Imposing Strong Energy Slews with Transverse Deflecting Cavities

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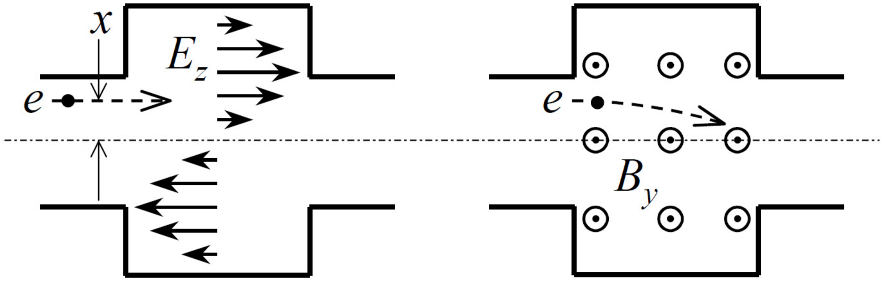
Transverse deflecting cavity

$$\varsigma_{out} = R\varsigma_{in} \qquad \varsigma = \left(x, x', z, \frac{\Delta\gamma}{\gamma}\right)$$



M. Cornacchia and P. Emma,
Phys. Rev. ST-AB **5**, 084001 (2002).

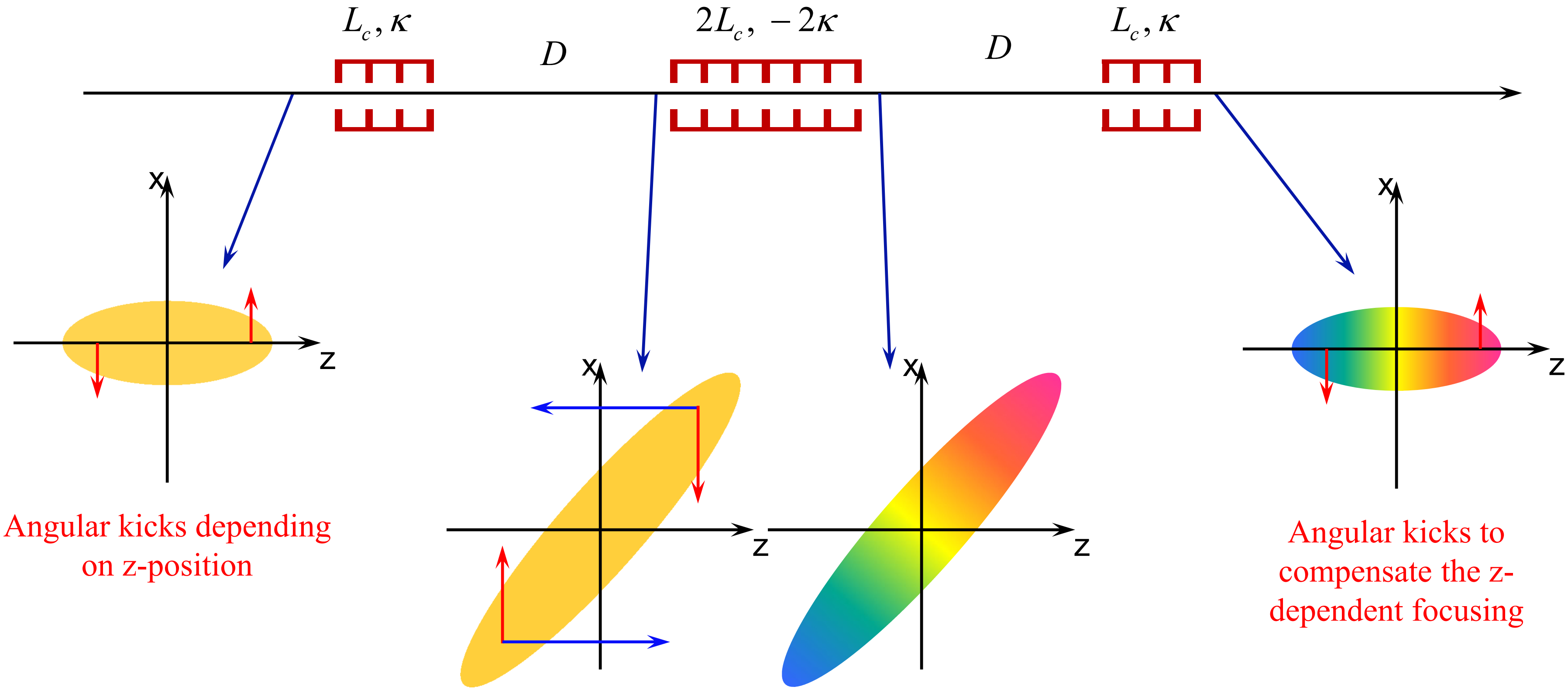
Angular divergence based on longitudinal position

$$R = \begin{pmatrix} 1 & L_c & 0 & 0 \\ 0 & 1 & \kappa & 0 \\ 0 & 0 & 1 & 0 \\ \kappa & 0 & 0 & 1 \end{pmatrix}$$

Energy change based on transverse position

This property can be used for imposing large energy slews inside the bunch since its transverse size can be quickly changed using conventional focusing beam optics unlike the longitudinal size which requires expensive compressors.

Transverse Cavity Based Chirper (TCBC)



Angular kicks depending on z-position

Opposite angular kicks depending on z-position.
Energy change depending on transverse position

Angular kicks to compensate the z-dependent focusing

Linear analysis

$$R_{cav}=\begin{pmatrix}1 & L_c & \frac{kL_c}{2} & 0\\ 0 & 1 & \kappa & 0\\ 0 & 0 & 1 & 0\\ \kappa & \frac{kL_c}{2} & \frac{k^2L_c}{6} & 1\end{pmatrix}$$

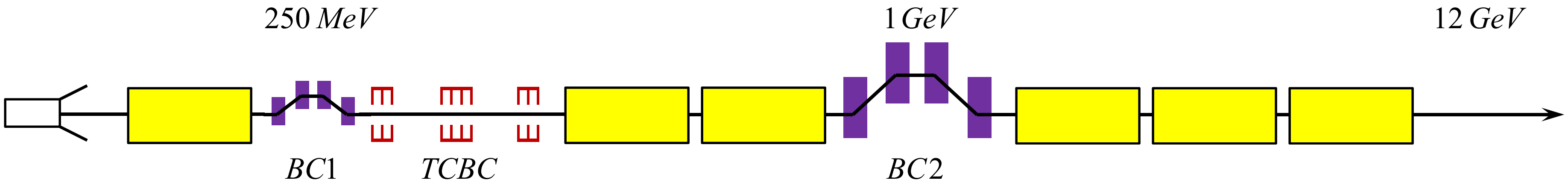
$$R_{TCBC}=R_{cav}(\kappa)R_{drift}R_{cav}(-2\kappa)R_{drift}R_{cav}(\kappa)=\begin{pmatrix}1 & 2D+4L_c & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & -\frac{2}{3}\kappa^2(3D+2L_c) & 1\end{pmatrix}$$

Large compressing energy slew can be imposed
if long drifts between the cavities are used

$$(\sigma_x)_{middle}\sim(kD)\sigma_z>>\sigma_z$$

The energy chirp is applied because of large transverse beam size in the middle of the beamline. Strong correlation between the longitudinal and transverse coordinates (imposed by the first cavity and the following drift) effectively substitutes short bunches with long ones, which makes their chirping extremely efficient.

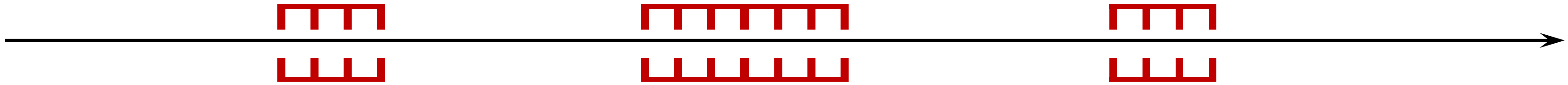
Typical parameters for MaRIE linac



TCBC parameters

Beam energy	E	250	MeV
Emittance	ε_n	0.15	mm-mrad
Beta-function	$\beta_{x,y}$	15	m
Bunch length (rms)	σ_z	90	μm
Energy spread	ΔE_{rms}	25	keV
RF frequency	f	2.85	GHz
RF gradient	G	22	MV/m
Cavity strength	k	2.1	m^{-1}
Cavity length	L_c	0.4	m
Drifts length	D	5	m
Total length	$2D+4L$	11.6	m
	kD	10	
Energy chirp	ΔE_{ind}	±1	MeV

Real Estate



Price of tunnel	P_1	~\$50 K/m
Price of RF	P_2	~\$500 K/m

$P_2/P_1 \sim 10$

For the same value of R_{65} the smallest overall cost is achieved when the cost of RF is twice the cost of drifts

$$\boxed{D \sim \frac{P_2}{P_1} L_c \sim 10 L_c}$$

Same RF can be used to apply energy chirp through conventional off-crest acceleration. Assuming that the spatial field gradient is the same for both modes at the same frequency, the TCBC is more efficient than off-crest acceleration if the induced energy chirp is large enough

$$\boxed{\Delta E > \sqrt{\frac{GE\sigma_z^2}{\lambda}}}$$

$\Delta E > 20\ keV$ @

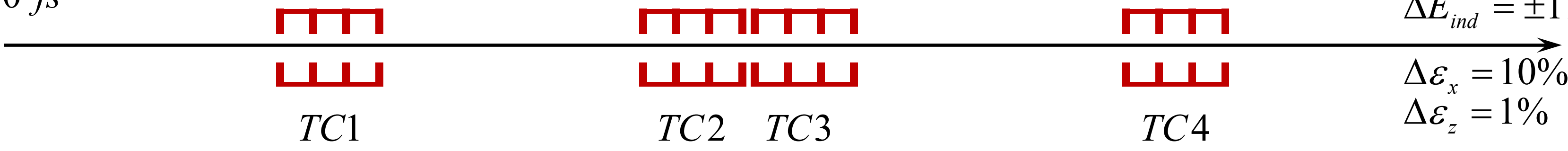
$E = 250\ MeV$
$G = 22\ MeV/m$
$f = 2.85\ GHz$
$\tau = 300 fs$

Implementation of TCBC at MaRIE between BC1 and BC2 will allow to reduce the number of **klystrons from 9 to 5** and reduce the length of linac from **50m to 35m**

ELEGANT simulations

$E = 250\ MeV$
$\varepsilon_n = 0.15\ \mu m$
$\tau = 300\ fs$

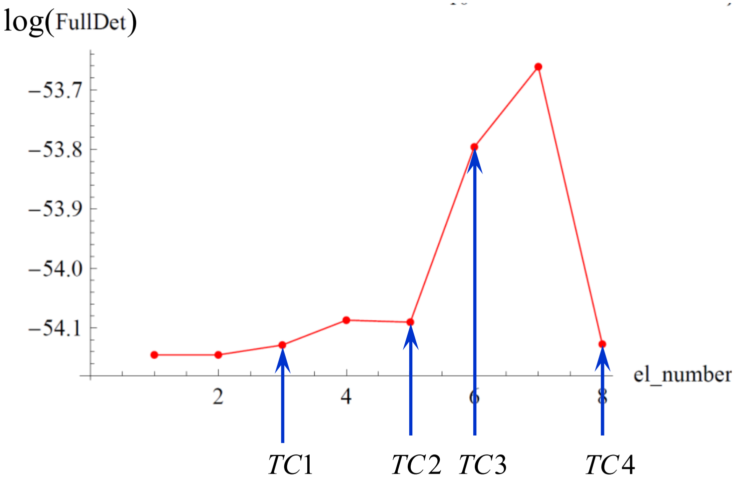
$f = 2.85\ GHz$
$L_c = 0.4\ m$
$G = 22\ MV/m$

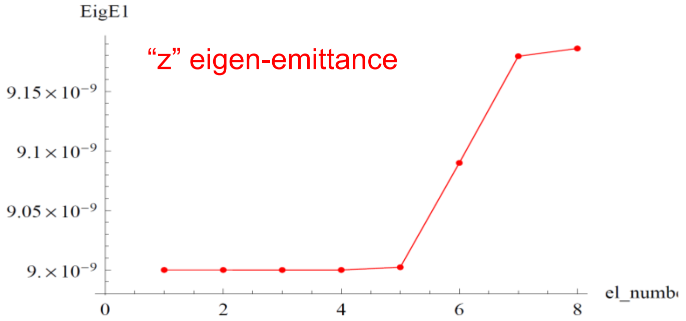


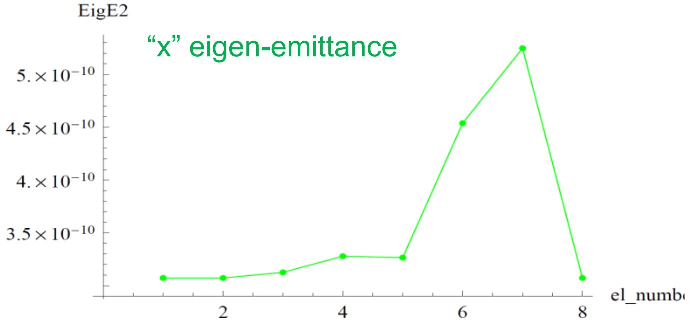
$D = 5m$

$TC1$ $TC2$ $TC3$ $TC4$

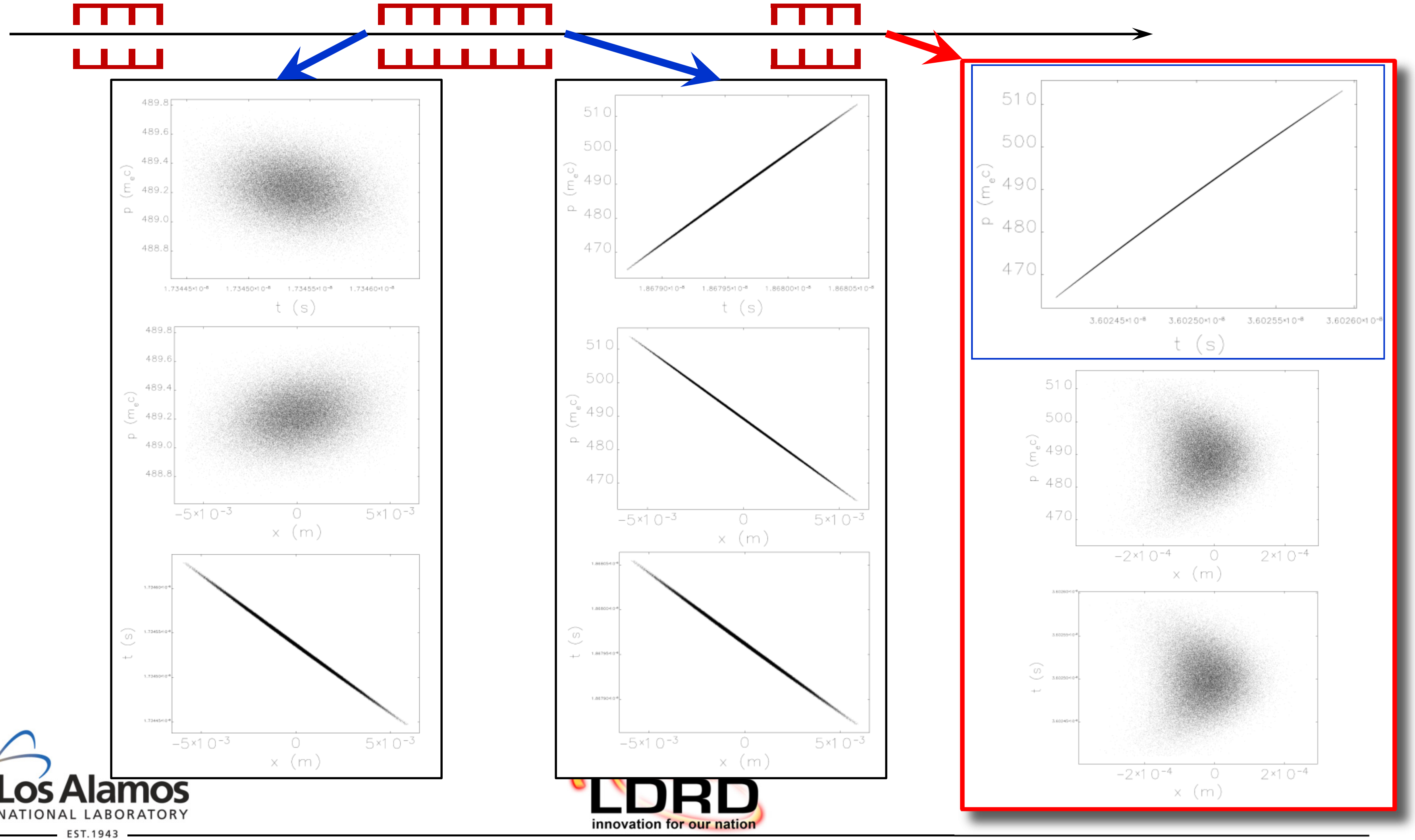
$\Delta E_{ind} = \pm 1\ MeV$
$\Delta\varepsilon_x = 10\%$
$\Delta\varepsilon_z = 1\%$



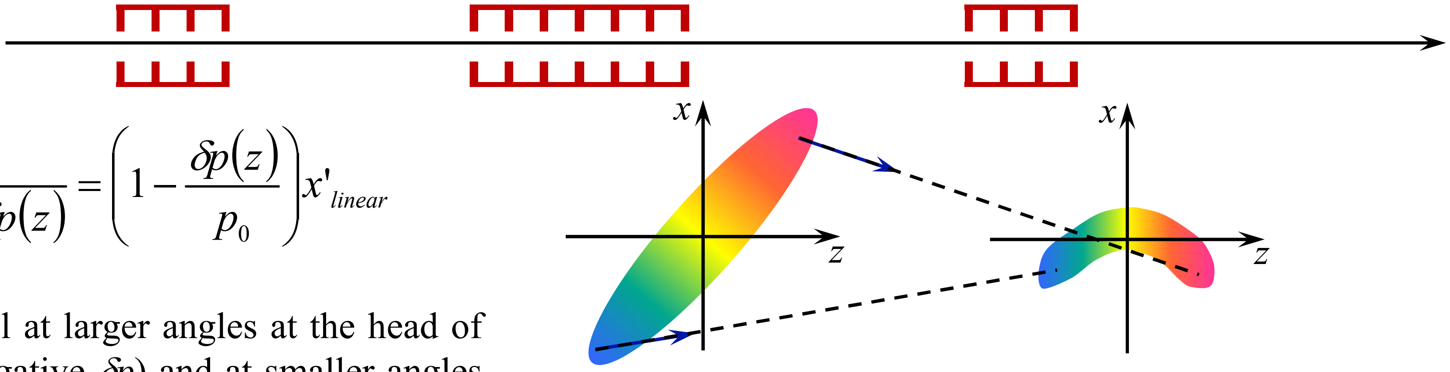




Phase space: TCBC at ±4MeV

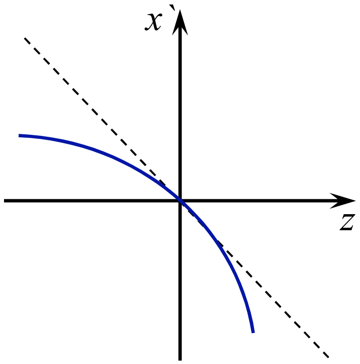


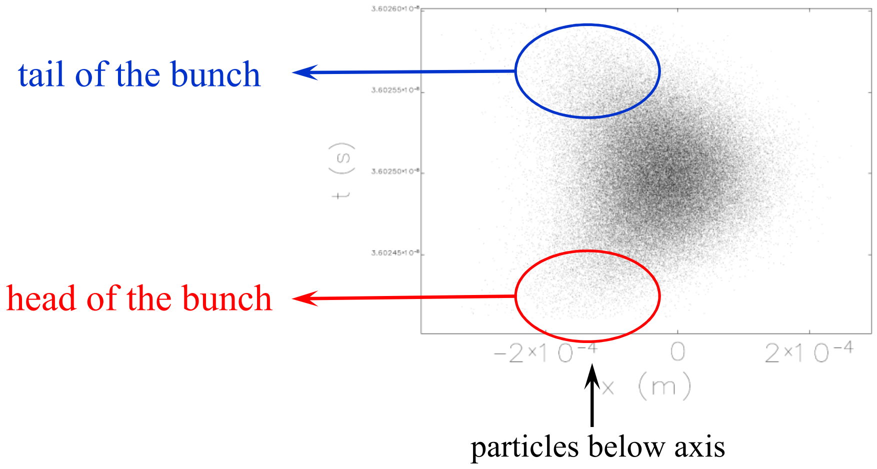
Phase space: underlying physics



$$x' = \frac{p_x}{p_0 + \delta p(z)} = \left(1 - \frac{\delta p(z)}{p_0}\right) x'_{linear}$$

Particles travel at larger angles at the head of the bunch (negative δp) and at smaller angles at the tail of the bunch (positive δp)





tail of the bunch

head of the bunch

particles below axis

$$x = x_0 - 2\kappa^3 D^2 z^2$$

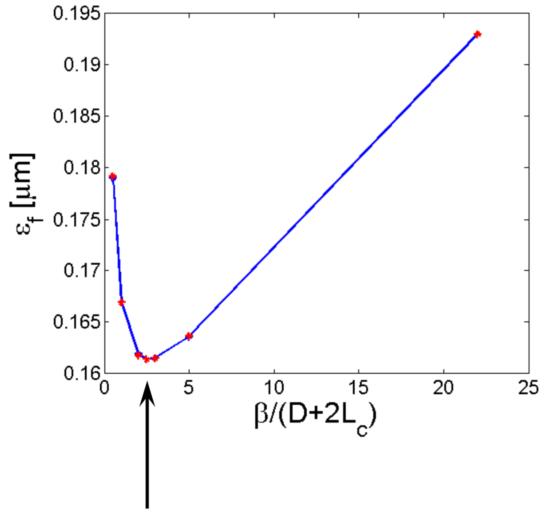
$$< x > = -\left(\frac{\Delta E_{ind}}{E}\right)^{3/2} \sqrt{\frac{D\sigma_z}{2}} = -30\ \mu m$$

Optimization

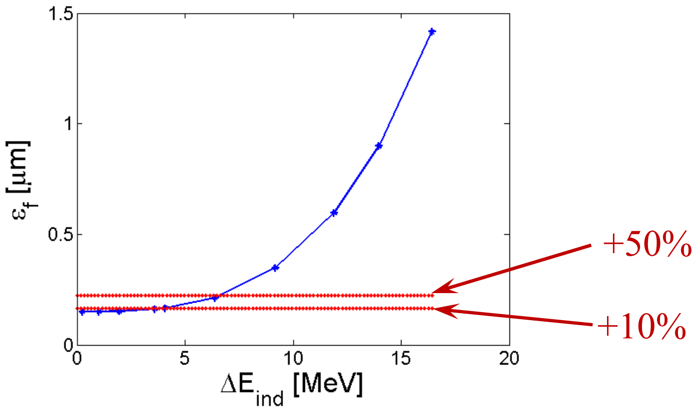
ELEGANT simulations for the case:

- Vacuum focus is in the middle of the of the central cavity
- Scan vs β-function at different frequencies

Emittance growth is the same
for different RF frequencies
f=2.85 GHz and *f=11.4 GHz*



β=2.5(D+2L_c)



No significant emittance growth due to vacuum nonlinearity up to several MeV energy chirp

$$\boxed{\frac{\Delta E_{ind}}{E} < \left(\frac{\varepsilon_n}{\gamma\sigma_z}\right)^{1/3} \sim \frac{5\,MeV}{250\,MeV}}$$

RF curvature

RF curvature cavity results in the smaller magnetic field at larger radius: $E,B \sim 1-(x/\lambda)^2$. As a result, particle acquires less than expected angular kick and travels at different trajectory. Both particle position and angle at the TCBC exit differ from the initial ones.

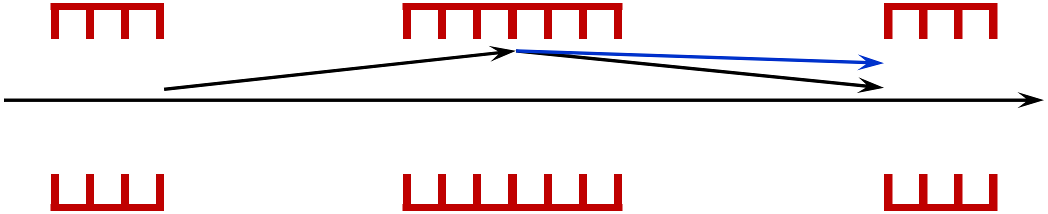
The beam size in the middle of the beamline is the smallest if this location correspond to the vacuum beam focal plane.

The emittance growth can be estimated as

$$\varepsilon^2 = \varepsilon_0^2 + 4(\kappa\sigma_z)^6 \frac{D^4}{\lambda^4}\left(\sigma_x^2 + 3D^2\sigma_{x'}^2\right)$$

extra angular spread

increase in beam size



$$\beta = \sqrt{3D} \longleftarrow$$

Optimum point is close to optimum for vacuum nonlinearity

$$\frac{\delta\varepsilon}{\varepsilon} \sim \left[\left(\frac{\Delta E_{ind}}{E}\right)^3 \frac{\gamma D^2 \sigma_z^3}{\varepsilon_n \lambda^4}\right]^{1/2}$$

$\delta\varepsilon/\varepsilon \sim 1\%$ for MaRIE

$\varepsilon_n = 0.15\,\mu m$, $\tau = 300 fs$

$D = 5m$, $f = 2.85 GHz$

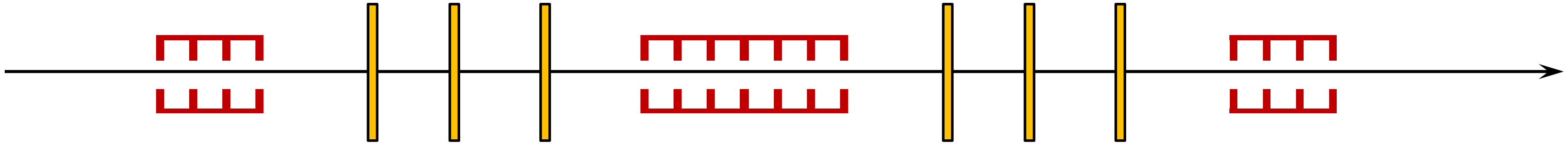
$\Delta E_{ind}/E = (1 MeV) / (250 MeV)$

$$\boxed{\lambda < \sqrt{D\sigma_z} = 2\,cm}$$

for RF curvature to be dominant effect of emittance growth. Careful cavity design may reduce emittance growth due to RF curvature

Further improvements

$$R_{FODO} = \begin{pmatrix} 1 & D_{eff} \\ 0 & 1 \end{pmatrix}$$



$$R_{TCBC} = \begin{pmatrix} 1 & 2D_{eff} + 4L_c & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -\frac{2}{3}\kappa^2\left(3D_{eff} + 2L_c\right) & 1 \end{pmatrix} \approx \begin{pmatrix} 1 & 2D_{eff} & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -2\kappa^2 D_{eff} & 1 \end{pmatrix}$$

Including FODO lattice which transform matrix is identical to a drift matrix will allow:

- reduce system size
- "negative drifts" will allow for dechirping beams after the bunch compression

TCBC vs off-crest acceleration

Better

- **Reduced cost.** The most efficient for short bunches at small energy
- Decoupling between the beam energy and compression factor
- On-crest acceleration results in zero curvature of longitudinal phase space after RF linearizer both in 2nd and 3rd orders

Same

- Time jitter results in the same energy jitter (final energy linearly depends on longitudinal coordinate in both schemes)

Worse

- Vertical offset of the final beam. Offset increases with time jitter
- Emittance growth strongly depends on the time jitter
- Initial transverse offset results in final energy jitter