New Method for Point-Charge Wakefield Calculation

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Office of Science
U.S. DEPARTMENT OF ENERGY

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References and Outline

- **Motivation**
- **New Method of Short-Bunch Wake Calculations**
  - **Singular wake models**
  - **Basic idea of the method (for step-out)**
  - **Parameter λ_g**
  - **How to apply step-by-step**
- **Illustrative Examples**
  - **Simple cavity**
  - **NSLS-II Harmonic Cavity**
  - **3D collimator**
- **Summary**

This work is done in collaboration with G. Stupakov (SLAC)

References

- *BP, GS, PRST-AB 16, 024401 (2013),*
  DOI: 10.1103/PhysRevSTAB.16.024401

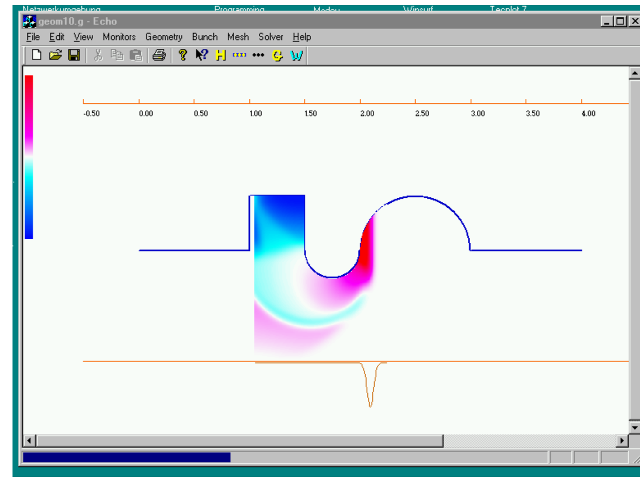
  Longitudinal wakes for 2D structures

- *BP, GS, WEODB1, this conference*

  Extension to transverse wakes

Motivation

- Knowledge of wakefields, incl. geometric ones, is critically important for accelerator beam dynamics.
- Detailed wakefield calculations for realistic vacuum chambers are done with time domain EM solvers, which calculate the fields due to finite length bunches.
- Extremely fine meshes are needed to compute wakes at small distances, where wake <u>singularities</u> dominate => calc's are slow and lots memory is req'd.

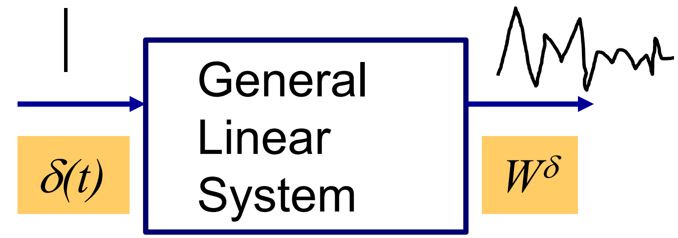


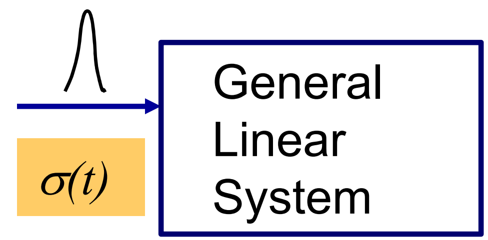
Electromagnetic
Code for
Handling
Of
Harmful
Collective
Effects

We suggest how to calculate short bunch wake-potentials, and even point-charge wakefields, from EM solver results for a long bunch. This saves greatly on calculation speed and provides physics insights.

Point-Charge Wakes from Finite Length Bunches – Is it even Possible?

- Can one get a δ-function impulse response, W^{δ}, using a finite duration Gaussian input, $\sigma(t) \sim \exp(-t^2/2\sigma^2)$?
- Or, equivalently, a frequency response, $Z(\omega)$, over infinite freq. range with finite BW excitation?





  - No, if the system is a black-box.
  - Yes, if the system is a gray-box, for example a set of harm. oscillators with (all normal modes) $\omega_n < \omega_{max}$

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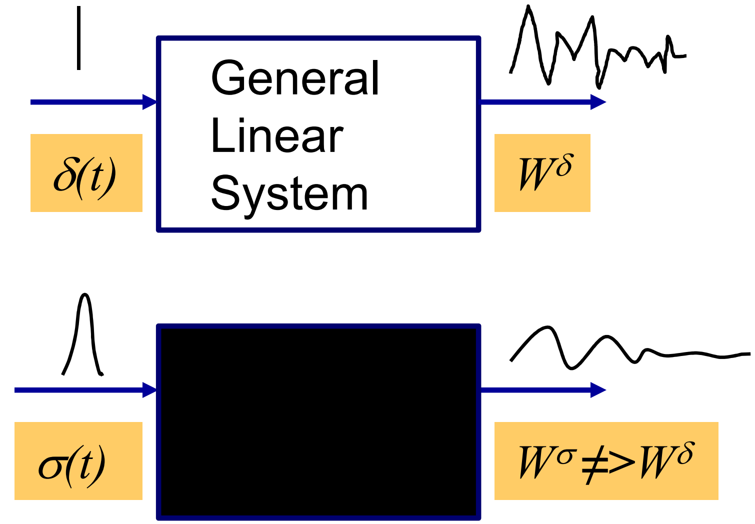

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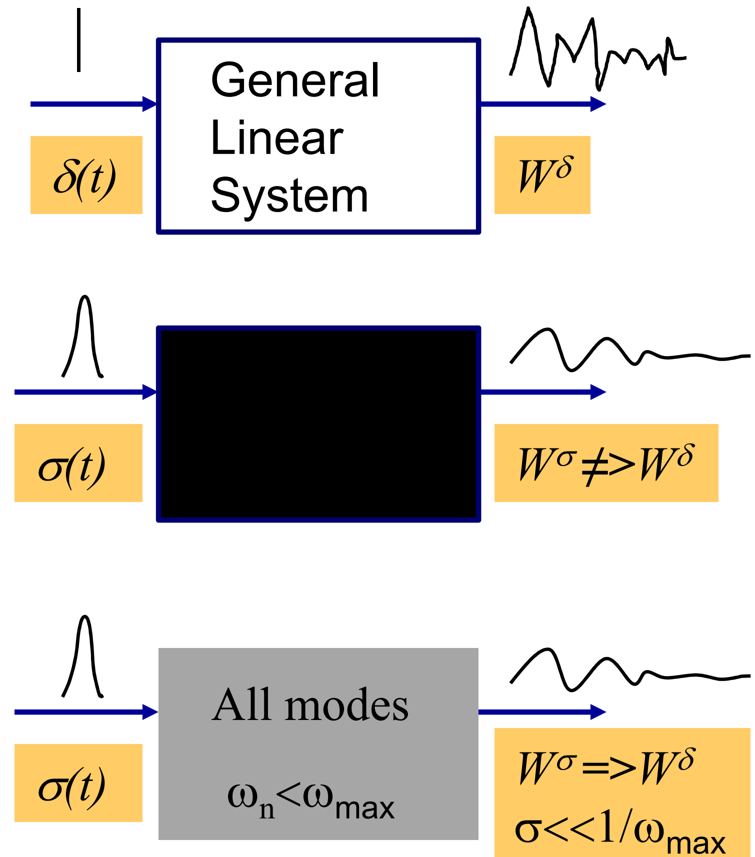

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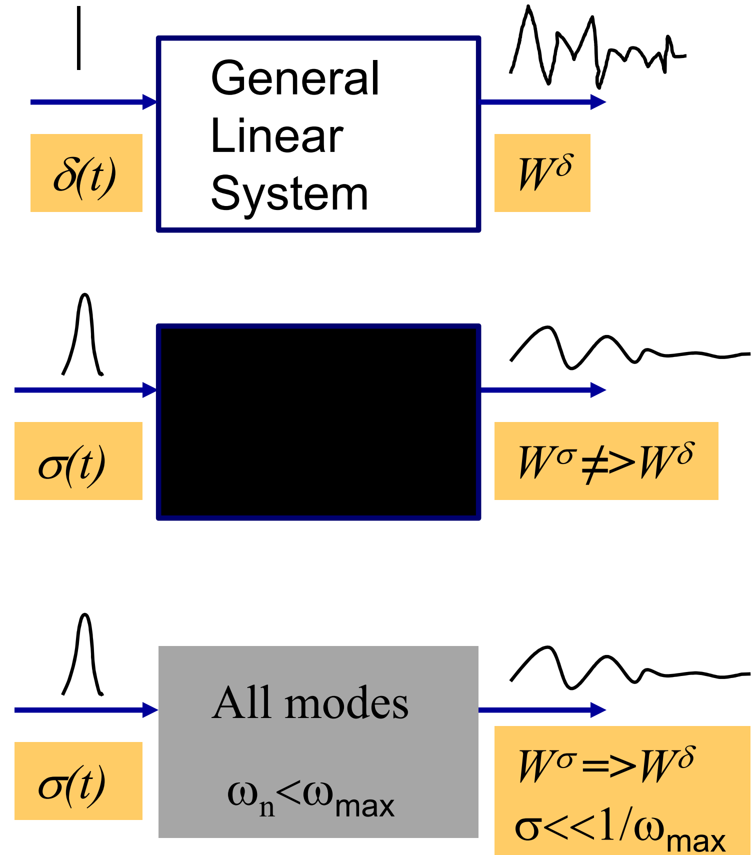
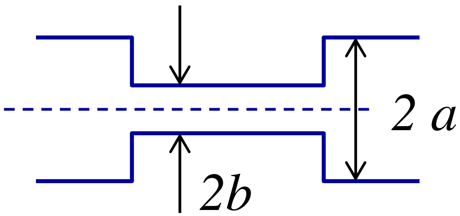


We claim that the problem of geometric impedance is similar to "gray box", since ω_{max}, as well as $Z(\omega \rightarrow \infty)$ asymptotic are known. Similarly true in time-domain for point-charge wakes.

Asymptotic Model for Short-Bunch Wakefields of Collimator-like Structures

- For collimator-like structures use the optical model:



$$W^{\delta}_{opt}(z) = k_{opt}\,\delta(z)$$

wake-function

$$W^{\sigma}_{opt}(z) = k_{opt}(2\pi)^{-1/2}\sigma^{-1}e^{-\frac{z^2}{2\sigma^2}}$$

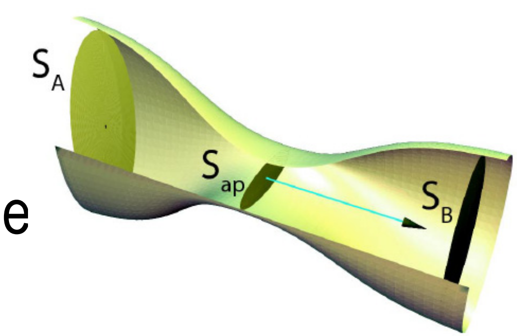
wake-potential

$$k_{opt} = -Z_0\,c\,Ln(a/b)/\pi$$

$$W^{\delta}_{opt}(z \to 0) = \infty$$

$$W^{\sigma \to 0}_{opt}(z) = \infty$$

- Turns out this model describes all collimator-like structures, including 3D; A recipe to calculate geometry-dependent k_{opt} exists [see Stupakov, Bane Zagorodnov, PRST-AB 10, 054401 (2007)]



Asymptotic Model for Short-Bunch Wakefields of Cavity-like Structures

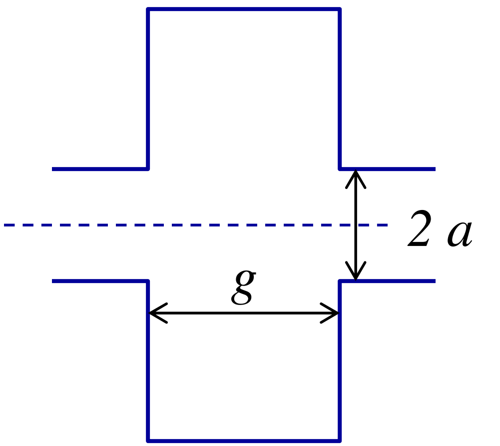
- For cavity-like structures we use the diffraction model:

$$W_d^{\delta}(z) = k_d \, z^{-1/2}, \quad z > 0$$

wake-function

$$W_d^{\sigma}(z) = \frac{k_d}{\sqrt{\sigma}} f\left(z/\sigma\right)$$

wake-potential



$$k_d = -\frac{Z_0 c}{\pi^2 a} \sqrt{g/2}$$

$$f(s) = e^{-s^2/4} \sqrt{\tfrac{\pi}{8} |s|} \left(\mathrm{I}(-\tfrac{1}{4}, \tfrac{s^2}{4}) + \mathrm{sign}(s) \, \mathrm{I}(\tfrac{1}{4}, \tfrac{s^2}{4}) \right)$$

I(…) are Bessel functions

- Wake-potentials for all cavity shapes (tapered or not, deep or shallow, etc.) converge to this model for short enough bunches and distances.
- Model is easily expandable to 3D geometries.

Introducing the Method: Wake of a Step-Out

- Wake-potentials are singular at $\sigma \to 0$
- Subtracting singular part (optical model) we obtain a well-defined limit (black line) at $\sigma \to 0$

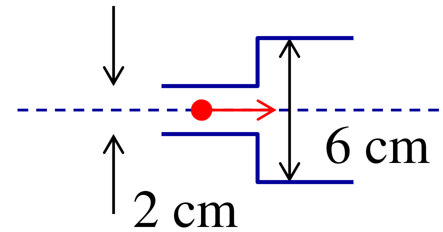
$$D^{\sigma}(z) = W^{\sigma}(z) - W_s^{\sigma}(z)$$

$$D^{\delta}(z) = \lim_{\sigma \to 0} D^{\sigma}(z)$$

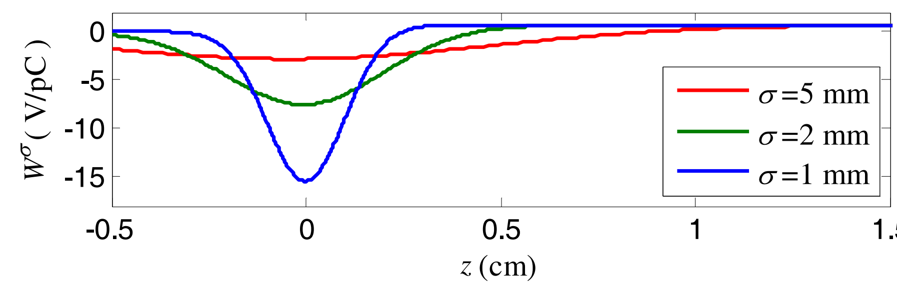
- This function is approximated by

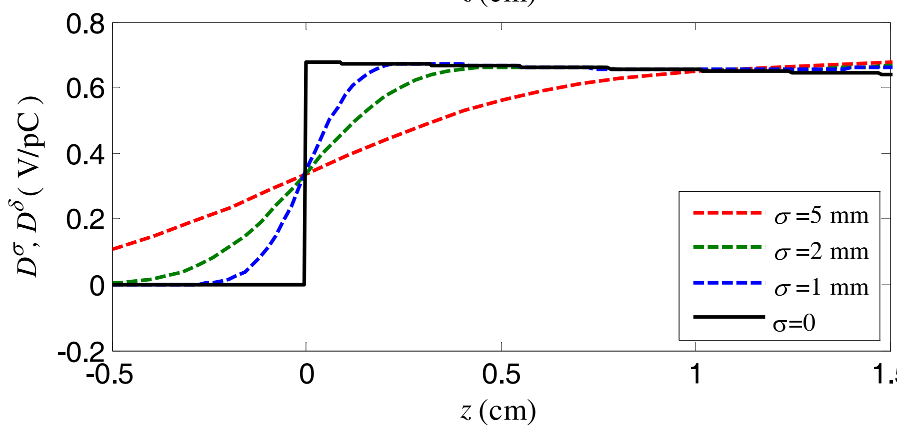
$$D^{\delta}(z) \approx (\alpha + \beta z) H(z)$$

- Coefficients α and β can be found by fitting (next VG).
- Thus we reconstruct point-charge wakefield (at short z-range)



$$W_s^{\sigma}(z) = W_{opt}^{\sigma}(z) = \frac{Z_0 c \, Ln(3)}{2^{1/2} \pi^{3/2} \sigma} e^{-\frac{z^2}{2\sigma^2}}$$



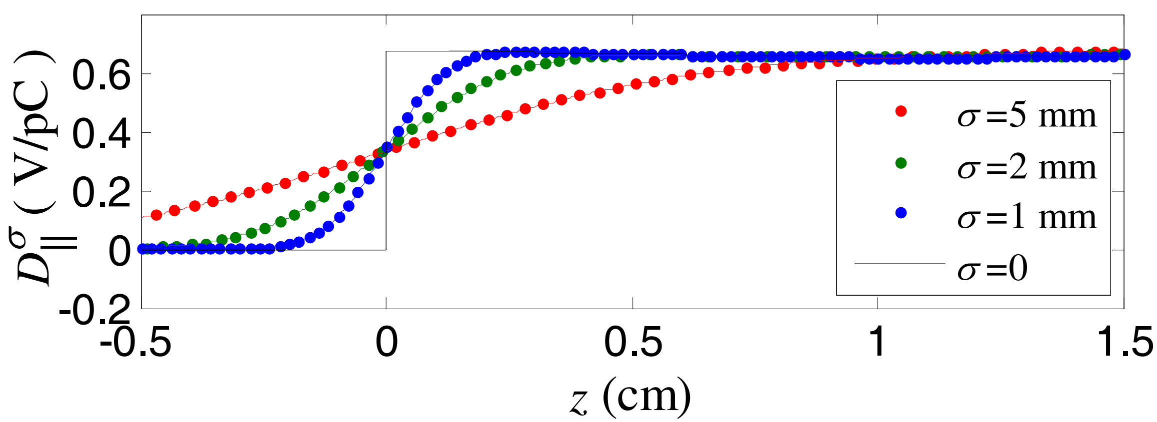


Wake of a Step-Out Con't: fitting for α and β

- Point-charge and Gaussian bunch functions are related:

$$D^{\delta}(z) = (\alpha + \beta z) H(z) \qquad D^{\sigma}(z) = \frac{\alpha+\beta z}{2}\left(1 + \mathrm{erf}\left(\frac{z}{\sqrt{2}\sigma}\right)\right) + \frac{\beta\sigma}{\sqrt{2\pi}} e^{-\frac{z^2}{2\sigma^2}}.$$

- α and β can be found by fitting $D^{\sigma}(z)$ from EM solver for i.e. $|z/\sigma|<3$.
- Take σ_0=2 mm and apply the fitting. Then use α and β obtained to reconstruct wakes for other values of σ:

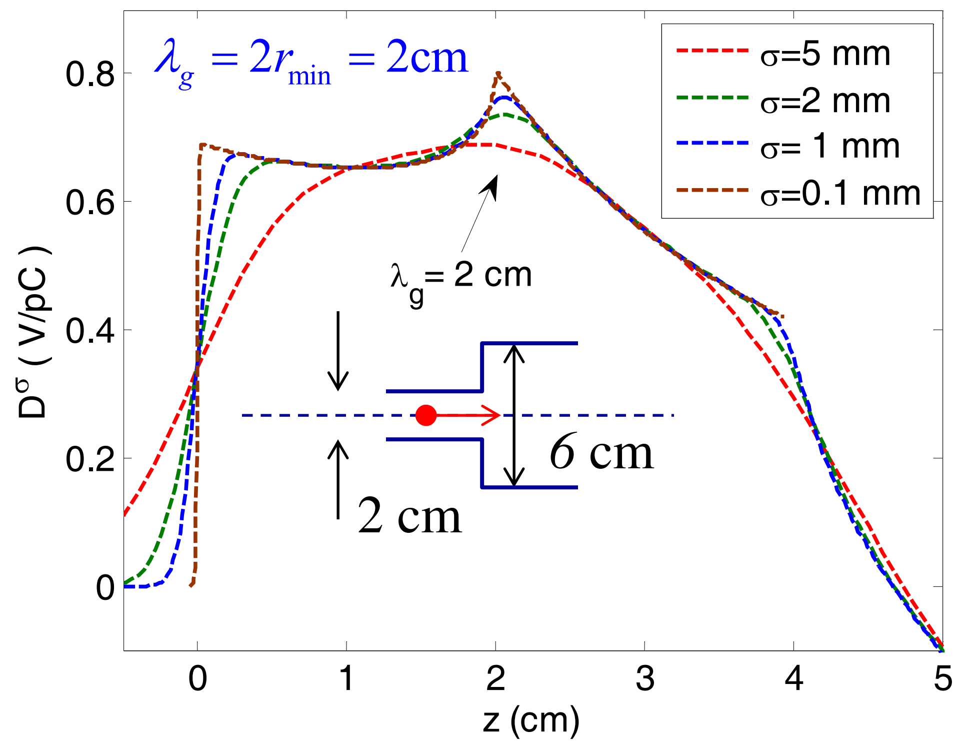


dots: ECHO
lines: rec'd from $W_{\mathrm{ECHO}}^{2\,\mathrm{mm}}$

- Reconstructed wakes agree well with direct ECHO calculation

How to Pick σ_0 in EM solver

- Why did the σ_0=2 mm fit work well? Because $\sigma_0 << \lambda_g$.
- Parameter λ_g>0 is the first location of the wake singularity (or singularity of its derivatives) closest to z=0.
- $D^\delta(z) = (\alpha + \beta z) H(z)$ cannot be extended beyond $z = \lambda_g$ since the wake derivative is singular ("kink").



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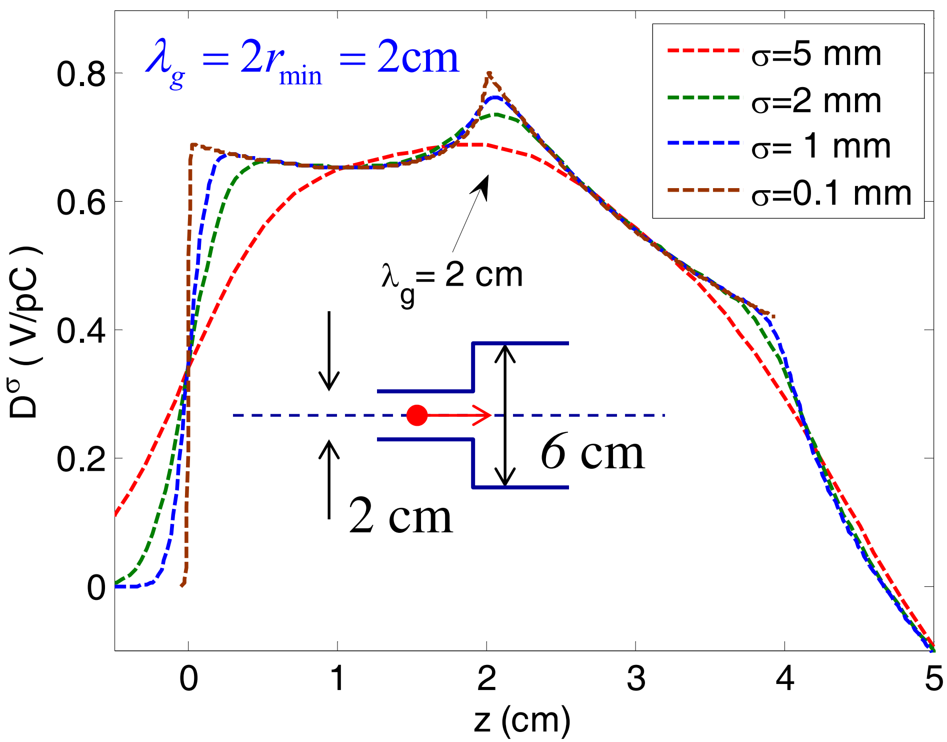
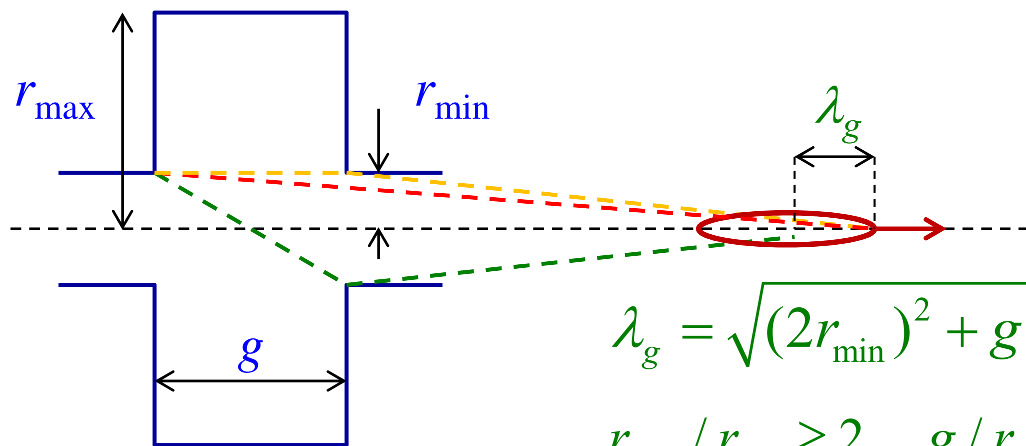


- Run EM solver with $\sigma_0 << \lambda_g$, typically σ_0/λ_g=0.1-0.15 is O.K.
- Running with shorter bunch gives no new information about the wake!
- λ_g can be found by simple geometry analysis.

λ_g Parameter for a Cavity



$$\lambda_g = \sqrt{(2r_{\min})^2 + g^2} - g$$

$$r_{\max} / r_{\min} \geq 2, \quad g / r_{\min} \geq 2^{-1/2}$$

- Red ray (spherical wave front) eventually catches up with ALL particles in the bunch, thus affecting the wakefield for all values of *z*.
- Green ray travels λ_g/c behind and it will never catch up with the front of the bunch, so λ_g emerges in the front portion of the wake.
- For other ratios between $r_{\min}$, $r_{\max}$, and g, other combinations may define λ_g , i.e. λ_g =2g for a short cavity or, for a shallow one,

$$\lambda_g = \sqrt{4(r_{\max} - r_{\min})^2 + g^2} - g$$

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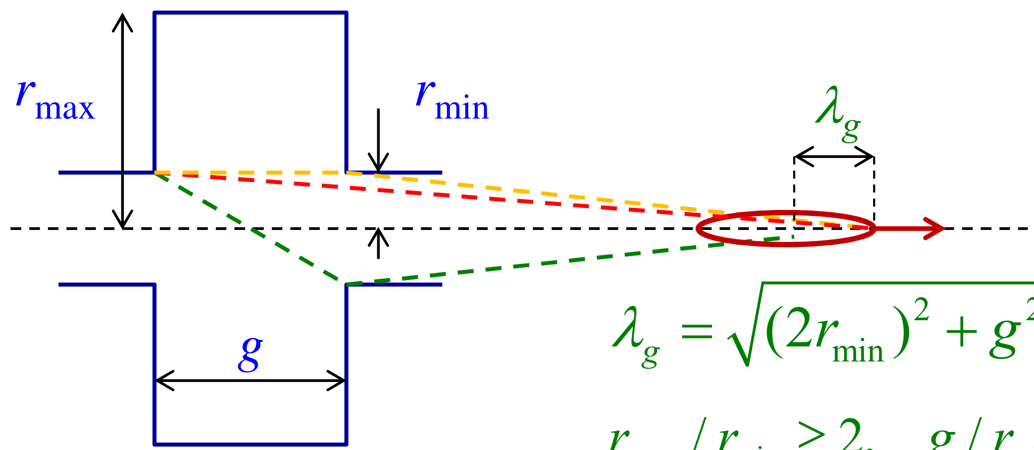

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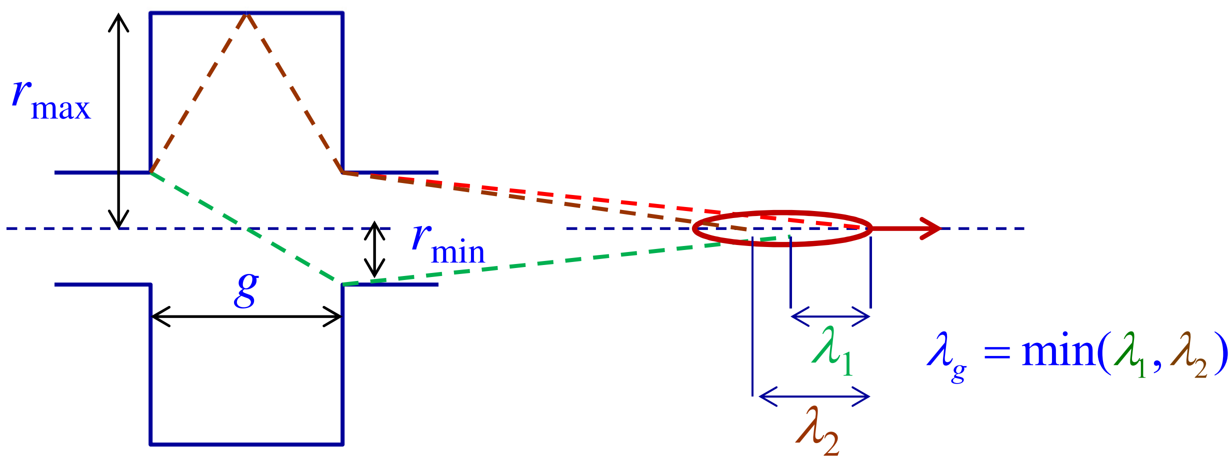
Similarly λ_g one can find for arb. geometry (see PRST-AB paper)

Why λ_g is Easy to Find for Arbitrary Geometry



- Green ray does not affect short-range wake for $z < \sqrt{(2r_{\min})^2 + g^2} - g$
- Brown ray does not affect short-range wake for $z < \sqrt{4(r_{\max} - r_{\min})^2 + g^2} - g$
  This is longer for deep cavities with $r_{\max} > 2r_{\min}$.
- By causality, any cavity with radial boundary, $r(s)$, that coincides with the figure for $r(s) < 2r_{\min}$, but otherwise is arbitrarily complex, must have the same short-range wake for $z < \lambda_g$.
- => λ_g is defined by the geometry near $r_{\min}$

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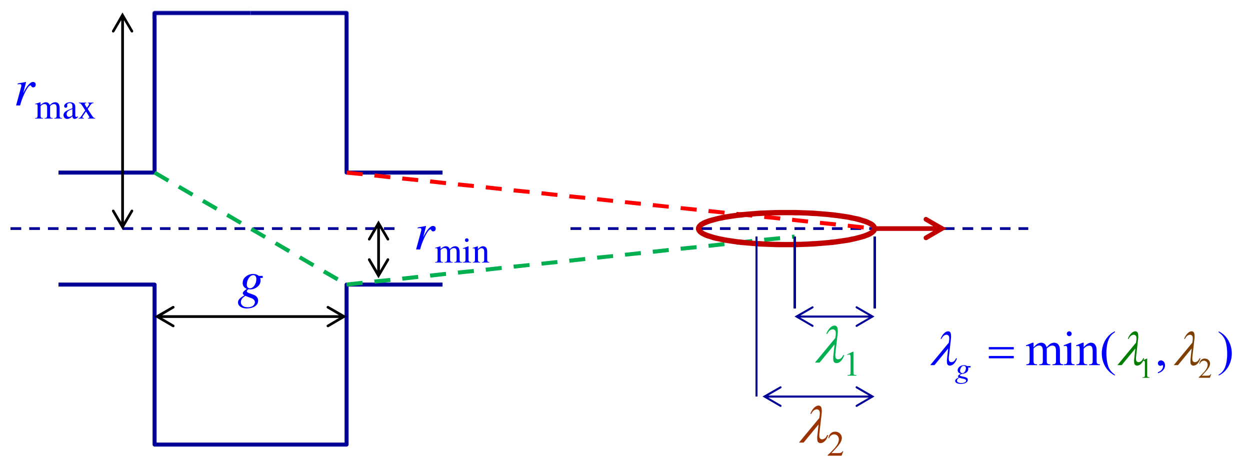

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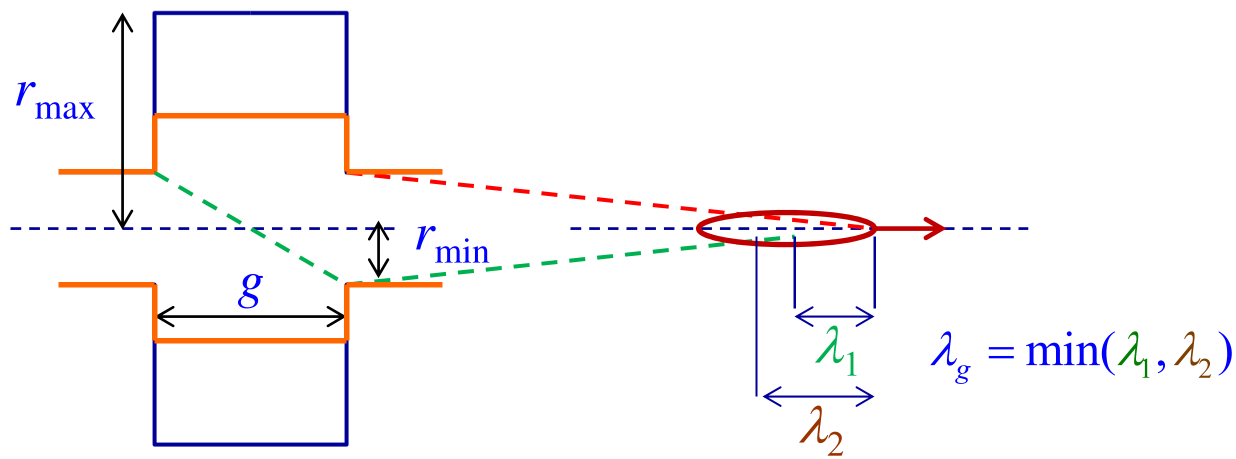

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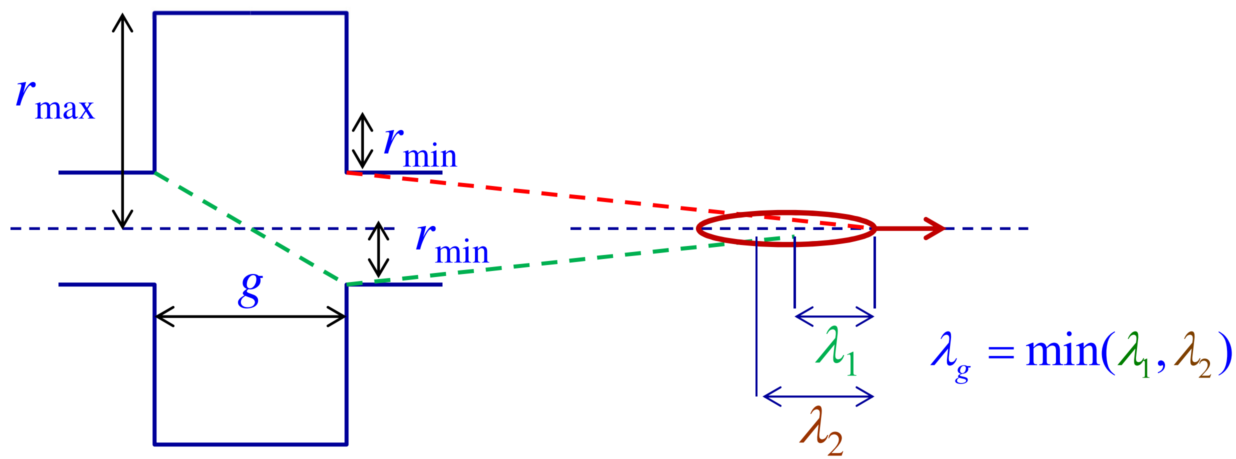

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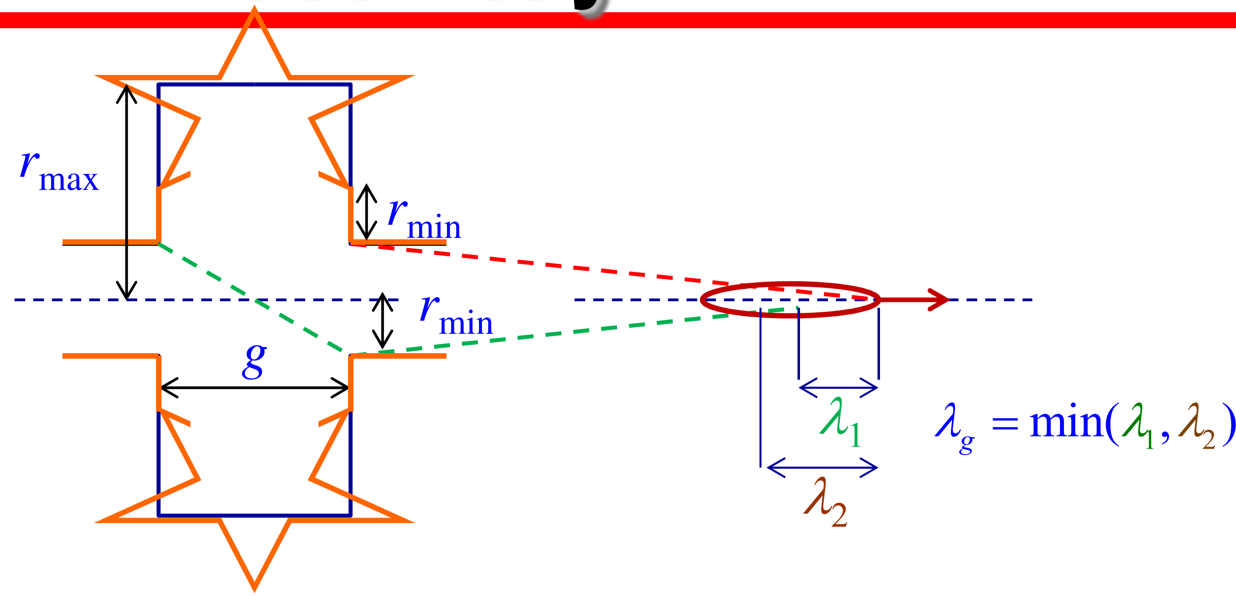

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Boris Podobedov, Oct. 2, 2013

How It All Works Together

1. Determine analytical singular wake model: $W_s^\delta(z)$ & $W_s^\sigma(z)$
2. Determine λ_g
3. Calculate the wake-potential with your favourite EM solver for $\sigma_0 << \lambda_g$: $W_{ECHO}^{\sigma_0}(z)$
4. Subtract the singular wake: $D^{\sigma_0}(z) = W_{ECHO}^{\sigma_0}(z) - W_s^{\sigma_0}(z)$
5. Fit the remainder, $D^{\sigma_0}(z)$, with the function: $\frac{\alpha+\beta z}{2}\left(1+\mathrm{erf}(\frac{z}{\sqrt{2}\sigma_0})\right)+\frac{\beta\sigma_0}{\sqrt{2\pi}}\mathrm{e}^{-\frac{z^2}{2\sigma_0^2}}$

   (fit range $|z/\sigma_0|<3$ works well)
6. Short-bunch wake (for arb. $\sigma \le \sigma_0$) is then:

$$W^\sigma(z \le 3\sigma_0) = \frac{\alpha+\beta z}{2}\left(1+\mathrm{erf}(\frac{z}{\sqrt{2}\sigma})\right)+\frac{\beta\sigma}{\sqrt{2\pi}}\mathrm{e}^{-\frac{z^2}{2\sigma^2}} + W_s^\sigma(z) \qquad W^\sigma(z > 3\sigma_0) = W_{ECHO}^{\sigma_0}(z)$$

7. For point-charge:

$$W^\delta(z \le 3\sigma_0) = (\alpha+\beta z)H(z) + W_s^\delta(z) \qquad W^\delta(z > 3\sigma_0) = W_{ECHO}^{\sigma_0}(z)$$

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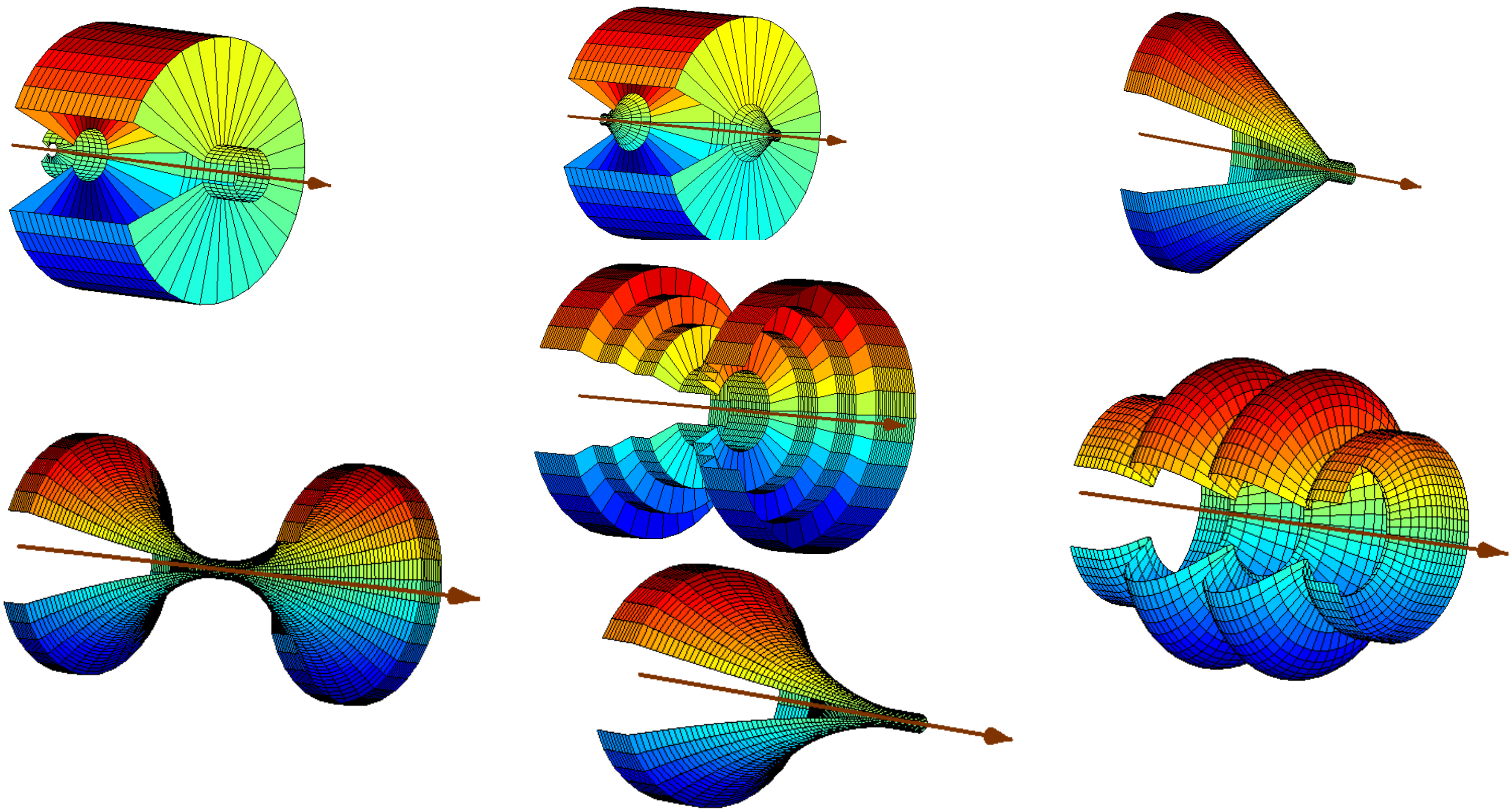
Transverse is very similar

The Method Was Applied to Many Geometries



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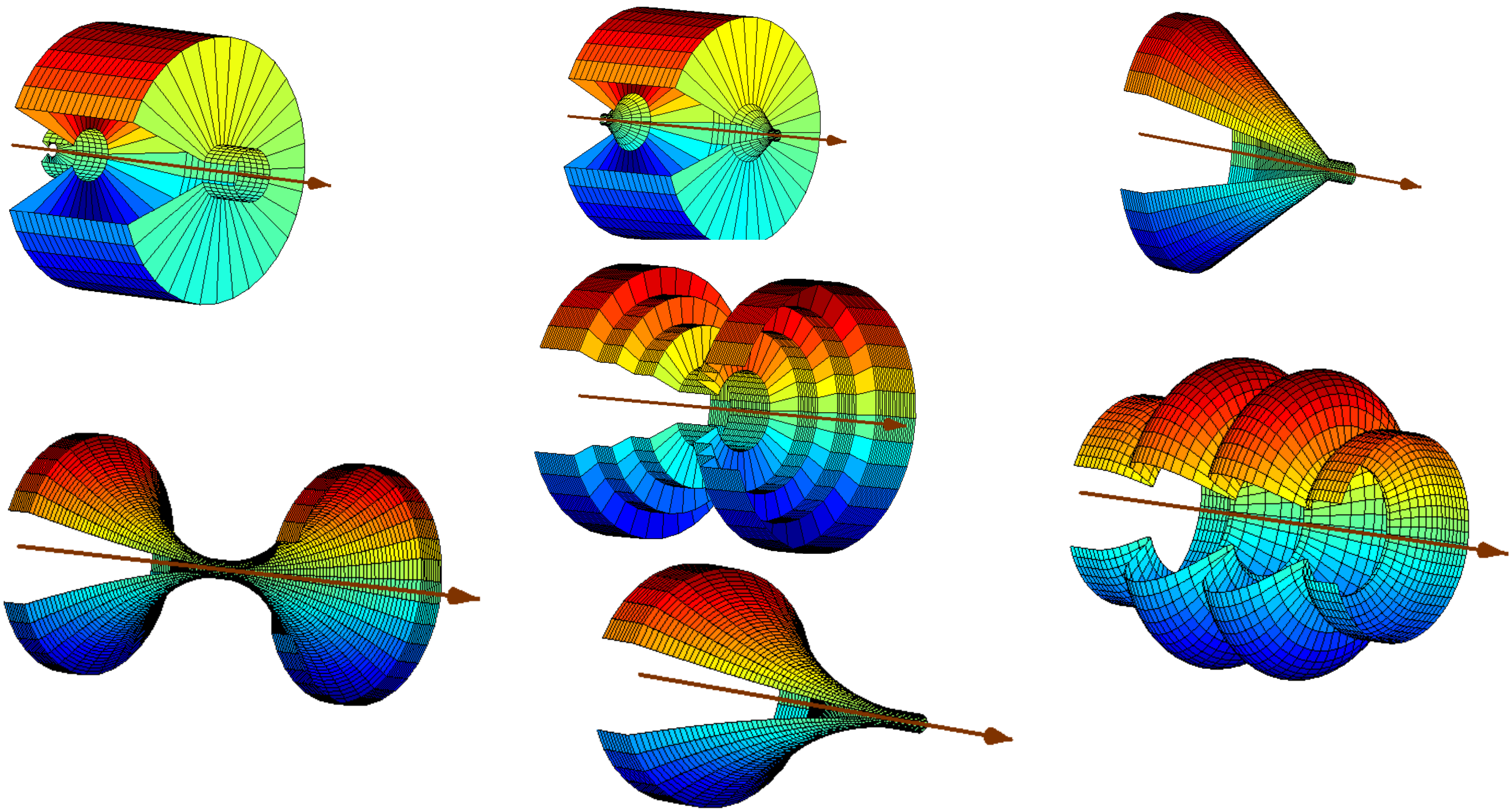
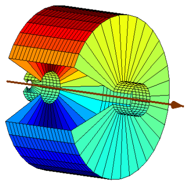


It worked well for all of them

Simple Cavity Example



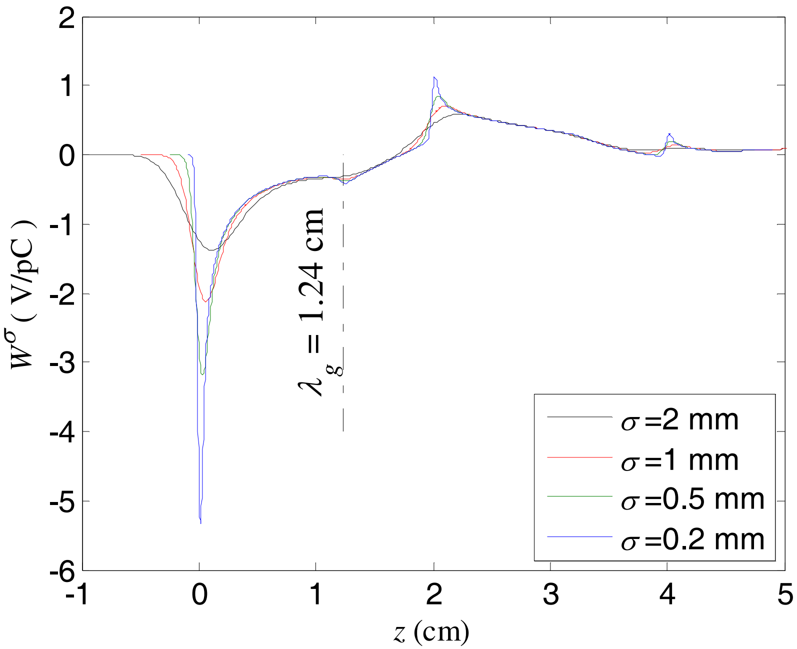
r_{min}=1 cm
r_{max}=5 cm
g=1 cm

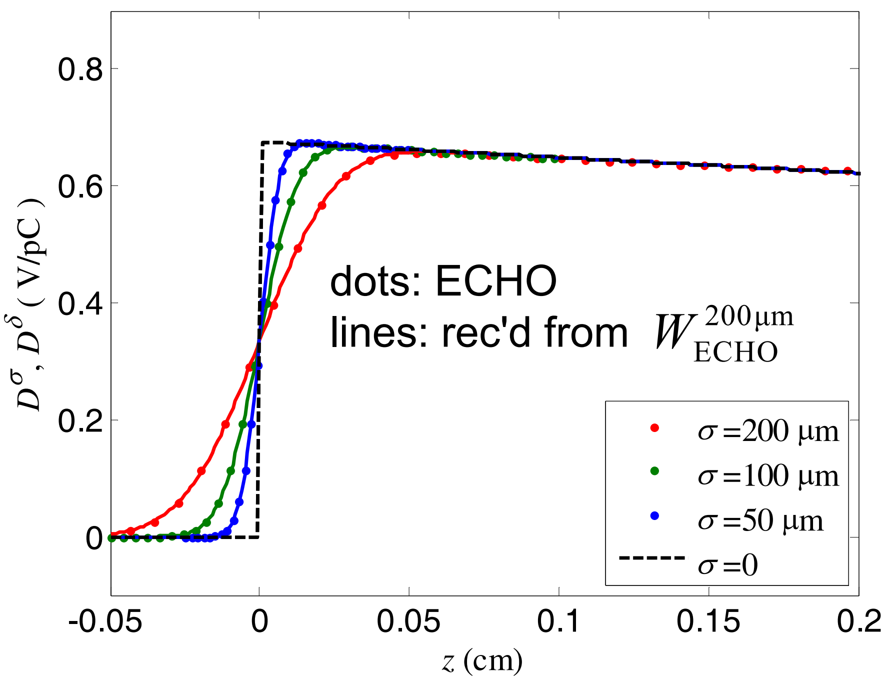
- Diffraction-model behaviour near z=0
- Pick σ_0=200μm<< λ_g

$$D^{\sigma_0}(z) = W^{\sigma_0}_{ECHO}(z) - W^{\sigma_0}_{d}(z)$$

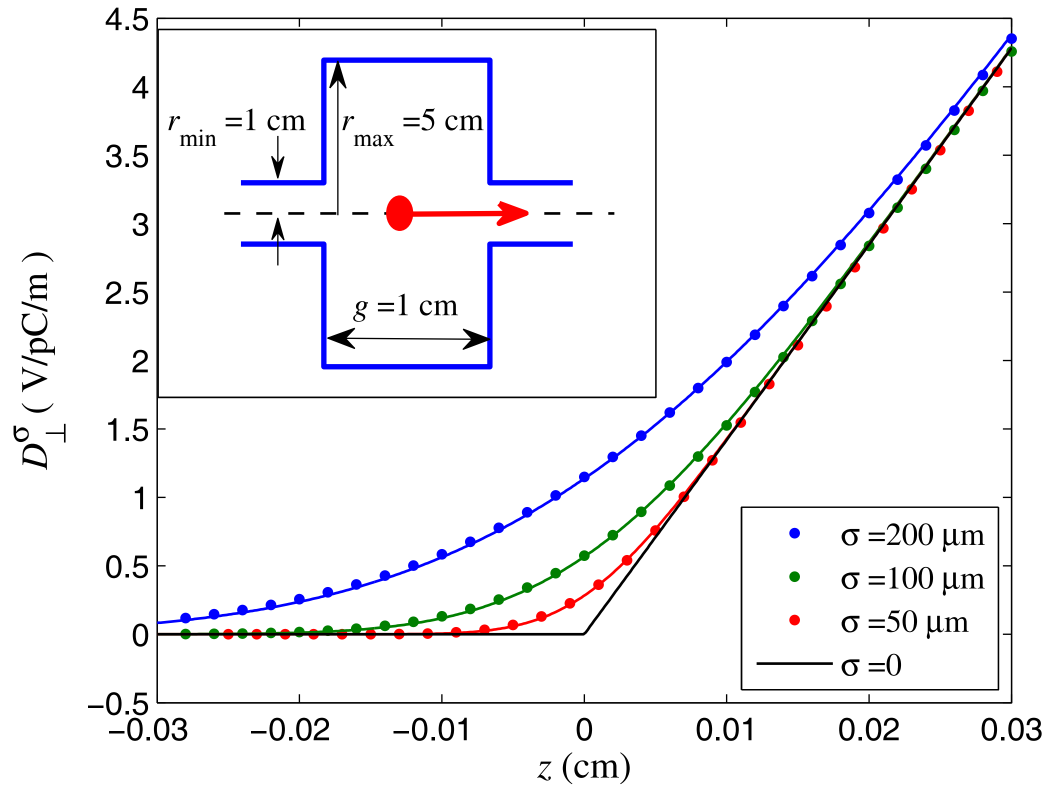
- Short-bunch wake reconstructed well.

$$\lambda_g = \sqrt{(2r_{min})^2 + g^2} - g = 1.24 \text{ cm}$$





Transverse Wake for the Same Cavity

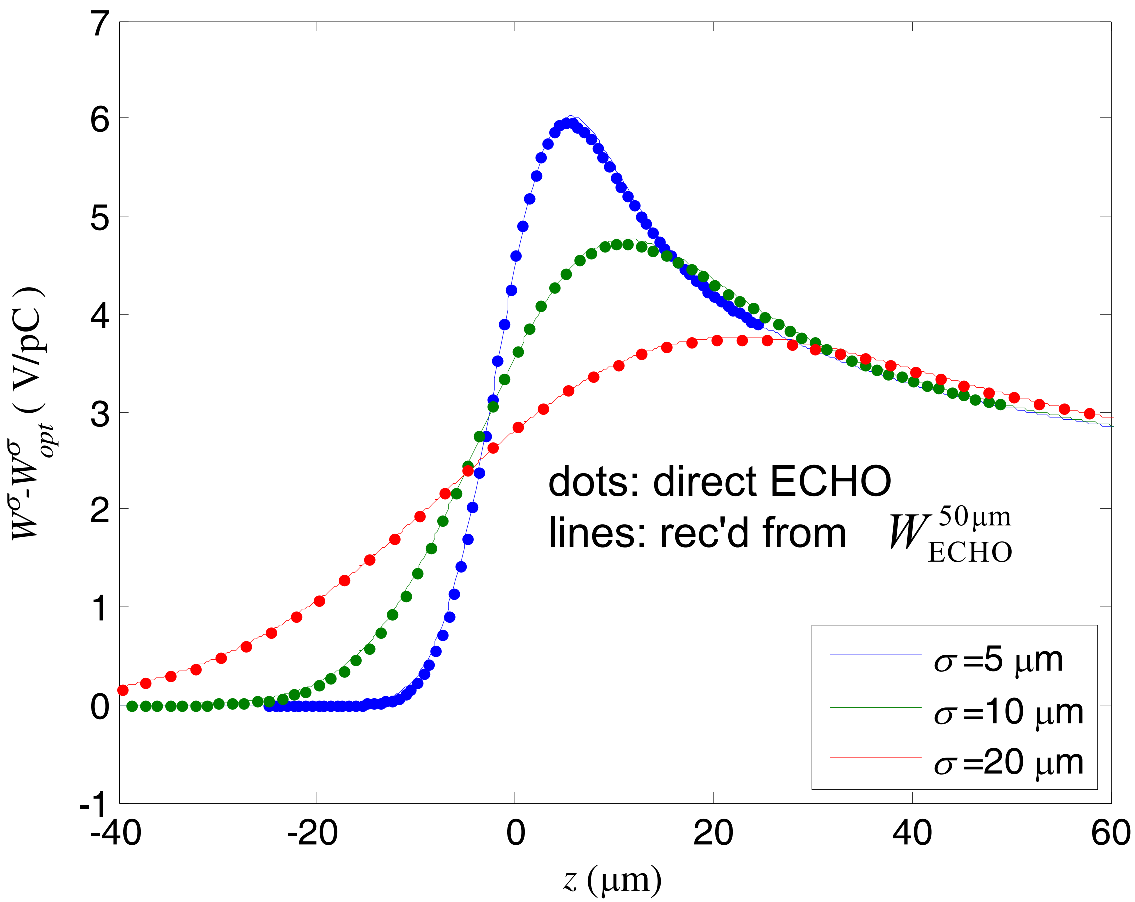


dots: direct ECHO
lines: rec'd from $W_{\text{ECHO}}^{2\text{mm}}$

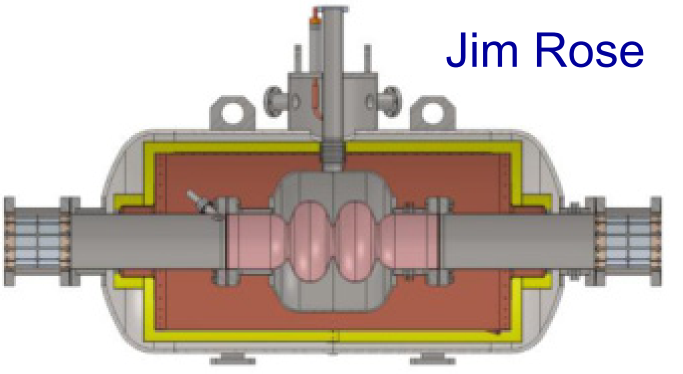
- The same algorithm works well in the transverse (except the transverse diffraction model is non-singular).
- Reconstructed wakes from σ_0 =2 mm agree perfectly with direct ECHO calculations

NSLS-II Landau Cavity

- 1.5 GHz dual cell cavity, r_{side_pipe}= 6 cm
- Final results for the short-range wakes:



Jim Rose



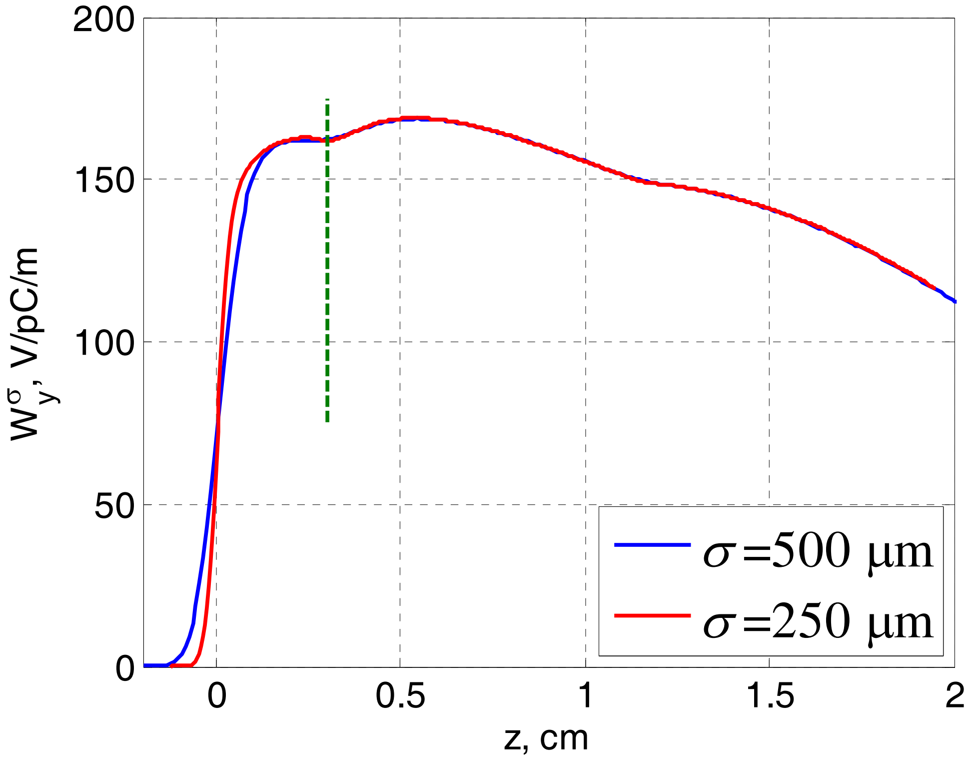
To find 10μm bunch wake

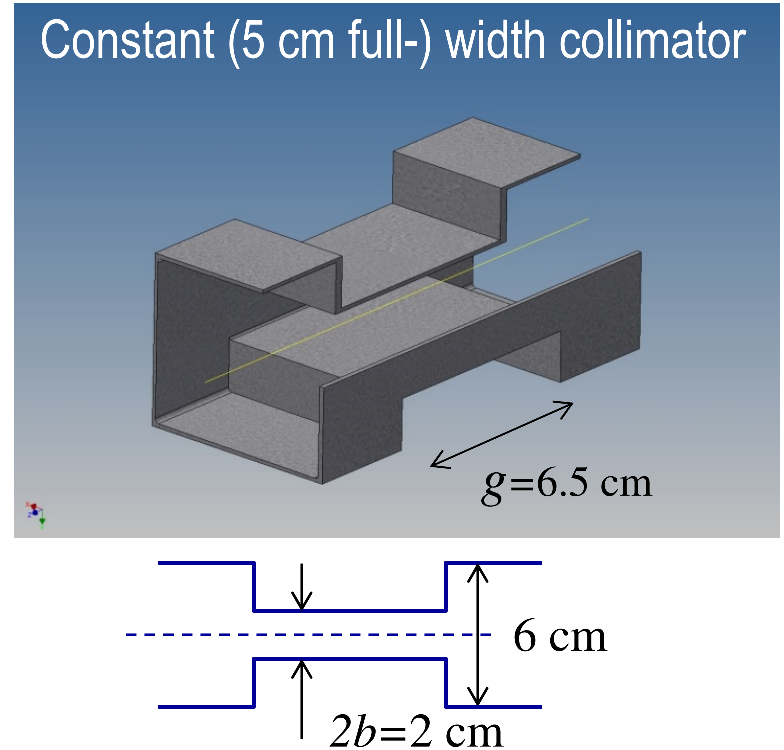
Brute force: ~480 hours of Intel(R) Xeon(R) 5570@2.93 GHz CPU to z_{max}=1 cm.

Our method: uses only σ=50μm calc's, saves a factor of 5^3 on CPU time and 5^2 on memory. Gives a model of the point-charge wake as a bonus.

3D Example

Wakes by I. Zagorodnov,
3D ECHO + CST mesher





- Observe λ_g, where expected
- Expected behavior near the origin; can easily fit point-charge wake α & β
- Same applies for logitudinal (+ optical model), and for quadrupolar wakes

3D Example

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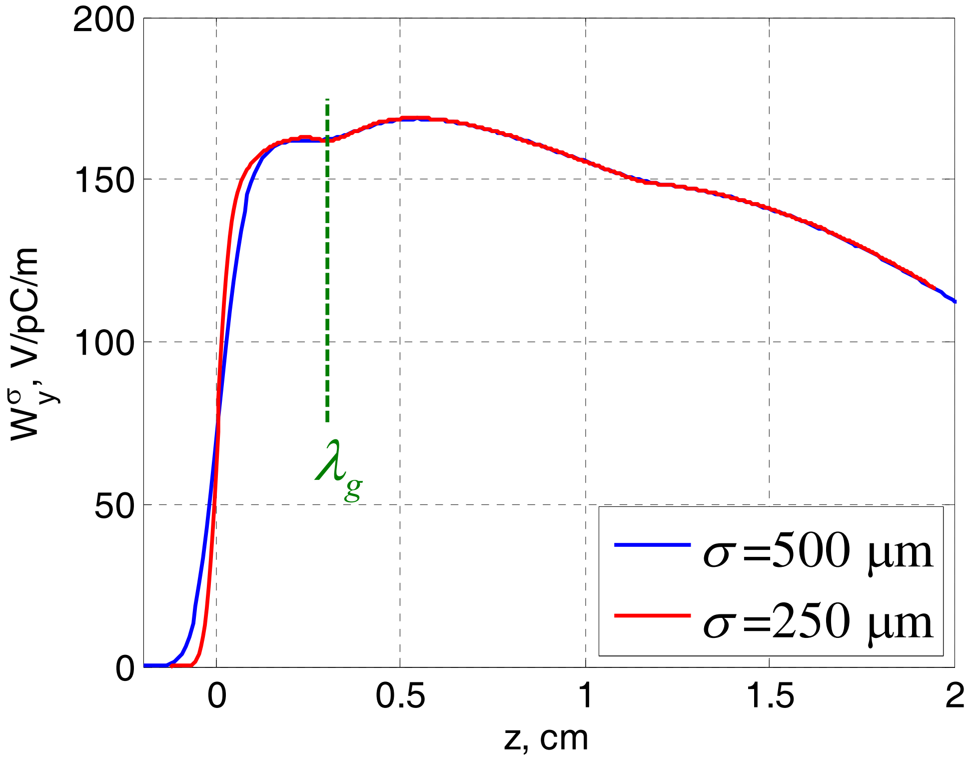

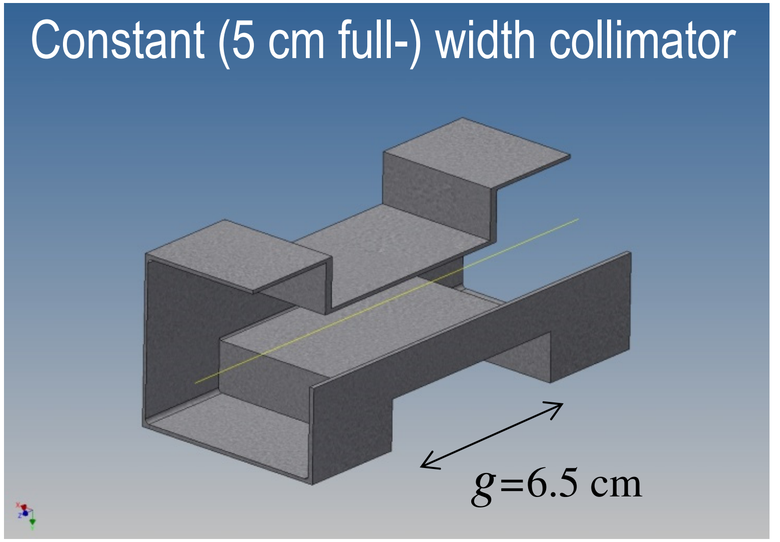

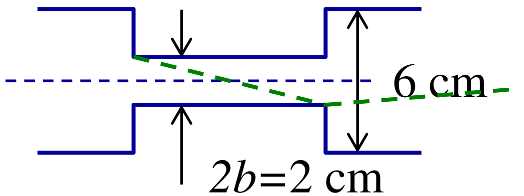

$$\lambda_g = \sqrt{(2b)^2 + g^2} - g = 3 \text{ mm}$$

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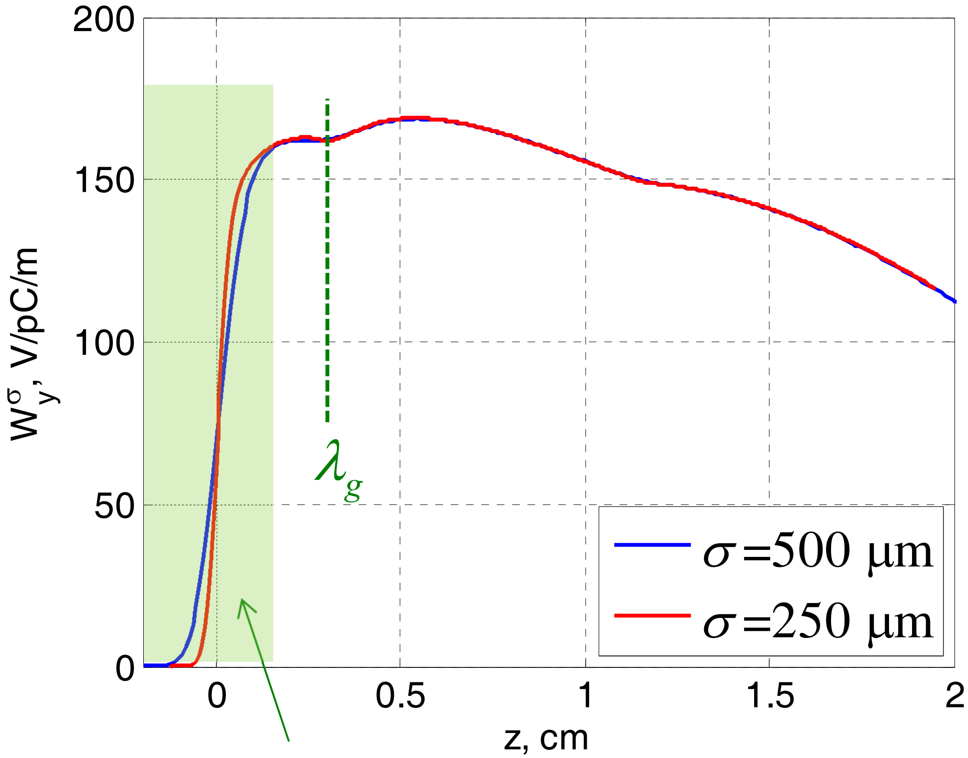
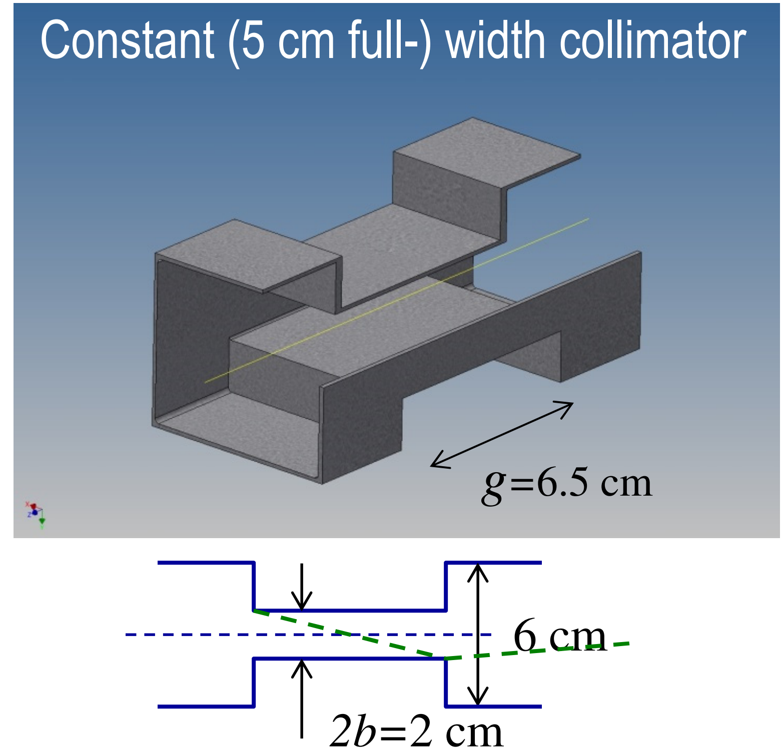


~(1+erf($z/\sigma/2^{1/2}$)) behavior



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Summary

- Wakefield calculation is important task for modern accelerators. For large and smooth accelerator structures and short bunches, direct EM solver calc's can be extremely time-consuming.
- We described a new method to accurately obtain wakefields of short bunches, including point-charge, by adding a (processed) long-bunch result from an EM solver and, if applicable, a singular analytical wake model.
- We showed that this method often provides great savings in computing time required to calculate wake-potentials due to very short bunches.
- The method resolves an important practical question, as to how short of a bunch one needs to use in an EM solver, so that shortening this bunch further would not result in any new information about the wake.
- In the future this work will be generalized to 3D geometries.

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Thank you