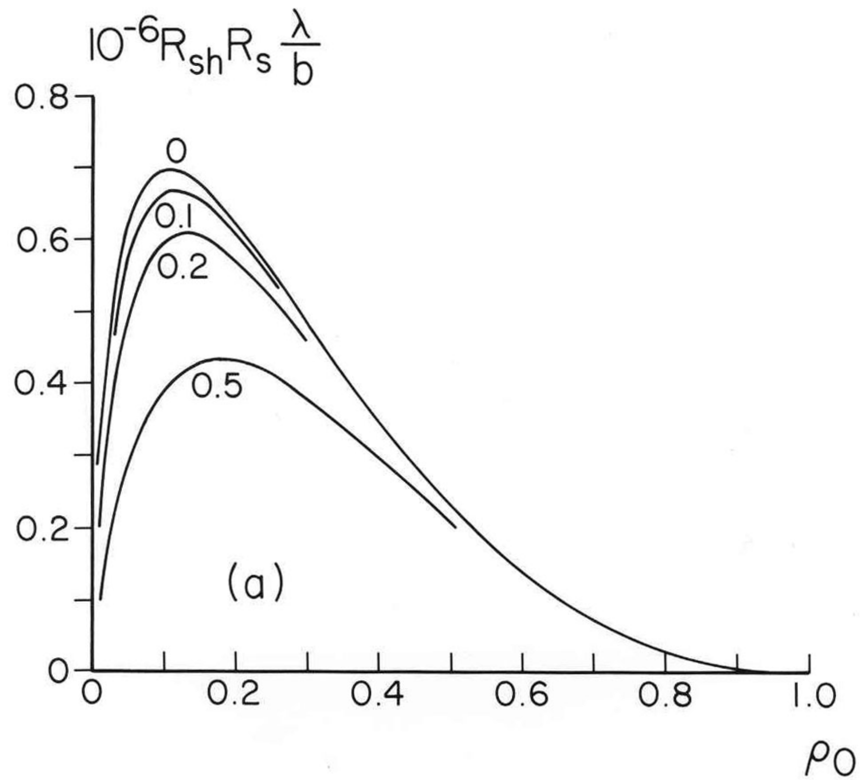
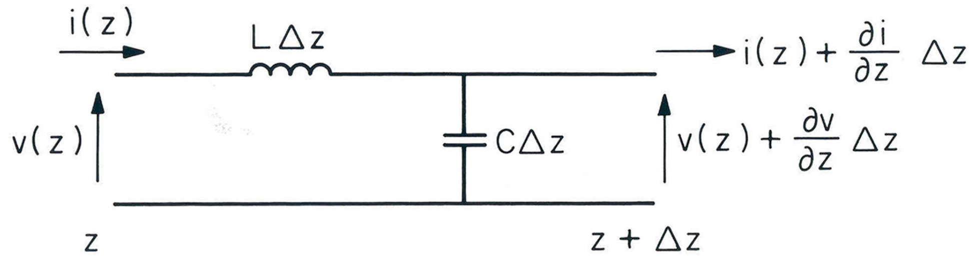




MKS units, lines of constant normalized loading capacitance $\Gamma/\lambda\varepsilon_0$

More Complicated Center Conductor Geometries



$$\frac{d^2 v}{d\zeta^2} - \frac{1}{\rho \ln \rho}\frac{d\rho}{d\zeta}\frac{dv}{d\zeta} + \frac{\pi^2}{4} v = 0$$

$$\frac{d^2 i}{d\zeta^2} + \frac{1}{\rho \ln \rho}\frac{d\rho}{d\zeta}\frac{di}{d\zeta} + \frac{\pi^2}{4} i = 0$$

$$\Gamma(z) = -C(z)\frac{i(z)}{di/dz}$$

More Complicated Center Conductor Geometries

Constant logarithmic derivative of line capacitance

Good model for linear taper

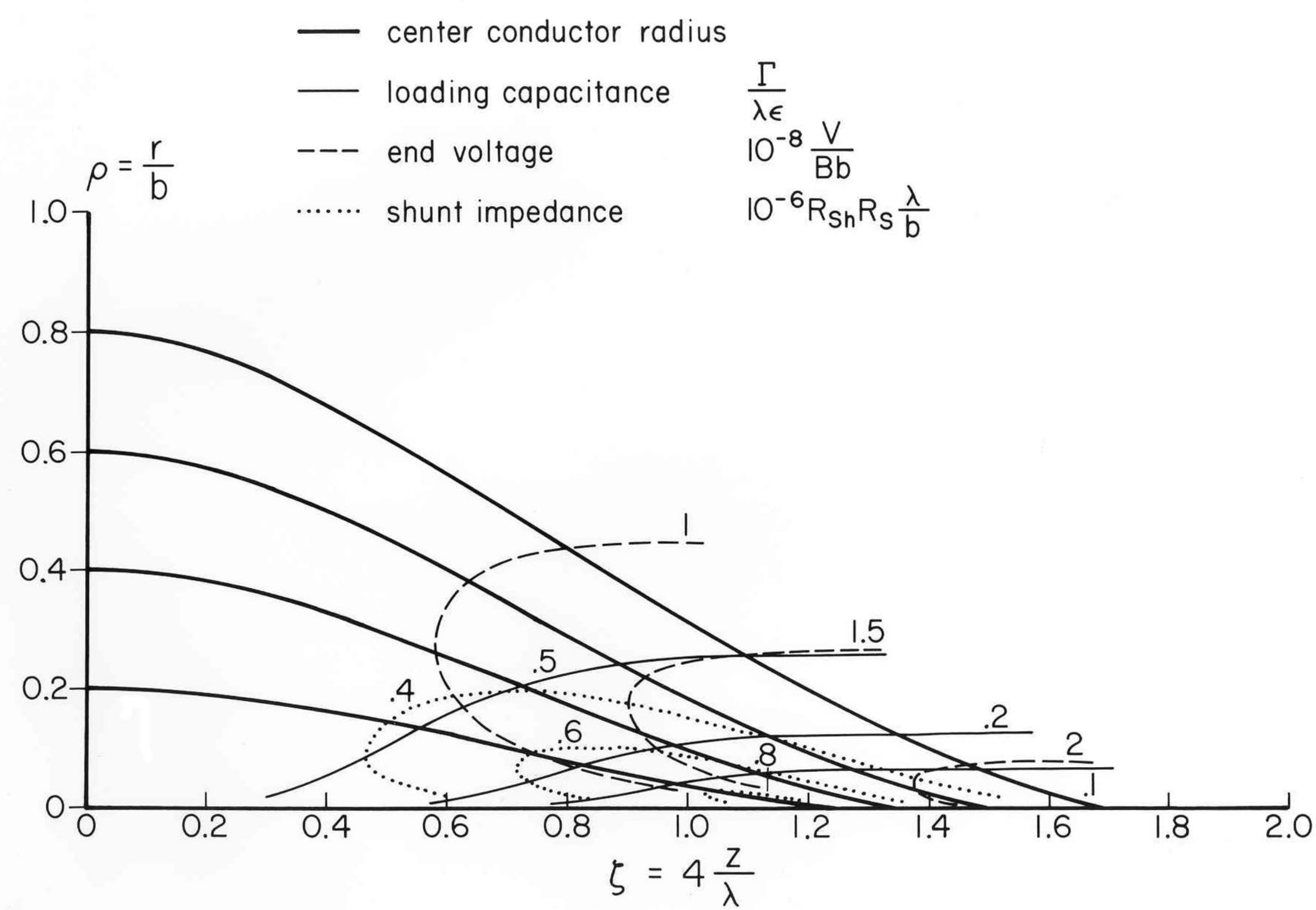
$$\frac{1}{C}\frac{\mathrm{d}C}{\mathrm{d}z} = -\frac{1}{d} \qquad\qquad r(z) = b\left(\frac{r_0}{b}\right)^{\exp(z/d)}$$

Constant surface magnetic field

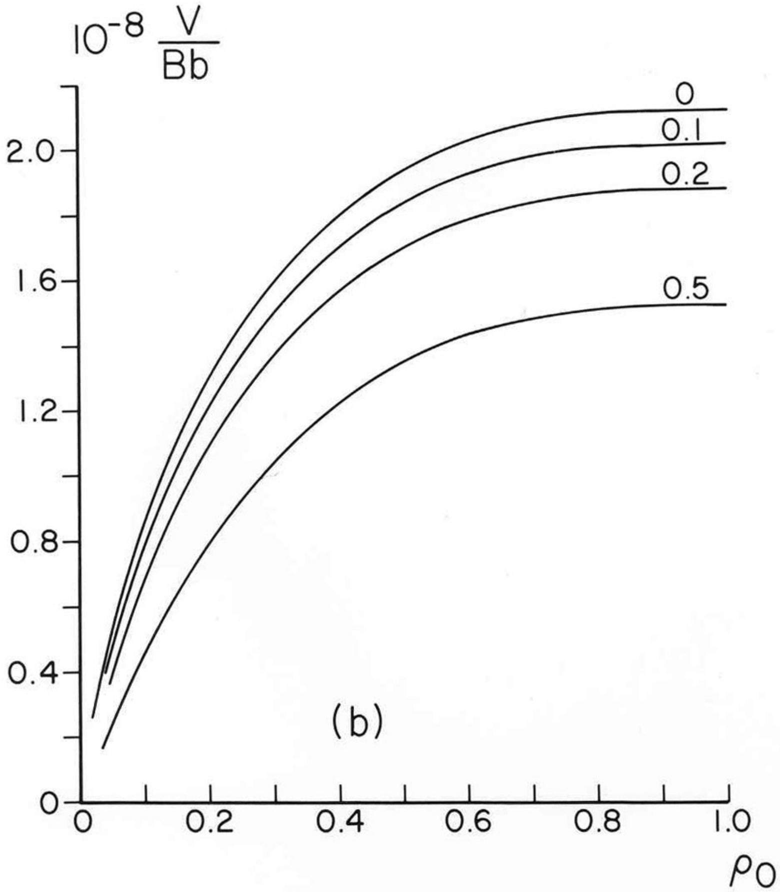
$$i(z) \propto r(z)$$

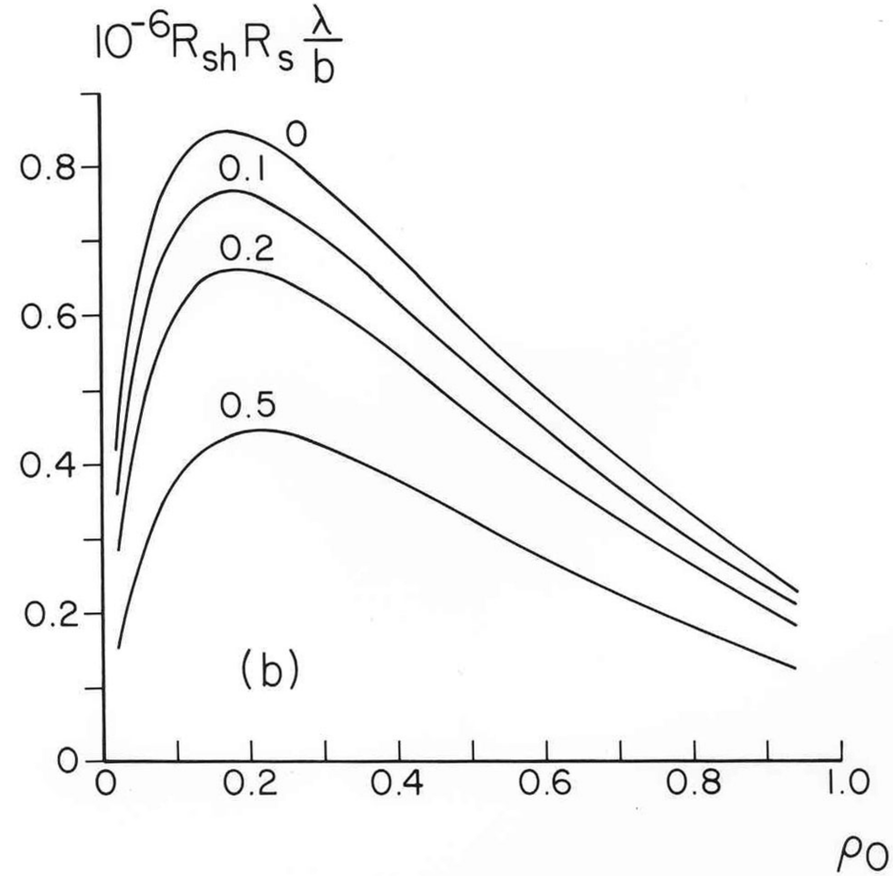
$$\frac{d^2 r}{dz^2} - \frac{1}{r\ln(b/r)}\left(\frac{dr}{dz}\right)^2 + \frac{4\pi^2}{\lambda^2} r = 0$$

Profile of Constant Surface Magnetic Field



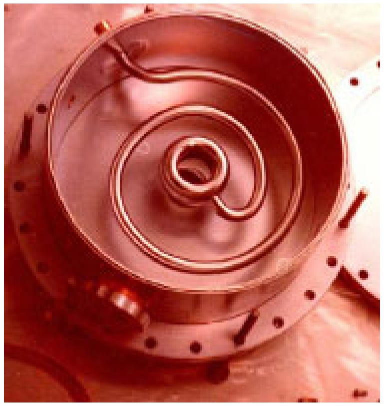
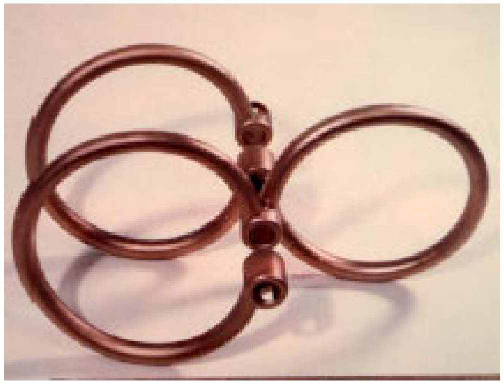
Profile of Constant Surface Magnetic Field

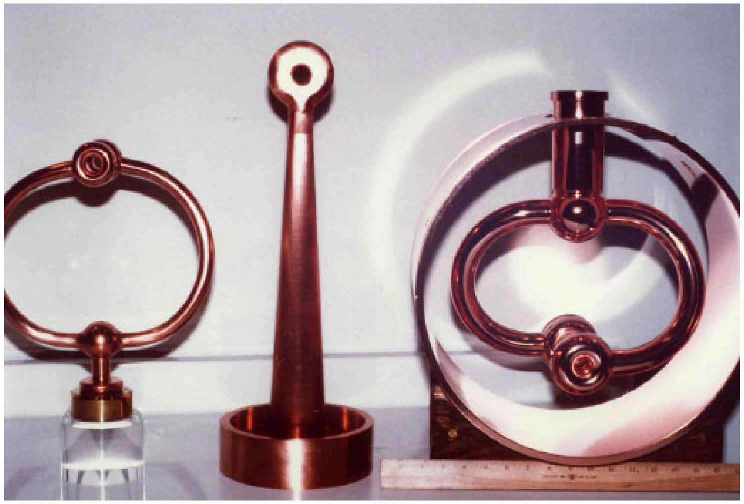


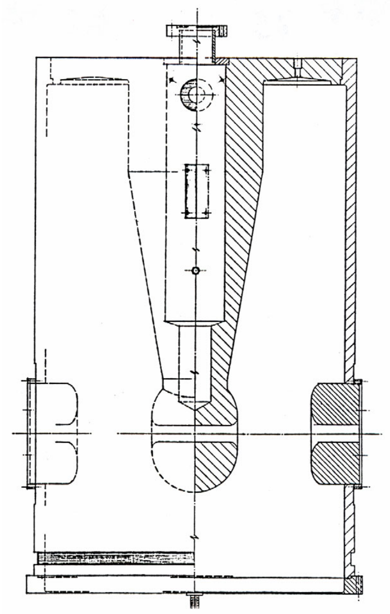


MKS units, lines of constant normalized loading capacitance $\Gamma/\lambda\varepsilon_0$

Some Real Geometries (λ/4)





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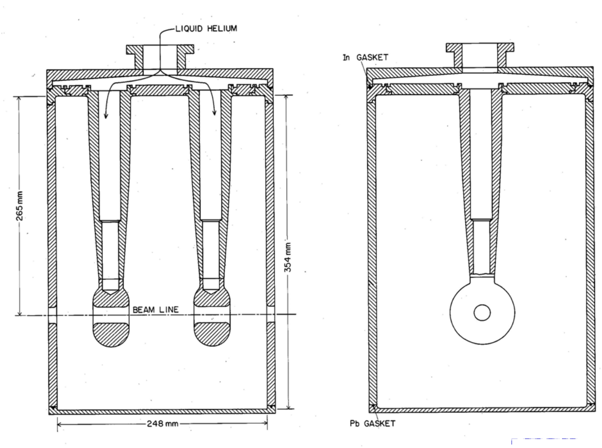
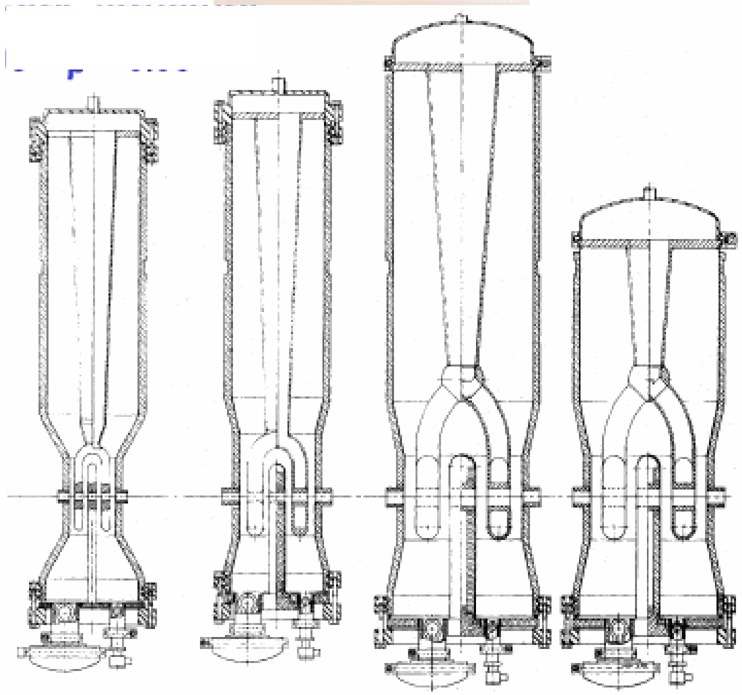
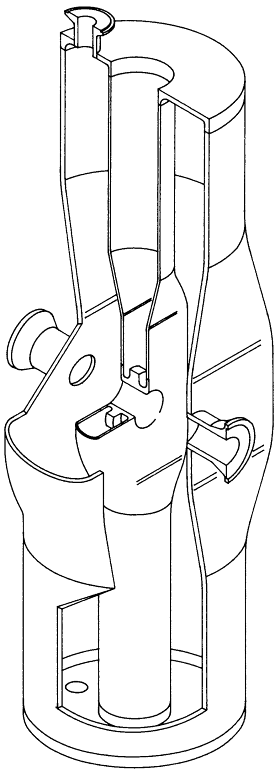
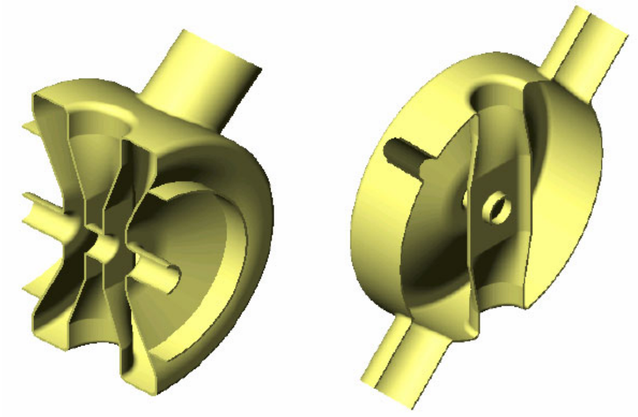
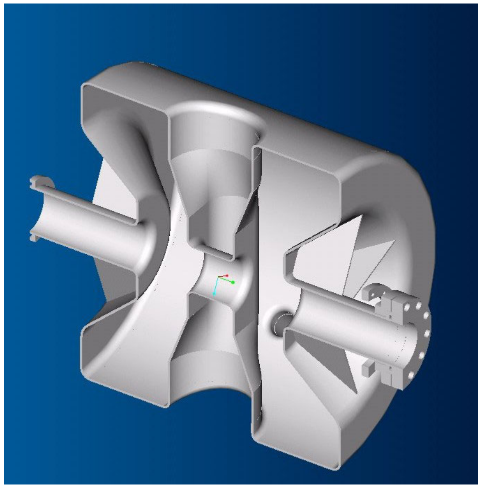




Some Real Geometries (λ/2)







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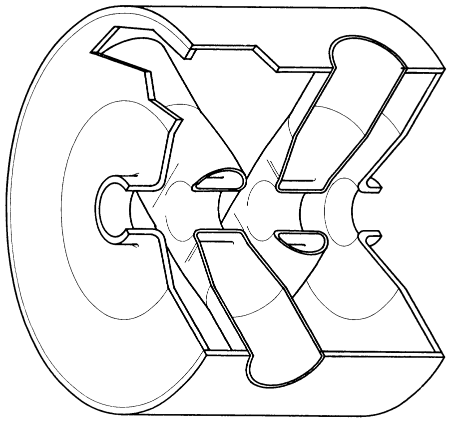

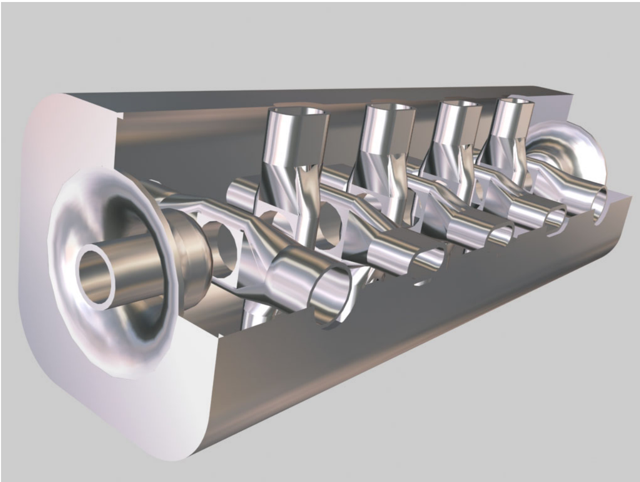

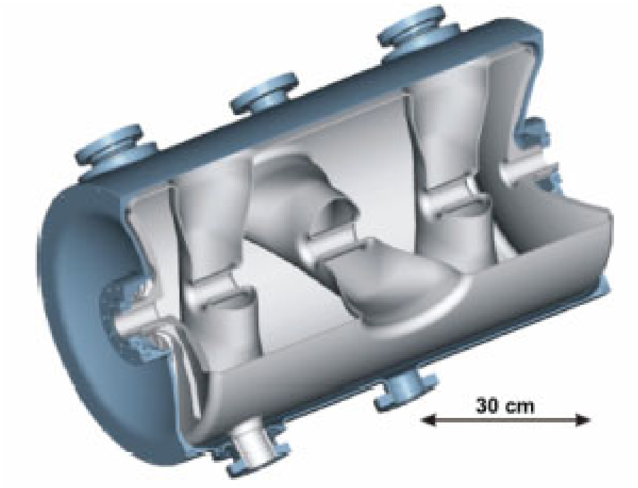

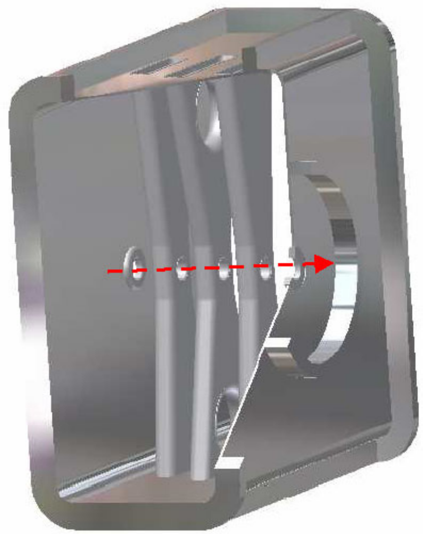

Parting Words

In the last 30+ years, the development of low and medium β superconducting cavities has been one of the richest and most imaginative area of srf

- **The field has been in perpetual evolution and progress**
- **New geometries are constantly being developed**
- **The final word has not been said**

The parameter, tradeoff, and option space available to the designer is large

- **The design process is not, and probably will never be, reduced to a few simple rules or recipes**
- **There will always be ample opportunities for imagination, originality, and common sense**