NONLINEAR EFFECTS IN THE TTX-LESR*

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Abstract

We are designing a Laser Electron Storage Ring (LESR) for Tsinghua Thomson scattering X-ray source (TTX). A very compact LESR is designed to increase the average flux of the generated X-ray photons through the way of increasing the repetition frequency of scattering. Considering to reduce the lengths and the number of the magnets is necessary in designing a small ring. However, we find that the nonlinear effects of dipole field become significant when the bending radii of the dipole magnets are small. In this paper, we first present the baseline design of the LESR for TTX. Both analytical analysis and simulation are carried out to study the particles' behavior around the third-order systematic resonance line $3\nu_x = 4$. The analytical results of the resonance strengths are found to agree well with results from numerical particle tracking.

INTRODUCTION

In the inverse Compton scattering (ICS) process, energy can be transferred from high energy electrons to low energy photons. Higher energy photons can be therefore generated, e.g., the head-on collision between 50 MeV electrons and 800 nm laser pulses can produce hard X-ray photons up to about 59 keV. However, the cross section of ICS is small [1] which is a limitation of this kind of X-ray source. To increase the flux of X-ray photons, Huang and Ruth proposed to use a compact ring for storing electron beams and an optical cavity for storing the laser pulses [2]. In this scheme, the scattering happens every turn. The repetition frequency of the interaction between electrons and laser is determined by the revolution frequency of the electron beam in the ring, of the order of tens of MHz.

Currently, TTX [3] consists of an S-band photocathode RF gun, an S-band 3 m linac, and a TW laser system. We propose a LESR in order to extend the capability of TTX. A four-mirror Fabry-Perot cavity is designed for TTX-LESR to store laser pulses inside [4]. In this paper, we focus on the study of the ring with the circumference of 4.8 m, consisting of 4 dipoles and 2 quadrupole magnets. The nonlinear effects induced by dipole field are studied based on this lattice. We organize this paper as follows. First, we show the baseline design of this 4.8 m ring. The basic parameters are also shown in this section. Then, we carry out the study of particles' motion influenced by the $3\nu_x = 4$ resonance. The existence of nonlinearity of dipole field is proved by the derivation. Strong evidence is also observed in the simulation. The discussion of the study is given at the end of this paper.

BASELINE DESIGN OF THE RING

Four dipoles with nonzero edge angles make up the basic lattice of the TTX-ring. Horizontal focusing is provided by the main body of the dipoles and vertical focusing comes from the edge angles of them. The length of each dipole is 0.4 m, corresponding to the bending radius 0.2546 m. Two quadrupoles are located at the centers of the two long straight sections. The optics of baseline design is shown in Figure 1 [5]. In the baseline design, the edge angles at both the entry and the exit of each dipole magnet are 37°. The vertical gap of the dipole magnets is chosen as 2.54 cm. Only the first order fringe field effect is included via K. L. Brown's theory. The FINT parameter is 0.9 in the calculation. The quadrupole focusing strength K_1 is 30 m^{-2} while the length of each quadrupole is 0.1 m.

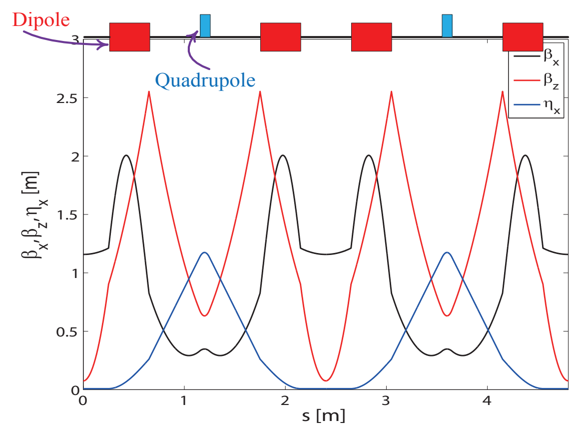

Figure 1: The beta functions and dispersion function in the baseline design of the 4.8 m ring.

The betatron tunes in the baseline design are $\nu_x = 1.21$ and $\nu_z = 1.28$. The horizontal chromaticity and vertical chromaticity are about -2.1 and -1.0, respectively. The length of each short straight section is 0.5 m. The RF cavity is located at one of them, and in the mean time, the optical cavity is located at the other one. The schematic of the ring is illustrated in Figure 2.

STUDIES OF NONLINEAR EFFECTS

The Frenet-Serret coordinate system (as shown in Figure 3) is utilized in the analysis in this section. The Hamiltonian in Frenet-Serret Coordinate System is

$$\widetilde{H} = -(1+\frac{x}{\rho})[\frac{(H-e\phi)^2}{c^2} - m^2c^2 - (p_x - eA_x)^2$$

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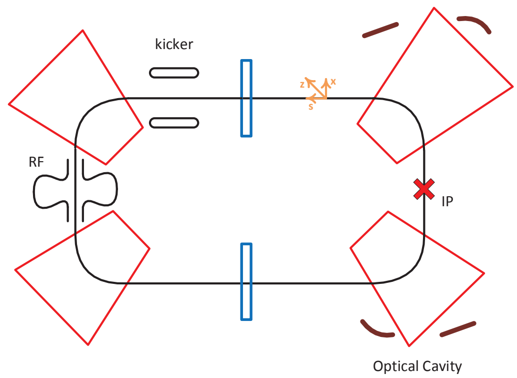

Figure 2: Schematic top-view of the 4.8 m ring.

The Frenet-Serret coordinate system is expressed as(x, s, z)in this Figure(z axis is perpendicular to the paper). The RF cavity and the interaction point (IP) are located at the center of the short straight sections

$$-(p_z - eA_z)^2]^{1/2} - eA_s \tag{1}$$

where the phase-space coordinates are $(x, p_x, z, p_z, t, -H)$. The total energy and momentum of the particle are $E = H - e\phi$ and $p = \sqrt{E^2/c^2 - m^2c^2}$. Since the transverse conjugate momenta p_x and p_z are much smaller than the total momentum, we expand the Hamiltonian up to second order in p_x and p_z

$$\begin{aligned} \widetilde{H} &\approx \frac{1+x/\rho}{2p}[(p_x - eA_x)^2 + (p_z - eA_z)^2] \\ &\quad -p(1+\frac{x}{\rho}) - eA_s \end{aligned} \tag{2}$$

where $A_x = A_z = 0$ in a normal circular accelerator which contains transverse magnetic field only.

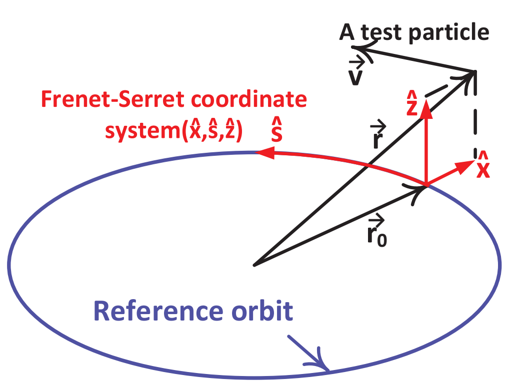

Figure 3: Frenet-Serret coordinate system for particle motion in a circular accelerator. $\hat{x}$, $\hat{s}$, and $\hat{z}$ form the basis of the Frenet-Serret coordinate system. $\hat{x}$ indicates the horizontal direction. $\hat{z}$ indicates the vertical direction.

Disregarding the synchrotron motion, Hamilton's equations of betatron motion are

$$x' = \frac{\partial \widetilde{H}}{\partial p_x},\ p_x' = -\frac{\partial \widetilde{H}}{\partial x},\ z' = \frac{\partial \widetilde{H}}{\partial p_z},\ p_z' = -\frac{\partial \widetilde{H}}{\partial z} \tag{3}$$

We can therefore get:

$$\begin{aligned} x' &= \frac{\partial \widetilde{H}}{\partial p_x} = \frac{1+x/\rho}{p}p_x \\ p_x' &= -\frac{\partial \widetilde{H}}{\partial x} = -\frac{p}{\rho} - \frac{p_x^2 + p_x^2}{2p\rho} + e\frac{\partial A_s}{\partial x} \\ z' &= \frac{\partial \widetilde{H}}{\partial p_z} = \frac{1+x/\rho}{p}p_z \\ p_z' &= -\frac{\partial \widetilde{H}}{\partial z} = e\frac{\partial A_s}{\partial z} \end{aligned} \tag{4}$$

If there are dipole magnets and quadrupole magnets only, the magnetic field is expanded up to the quadrupole component only (ignoring the nonlinear field components in the fringe field region of the dipole and quadrupole magnets). which means:

$$\begin{aligned} \frac{\partial A_s}{\partial z} &= -h_s B_x = -(1+\frac{x}{\rho})B_x = -B_1 z \\ \frac{\partial A_s}{\partial x} &= h_s B_z = (1+\frac{x}{\rho})B_z \\ &= B_0 + B_0\frac{x}{\rho} + B_1 x \end{aligned} \tag{5}$$

The obtained equations of betatron motion are

$$\begin{aligned} x'' &= \left[-\frac{1}{\rho^2} + \widetilde{p_x}^2\frac{1}{2\rho^2} - \widetilde{p_z}^2\frac{1}{2\rho^2}\right]x \\ &\quad + \left[\widetilde{p_x}^2\frac{1}{2\rho} - \widetilde{p_z}^2\frac{1}{2\rho}\right] - \left[\frac{1}{\rho^3}\right]x^2 + \frac{B_1}{B\rho}x \\ z'' &= \widetilde{p_x}\widetilde{p_z}\left(\frac{1}{\rho} + \frac{x}{\rho^2}\right) - \frac{B_1}{B\rho}z \end{aligned} \tag{6}$$

where $\widetilde{p_x} = p_x/p$ and $\widetilde{p_z} = p_z/p$ indicating the normalized horizontal momentum and normalized vertical momentum, respectively. The equations of betatron motion shown in Eq. 6 contain the nonlinear terms introduced by dipole magnets. We can therefore build a new Hamiltonian from these equations.

$$\begin{aligned} H_{\text{new}} &= \left[\frac{1}{2}\left(\frac{1}{\rho^2} - \frac{B_1}{B\rho}\right)x^2 + \frac{1}{2}\widetilde{p_x}^2 + \frac{1}{2}\frac{B_1}{B\rho}z^2 \right. \\ &\quad \left. + \frac{1}{2}\widetilde{p_z}^2\right] + \frac{1}{2}\frac{x}{\rho}\left(\widetilde{p_x}^2 + \widetilde{p_z}^2\right) \end{aligned} \tag{7}$$

We can then carry out the coordinates transformation. Convert the new Hamiltonian into new action-angle variables and expand it around the third order resonance $3\nu_x = 4$. The approximated Hamiltonian under this condition can be expressed as

$$\begin{aligned} H &\approx \nu_x J_x + \nu_z J_z \\ &\quad + g_{3,0,3,0,4} J_x^{3/2} \cos(3\phi_x - 4\theta + \xi_{3,0,3,0,4}) \end{aligned} \tag{8}$$

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where $\xi_{3,0,3,0,4}$ is the phase, the resonance strength $g_{3,0,3,0,4}$ can be expressed as

$$g_{3,0,3,0,4} \cdot e^{j\xi_{3,0,3,0,4}} = \frac{\sqrt{2}}{8\pi} \oint \frac{\alpha_x^2 - 1 - 2j\alpha_x}{\rho \beta_x^{1/2}} e^{j[3\chi_x(s) - (3\nu_x - 4)\theta]} ds \quad (9)$$

In order to use particle tracking method to study the particles' behavior around the resonance line $3\nu_x = 4$, we first have to move the bare horizontal tune closed to it. By adjusting the focusing strength of the two quadrupole magnets and the edge angles of dipole magnets, we can change the betatron tunes. For example, when we set the edge angle of dipole magnets as 29° and the focusing strength of quadrupole magnets at $K_1 = 4$ [m^{-2}], the horizontal tune and vertical tune will be 1.2783 and 1.5334, respectively. Since the horizontal tune is closed to $4/3$, we predict that the particles' motion under this condition will be dominated by the third-order resonance line $3\nu_x = 4$.

We then put several particles with different initial horizontal deviations at the beginning of the lattice shown in Figure 4, which indicates the center of one of the straight sections. The particles have zero horizontal momentum initially. In the tracking process, the particles' coordinates in horizontal phase space (x, p_x) are recorded turn by turn. By observing the Poincaré map in the normalized horizontal phase space shown in Figure 5, we can find the triangle shape distortion which indicates that the particles' motion are dominated by a 3^{rd} order resonance line ($3\nu_x = 4$). By converting the turn-by-turn data from the normalized horizontal phase space (X, P_x) to proper action-angle variables (J, ϕ), we can fit the Hamiltonian tori of the tracking data points to obtain the corresponding resonance strength. The fitted result is $g_{3,0,3,0,4,fitted} = 1.1079$ $[(\pi \mathrm{m})^{-1/2}]$, which agrees well with the analytical result $g_{3,0,3,0,4,analytical} = 0.8685$ $[(\pi \mathrm{m})^{-1/2}]$.

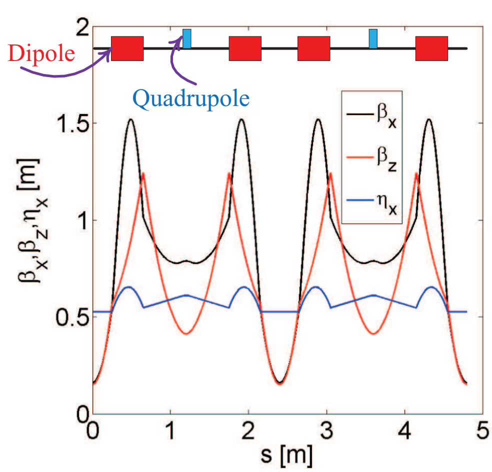

Figure 4: The optics used in the tracking for studying the resonance $3\nu_x = 4$.

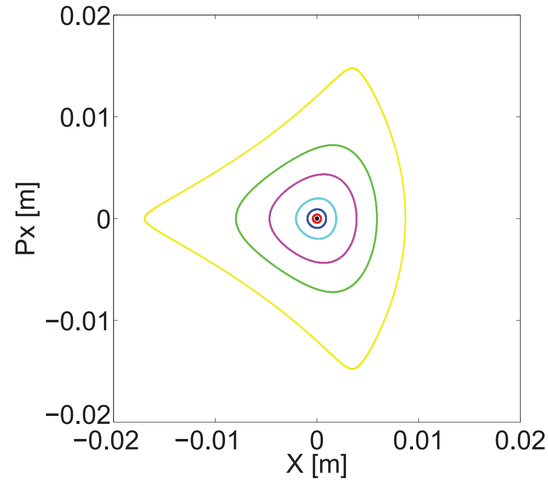

Figure 5: This plot shows the Poincaré map of the normalized phase space coordinates (X, Px) of betatron motion near the 3^{rd} order resonance $3\nu_x = 4$ at the 4.8 m ring.

DISCUSSION

In this paper, we point out that the nonlinear effects induced by the dipole field is remarkable when the bending radius of dipole magnets is small. We obtain the expression of the resonance strengths of the 3^{rd} order resonance $3\nu_x = \ell$, which are considered as the most significant low order nonlinear resonances induced by dipole magnets, via the Hamiltonian approach. To compare with the analytical results, we apply particle tracking method. By transforming the particles' coordinates from the horizontal phase space (x, p_x) to the proper action-angle variables (J, ϕ), we can obtain the resonance strengths through fitting the tracking data. With this method, we show that the fitted resonance strengths of the 3^{rd} order resonances $3\nu_x = 4$ agree with the analytical results well. However, the systematic studies still need to be carried out to demonstrate the analytical analysis under different conditions. We will continue to work on this problem in the future.

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