

## OPTIMIZED SHAPE SLIDING PHASE STRUCTURE FOR TESLA<sup>#</sup>

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### Abstract

Optimization of the cavity shape gives a possibility to increase the accelerating rate with the same maximal magnetic field as in the TESLA structures now. Some increase of the electric field is acceptable because the electric field is not a severe limit in the SC cavities. Usage of a sliding phase in the  $7 \div 9$  cells cavity gives additional possibilities to increase the accelerating rate and even to reduce the number of cells in the cavity keeping the same energy gain per cavity. The shorter cavity has smaller wakefields and allows smaller cell-to-cell coupling, so it can have smaller iris aperture that gives further possibility to increase the accelerating rate. Description of the proposed geometry, some results of the optimization and fabrication are presented. A considerable gain can be obtained by using this approach for the TESLA structure, about  $10 \div 20$  % in accelerating rate with the corresponding increase of the final energy, or reduction in the length.

### INTRODUCTION

The shape of the TESLA structure was thoroughly and repeatedly optimized [1] because the sizable cost of the superconducting linear collider is dominated by the accelerating structure. Both this optimization and the recent technological progress in surface preparation made possible to raise the accelerating rate up to 37 MeV/m [2].

Further increase of the accelerating field is limited now by magnetic losses whereas the electric field is still not a cause of considerable emission or breakdown.

This experimental fact is an incentive to revise the existing shape of the cavity cells with the aim to equalize the limits connected with both the electric and magnetic field.

To decrease the *electric* field in the iris area, having at the same time a high acceleration rate, one needs to make the iris shape elliptic. The same approach can be used for the equatorial region of the cavity when we have to decrease the *magnetic* field [3]. The elliptic dome of the cavity keeps the cavity not liable to multipactoring. The optimized shape becomes the re-entrant one that presents a technological challenge. However, first results of fabricating such a cell look promising.

One more idea is used here for further increase of the acceleration rate of the TESLA structure: a sliding phase structure (SPS) [4]. Its essence consists in the fact that the increase in length of the cell causes the loss of the energy

gain as the second order of the length increase because of the transit time factor. At the same time the increase of energy gain due to a longer interval where the field is close to maximum can be of the first order of the increase in the cell length.

### DEFINITIONS

For comparison of different shapes one can use the ratios of the peak electric and magnetic field strength on the cell surface to the acceleration rate achievable in the given cell:

$$\frac{E_{pk}}{E_{acc}} = \frac{E_{pk}}{\Delta W/L} = \frac{E_{pk}}{2\Delta W/\lambda}, \quad \frac{H_{pk}}{E_{acc}} = \frac{H_{pk}}{2\Delta W/\lambda}.$$

Here  $\Delta W$  is the energy gain (in volts) obtained at the cell length  $L$  that can be not equal to half wavelength (even for the  $\pi$ -mode). For the TESLA accelerating cavity these values are [4]:

$$E_{pk}/E_{acc} = 2.0, \quad H_{pk}/E_{acc} = 42 \text{ Oe}/(\text{MV}/\text{m}).$$

We will compare values of calculated fields with these values and introduce for this purpose the normalized peak electric and magnetic fields in a separate cell:

$$e' = \frac{E_{pk}}{2E_{acc}}, \quad h' = \frac{H_{pk}}{42E_{acc}}.$$

When considering the whole structure we introduce the transit time factor  $M$  and define one more pair of normalized fields:

$$e = \frac{E_{pk}}{2M \cdot E_{acc}}, \quad h = \frac{H_{pk}}{42M \cdot E_{acc}}.$$

The so defined transit factor  $M$  takes into account the “incorrect” phases of enter in and exit out of the cell, and is equal to 1 for the cells with length  $\lambda/2$  or for longer cells with the right phases of transit: maximal electric field when the particle is at the middle plane of the cell.

For the regular TESLA cells [4]

$$e' = e = 1, \quad h' = h = 1.$$

### TRANSIT TIME FACTOR

The transit time factor is usually defined as a ratio of the maximal voltage “seen” by an electron crossing the accelerating gap to the momentary amplitude voltage applied to this gap. We will consider *this* transit factor included into the corresponding formulae. Here we are interesting in additional factor that takes into account the phase shift of the electron connected with a longer cavity length.

Let us consider a periodic string of cells such that in the central cell the particle experiences a maximal possible

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voltage. For a symmetric cell it means that it is in the middle of the cell when the voltage is maximal.

Field in the middle three cells of such a string can be written as

$$E(z) = \begin{cases} -E_0(z+L), & -3L/2 \leq z \leq -L/2 \\ E_0(z), & -L/2 \leq z \leq L/2 \\ E_0(z-L), & L/2 \leq z \leq 3L/2, \end{cases}$$

and the energy (in volts) gained by the particle in the middle cell is

$$U_0 = - \int_{-L/2}^{L/2} E_0(z) \cos kz dz.$$

The energy gain in the neighbor cell with negative values of  $z$ , is

$$\begin{aligned} U_{-1} &= \int_{-3L/2}^{-L/2} E_0(z) \cos kz dz = \int_{-L/2}^{L/2} E_0(\zeta) \cos k(\zeta - L) d\zeta = \\ &= -\cos \Psi \int_{-L/2}^{L/2} E_0(\zeta) \cos k\zeta = \cos \Psi \cdot U_0. \end{aligned}$$

$\Psi$  is a phase slide for one cell and can be defined from the expression

$$L = \frac{\pi + \Psi}{\pi} \cdot \frac{\lambda}{2}.$$

In a similar way, we can find that for the  $n$ -th cell if the central one is counted as the 0-th, the energy gain is

$U_n = U_0 \cos n\Psi$ . So, the additional transit factor for the  $n$ -th cell is

$$M'_n = \cos n\Psi.$$

The additional transit factor for the whole string of  $N$  cells, when  $N$  is an odd number, is

$$M_N = \frac{1}{N} \left( 1 + 2 \cdot \sum_{n=1}^{(N-1)/2} \cos n\Psi \right);$$

for an even  $N$ , it is easy to obtain

$$M_N = \frac{2}{N} \cdot \sum_{n=1}^{N/2} \cos \left( n - \frac{1}{2} \right) \Psi.$$

## OPTIMIZATION FOR A SLIDING PHASE

As it was shown in [3], by optimization of the cell shape we can decrease the magnetic field sacrificing the electric field and having the same acceleration per cell. It is more convenient to compare different structures having the same, for example, maximal magnetic field and trying to decrease the electric field.

For optimization we used as before [3] construction of the profile line of the cell by two conjugated ellipses. Such a simple shape gives 3 only independent variables (half-axes) for optimization for a given cell length and aperture radius. The fourth half-axis becomes predetermined by the cell frequency.

We are going to propose less number of cells per structure, 8 instead of 9, so it is reasonable to try to keep the same acceleration in this shorter cavity.

We will search for smaller as possible electric peak surface field in the optimized cavity under

abovementioned conditions. Let us look for this minimum with the same value of the aperture radius as in TESLA inner cells, and with smaller aperture if we will have a sufficient value of the cell-to-cell coupling.

We will have in this case

- shorter structure, of 8 not of 9 cells that is cheaper,
- presumably less wake fields,
- additional space: shorter accelerator or higher final energy.

To keep the same energy gain that in original TESLA structure, we should have

$$E_{acc} = \frac{9}{8} \frac{\pi}{\pi + \Psi} E_{acc,0}.$$

Here  $E_{acc}$  is the average acceleration field in the 8-cell cavity and  $E_{acc,0}$  - in the original cavity. It follows from definitions that

$$E_{acc} = \frac{E_{acc,0}}{h}.$$

Two last equations and definitions above give us

$$h' = \frac{8M_8(\Psi) \cdot (\pi + \Psi)}{9\pi} \text{ and } E_{pk} = \frac{e'}{h'} E_{pk,0}.$$

The function  $h' = h'(\Psi)$  has its maximum at  $\Psi = 3.4^\circ$ ,  $h'_{max} = 0.8973$ , and for minimizing  $E_{pk}$  we need to find minimum of  $e'$  under condition  $h' \leq h'_{max}$ . (Strictly speaking, we need to search  $\min(e'/h')$ , but it appeared that the minimum doesn't shift substantially from  $3.4^\circ$ .)

For comparison, the case of  $\Psi = 0$  was also considered for 8 cells. In this case  $\min(e')$  was searched under condition  $h' \leq 0.8889$ .

Results of calculations are presented in the Table 1 and the original and optimized shapes are shown in Fig. 1.

One can see that in the original TESLA cavity with the accelerating rate of 37 MeV/m the energy gain per cavity is 38.4 MeV and the peak surface magnetic field is 155.4 mT. We can have the same energy gain and magnetic field with a shorter by 9.4 % structure (so that  $E_{acc}$  increases by  $1/0.906 = 10.4$  %) and 13 % higher peak electric field using the optimized shape, the sliding phase and a smaller aperture. The smaller aperture leads to a smaller cell-to-cell coupling but the value of  $N^2/k$  doesn't increase. This is precisely the value responsible for sensitivity of field profile to an individual cells frequency perturbation. We didn't yet perform the analysis of HOMs distribution but it is known that a large number of cells favors some trapped modes. The smaller aperture leads to a smaller equatorial radius that shifts the whole HOM spectrum to higher values that is also favorable.

A change to an 8-cell optimized structure with the same energy gain and magnetic field leads to increase of the electric peak surface field by 42 %. Usage of smaller aperture decreases this value by 17-18 %. Further decrease, 3-5 %, adds the sliding phase (5 % for bigger aperture). It should be added that the optimized cells have

by 13 % higher value of  $R/Q$ , and by 3 % higher value of the geometry factor  $G$  that decreases RF losses and facilitate cooling of the cavity.

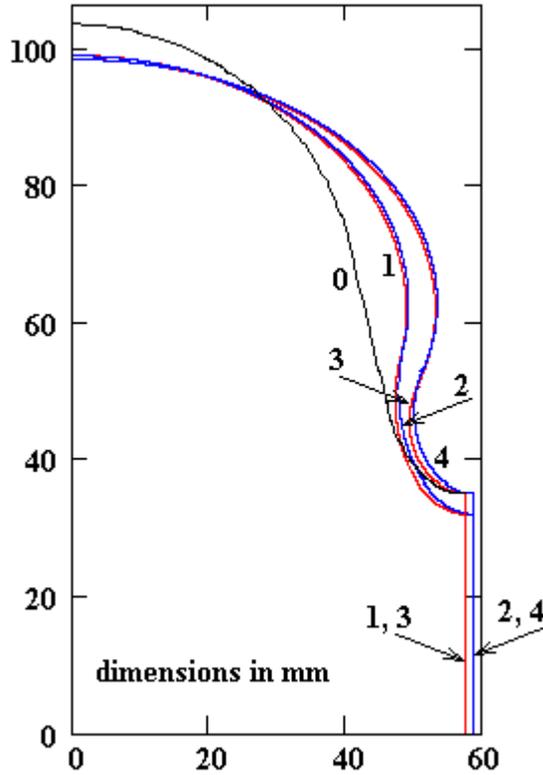


Fig. 1. Original TESLA cell shape (0) and optimized shapes: 1, 3 -  $\Psi = 0$ ; 2, 4 -  $\Psi = 3.4^\circ$ ; 1, 2 -  $R_a = 32$  mm,, 3, 4 -  $R_a = 35$  mm.

The optimization of end cells should be done separately, their effect is assumed to be the same as of the inner cells in this consideration. However, one can correct the influence of the phase shift in the end cells and

improve on this way the general features of the structure. Generally speaking, the end cells have one additional free parameter for optimization: their length.

### FABRICATION

The reentrant shape is a challenge for fabrication. However, two cups were successfully formed by deep drawing 3 mm thick RRR300 sheet Nb, annealed (1200°C, 4 hours, that increased its quality to RRR500), and welded to beam tubes, Fig. 2. We plan to electropolish them before the final equatorial weld.

The re-entrant cavity surface area is bigger, it will need 3 % more material for each cell but less number of cells gives more economy both in material and work.

The mechanical strength of the re-entrant cavity will need more attention because this shape is more vulnerable to Lorentz force detuning.

### CONCLUSIONS

At the cost of 13 % increase of the peak surface electric field, a number of the present TESLA cavity parameters can be improved. The main improvement is a change to an 8-cell cavity that makes its fabrication substantially cheaper. For the same energy gain per cavity and the same peak surface magnetic field, the length of the cavity will be decreased by 9 %, its geometric shunt impedance,  $R/Q$ , increased by 13 %, and geometry factor increased by 3 %. The proposed cavity cells are optimized using the elliptic shape of both electric and magnetic regions, smaller aperture radius justified by a smaller number of cells, and by using a new feature of a sliding phase.

Calculations show that the new shape should be free of multipacting.

The change of the shape leads to some technological complications. Re-entrant cups were successfully formed, heat-treated and prepared to electropolishing and welding.

The total number of cryomodules in the TESLA-800 is 2628 with 12 cavities in each of them. Our proposal is to shorten each cavity by 98 mm that gives for the total accelerator 3.1 km.

Table 1. Comparison of the original TESLA structure with the optimized structure without and with a sliding phase.

Number of cells	N = 9 (present)	N = 8 (optimized)			
		$\Psi = 0$		$\Psi = 3.4^\circ$	
Phase slide per cell	0				
Aperture, mm	$R_a = 35$	$R_a = 35$	$R_a = 32$	$R_a = 35$	$R_a = 32$
Gain per cavity, $W$ , MeV		38.4			
$H_{pk}$ , Oe		1554			
$E_{pk}$ , MV/m	74	105.9	86.6	100.5	83.7
$E_{acc}$ , MV/m	37	41.6	41.6	40.9	40.9
Cavity length, $L$ , mm	1038	922	922	940	940
$R/Q$ , Ohm (per cell)	114.5	120.8	128.0	122.2	129.4
$G$ , Ohm	271.0	285.3	278.6	287.2	280.4
$G \cdot R/Q$ , Ohm <sup>2</sup>	31030	34470	35650	35100	36270
Cell-to-cell coupling, $k$ , %	1.87	2.63	1.65	2.50	1.56
Coupling normalized to 9 cells, $k \cdot (9/8)^2$	-	3.33	2.09	3.16	1.97



Fig. 2. 1300 MHz re-entrant cavity cups.

### REFERENCES

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