Beam-Beam Limit in an Integrable System

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Introduction and Motivation

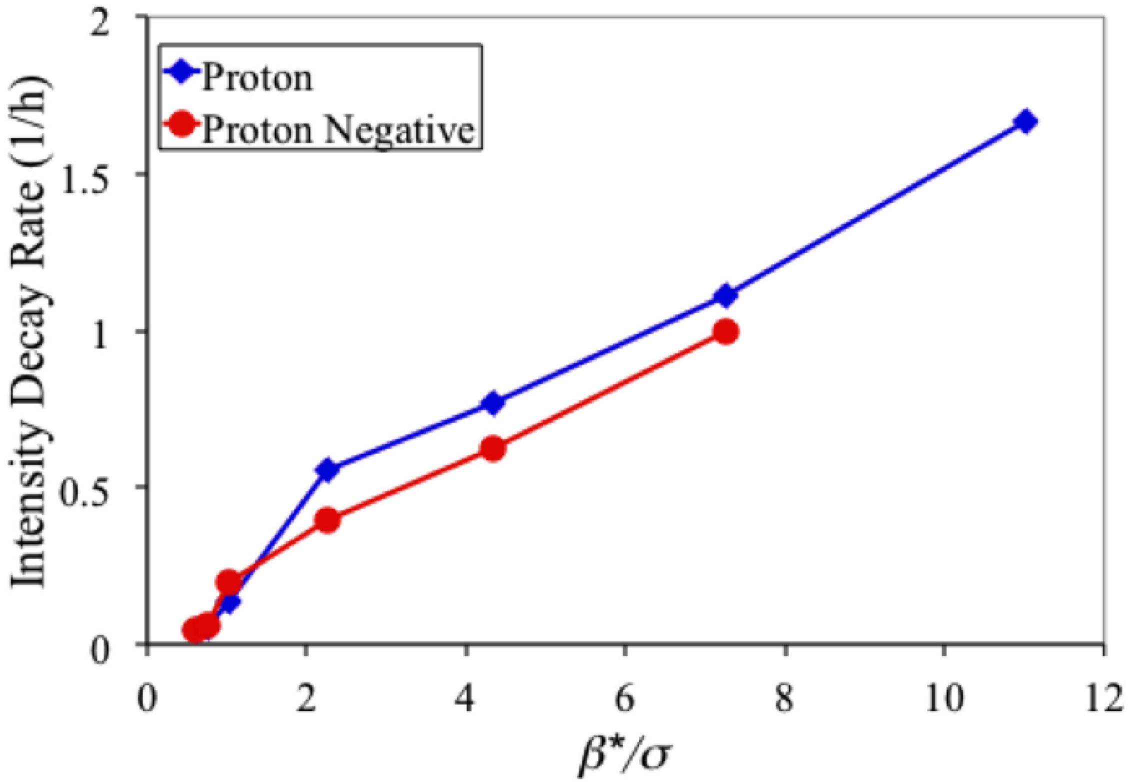
S. Krishnagopal and R. Siemann, “Bunch-length effects in the beam-beam interaction”, Phys. Rev. D 41, 2312 (1990)

- analysis of effect of the strong bunch length σ_z on the magnitude of resonances with the use of canonical perturbation theory
  - ‘…the finite longitudinal extent of the beam-beam interaction results in averaging of the betatron phase over the collision, which mitigates the destructive effects of resonances’
  - conversely, the synchrotron oscillations of the test particle lead to a greater depth of modulation and enhance the effect of resonances
  - optimum $\sigma_z \sim \beta^*$

Effect of Bunch Length

Tevatron experience: $\xi < 0.03$, $\sigma_z/\beta^* \sim 1.5$

T. Sen, FERMILAB-Pub-00/093-T (2000), Yu. Alexahin FERMILAB-TM-2148 (2001), A.Valishev et al., IPAC'12



Integrability in Round Beams

1. Axisymmetric beam-beam kick
   - $\beta_x = \beta_y$
   - $\varepsilon_x = \varepsilon_y$
2. Arc map with equal tunes, $Q_x = Q_y$

- Additional integral of motion $M = x\,p_y - y\,p_x$. Transverse motion equivalent to 1D. A.G. Ruggiero, Particle Accelerators, v.12 (1982)
- 2D integrability: V. Danilov and E. Perevedentsev, "Two Examples of Integrable Systems with Round Colliding Beams", PAC'97
  - System can be made integrable through proper longitudinal bunch shaping

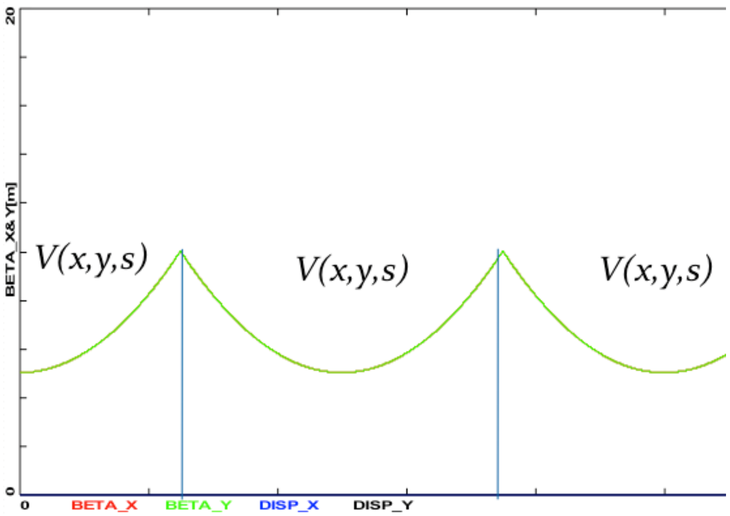
Recent Developments

- ξ = 0.25 at , σ_z/β^*~1. Round colliding beams at VEPP-2000 (BINP), A.Romanov et al., this Conference
- 2D integrability with Laplacian fields. V.Danilov and S.Nagaitsev, Phys. Rev. ST Accel. Beams 13, 084002 (2010)
- Advances in beam-beam simulation techniques. Frequency Map Analysis

Approach

- Construct the lattice
  1. Interaction Region $\beta_x = \beta_y$
  2. Arc cell, $Q_x=Q_y=n/2$

$$H = \frac{p_x^2 + p_y^2}{2} + \frac{x^2 + y^2}{2} + \beta(\psi) V\left(x\sqrt{\beta(\psi)}, y\sqrt{\beta(\psi)}, s(\psi)\right)$$



- Shape bunch density $\lambda(s)$
  1. *Ideal* distribution

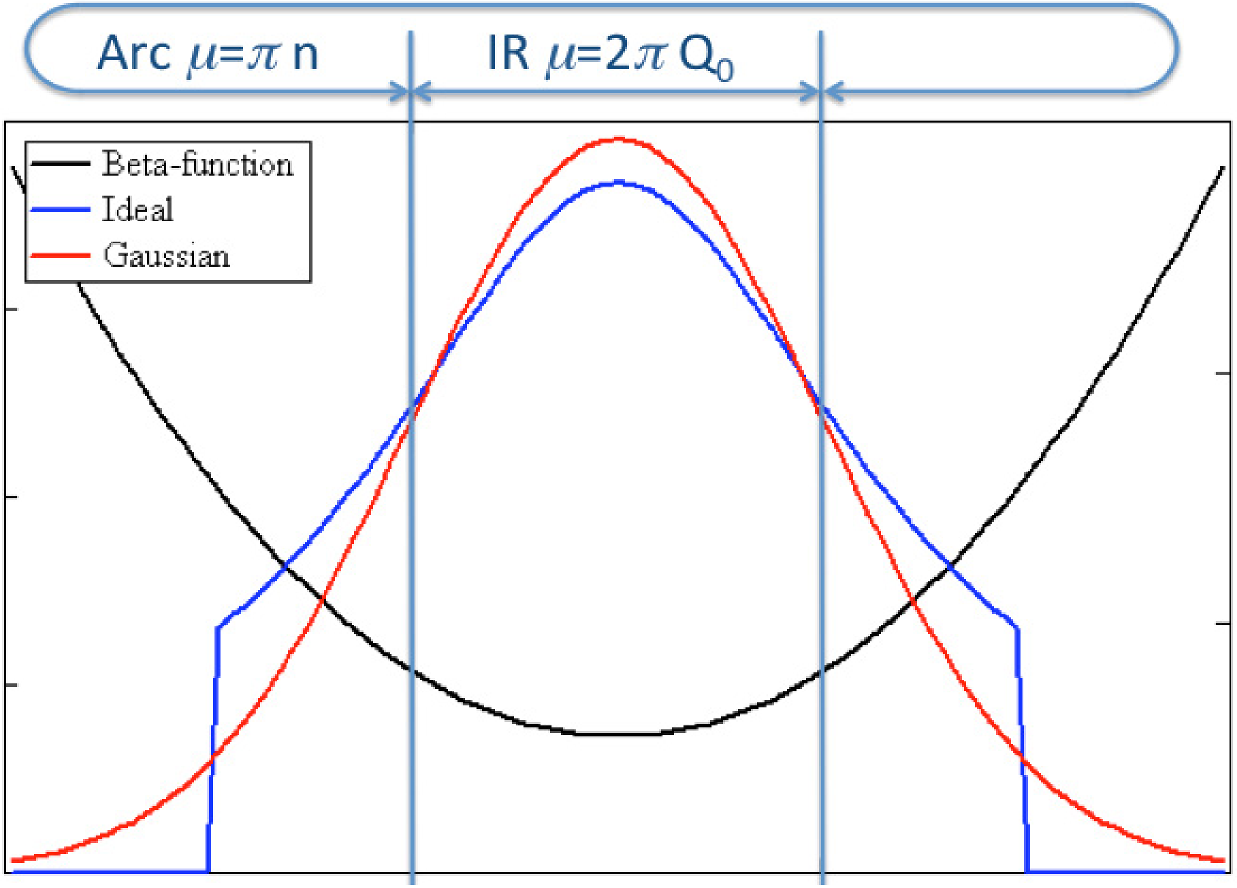
$$\lambda(s) \propto \frac{1}{\beta(s)} = \frac{1}{1+(s/2\beta^*)^2}$$

  2. Gaussian with $\sigma_z \sim \sqrt{2}\,\beta^*$

$$\lambda(s) \propto e^{-s^2/2\sigma_z^2} \approx 1 - s^2/2\sigma_z^2$$

- Weak-strong particle tracking code Lifetrac. FMA – qualitative analysis

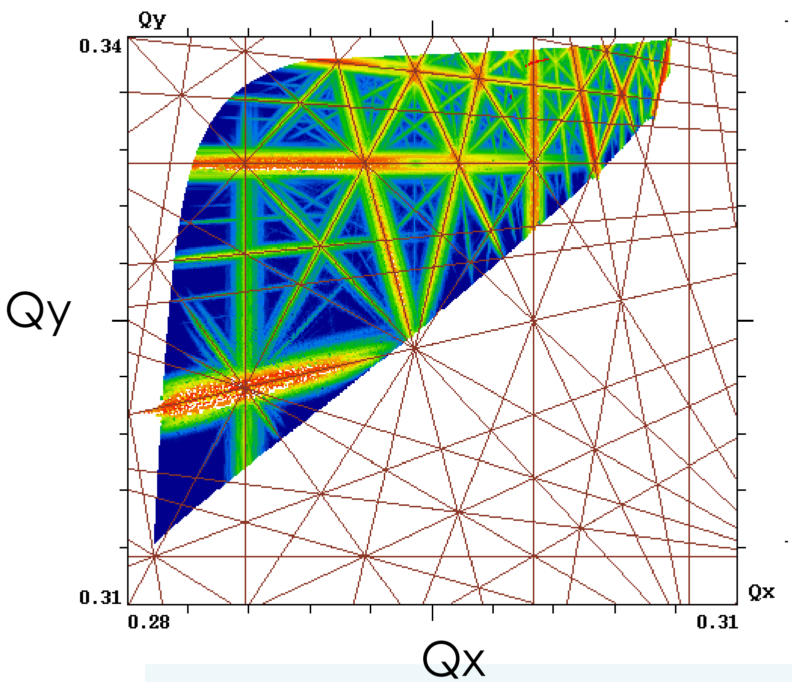
Model Lattice

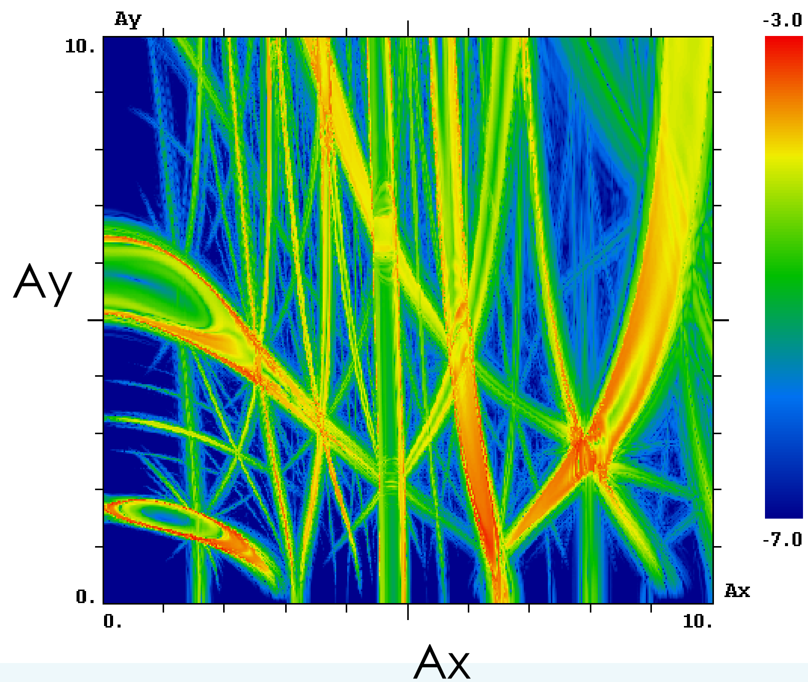


- Machine tune does not need to be close to integer or half-integer, $Q=n/2+Q_0$!

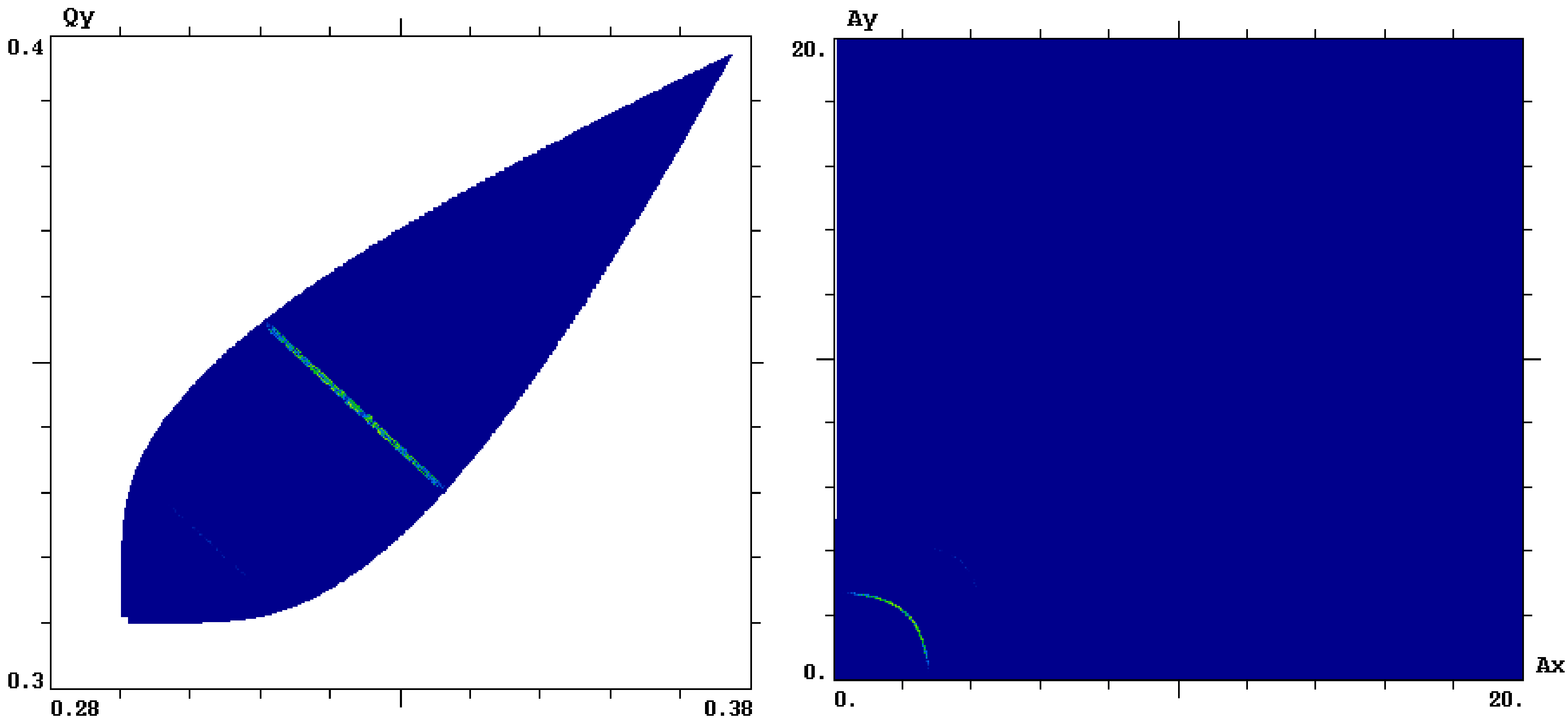
Frequency Map Analysis

- Based on precise tune determination from FFT of turn-by-turn particle coordinate (2D tracking)
- Evaluate tune jitter in sliding time window ➔ resonances
- When the dynamic system is (or very close to) integrable, nonlinear resonances disappear – this must be clearly seen in FMA plot
- Imperfections violate integrability. Then FMA easily allows to determine the regular and stochastic areas in phase space

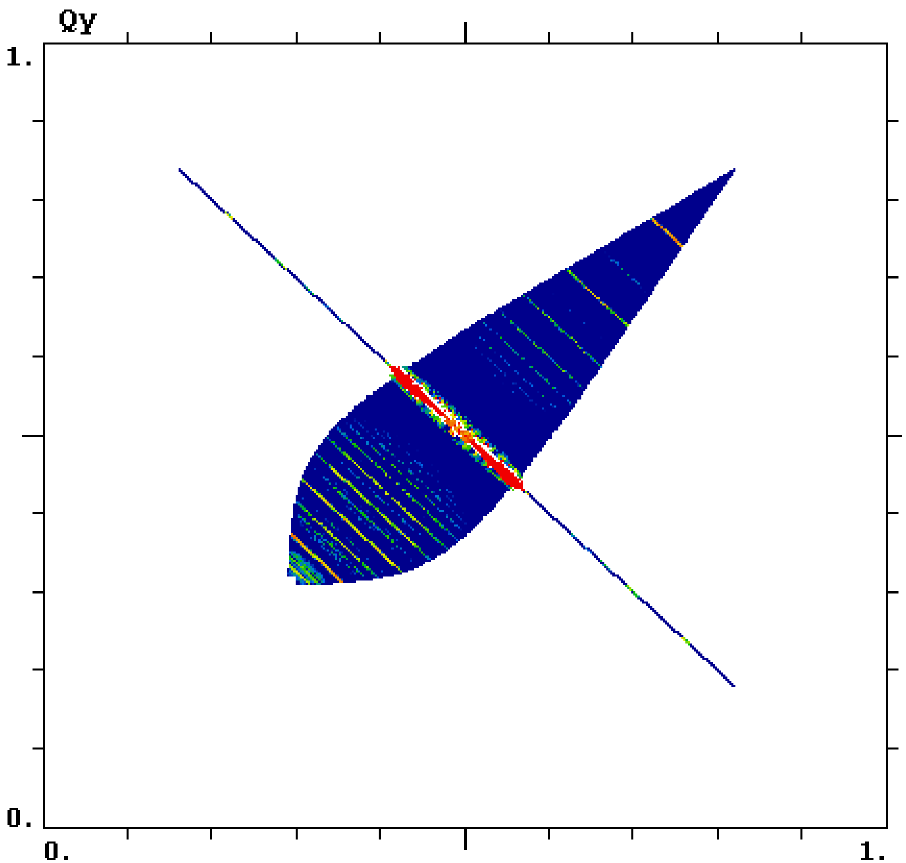


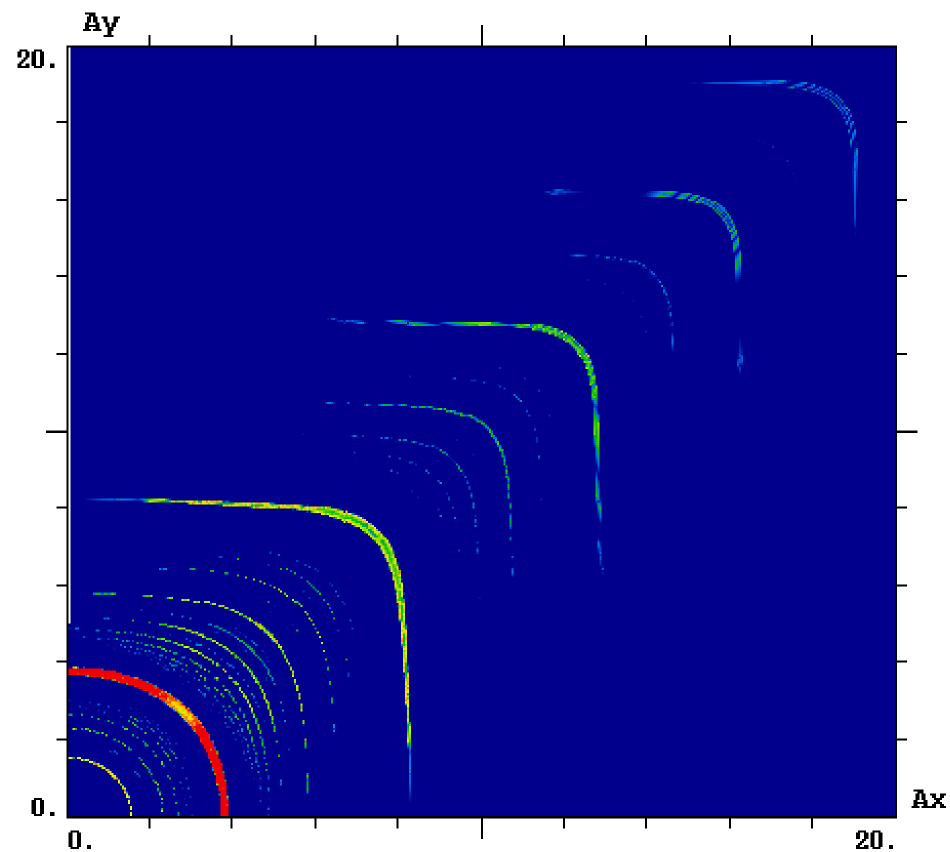


Ideal distribution Q = 0.3, $\xi = 0.1$

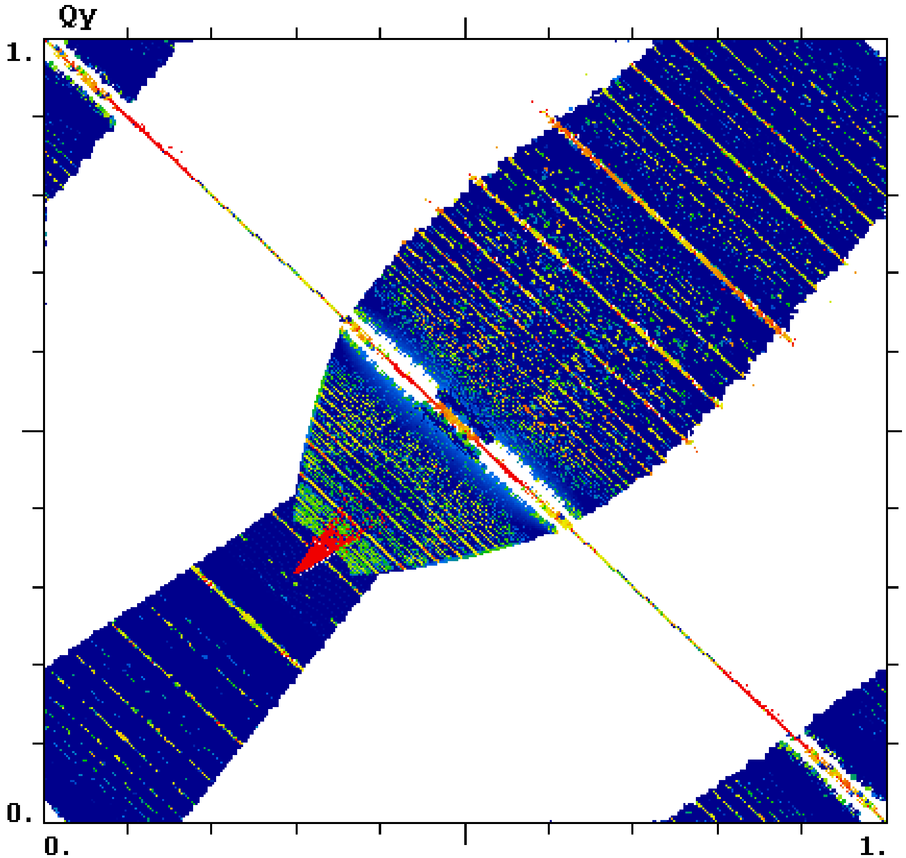


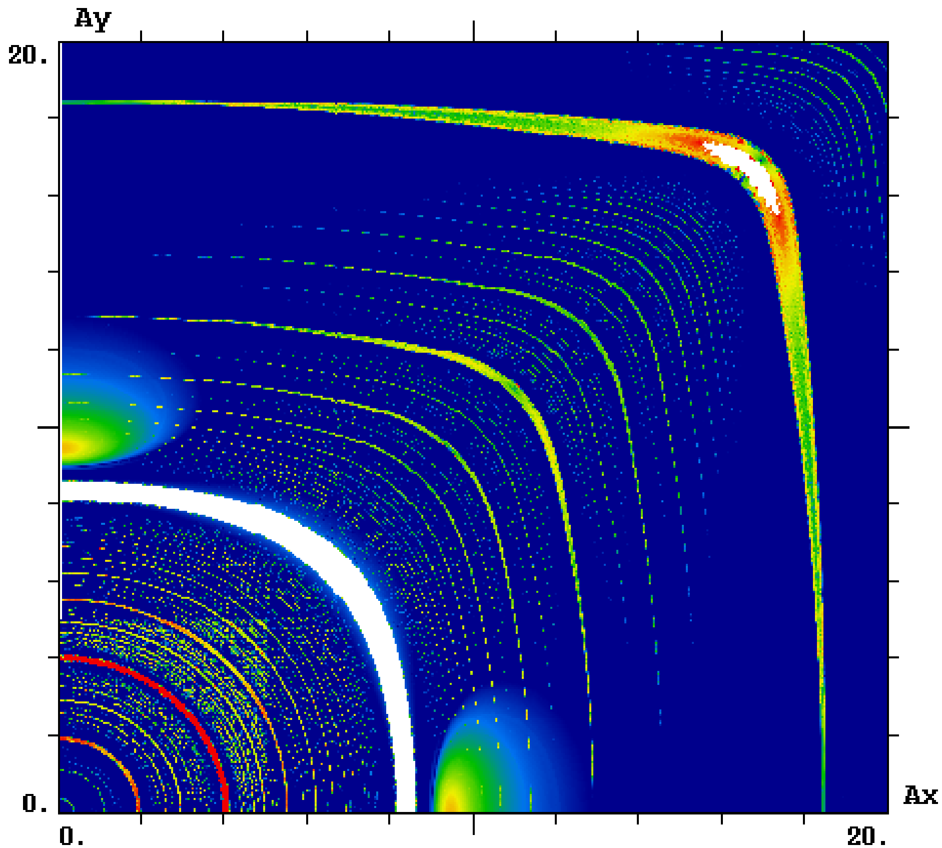
Ideal distribution Q = 0.3, $\xi = 1$



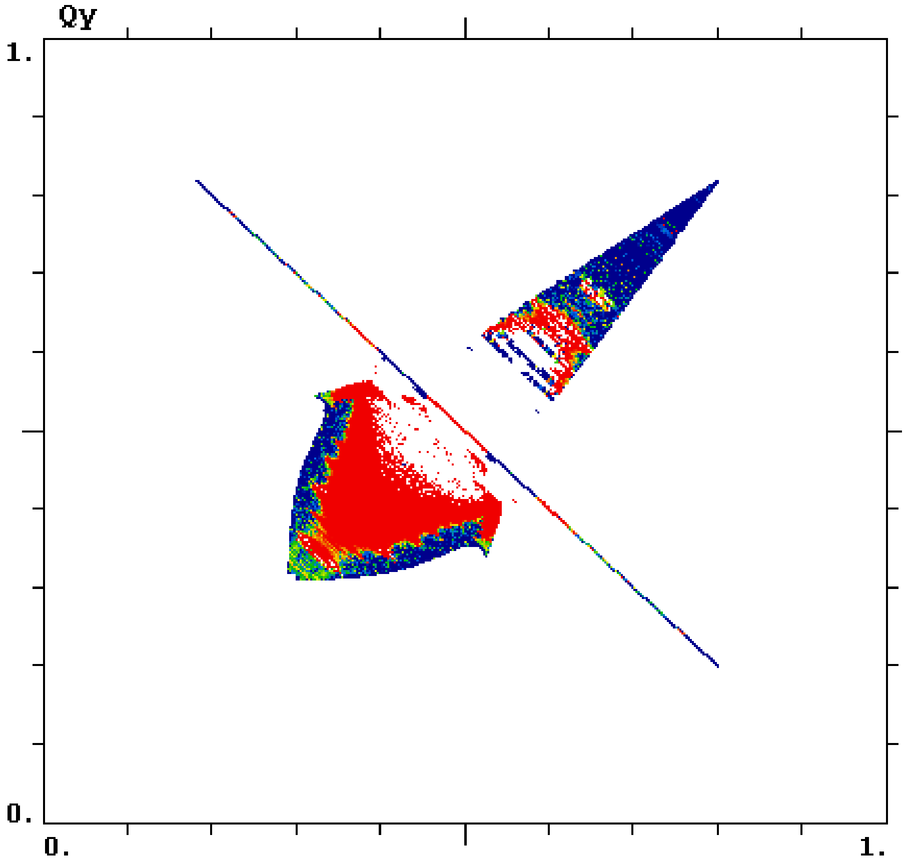


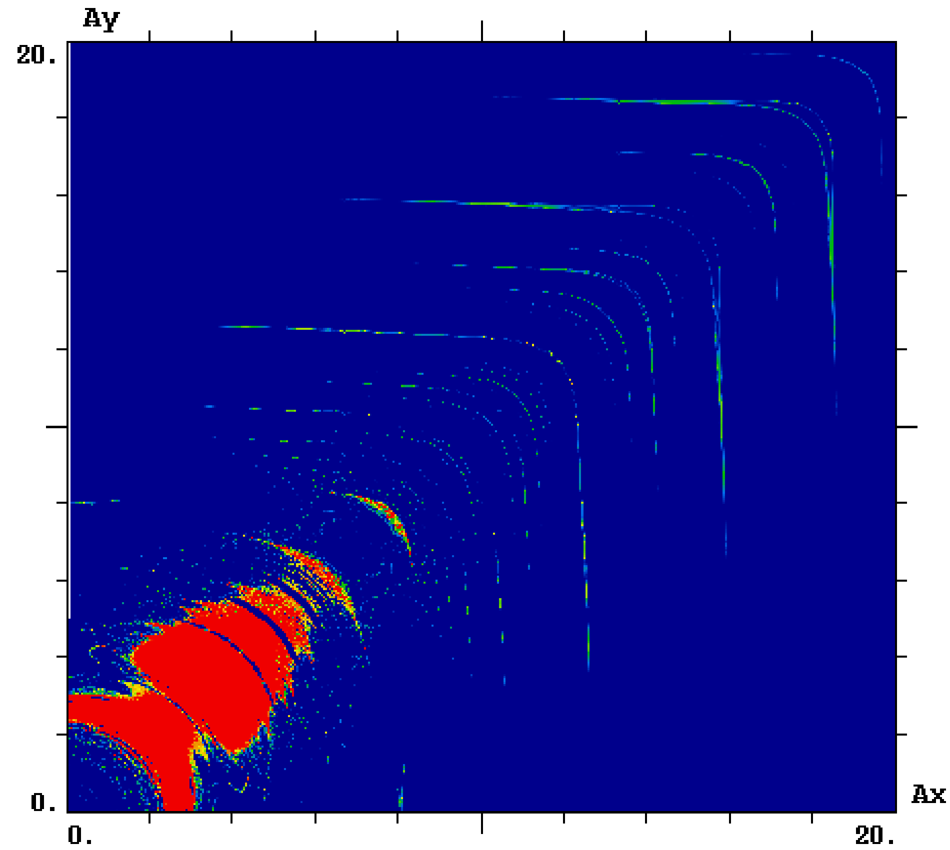
Ideal distribution Q = 0.3, $\xi = 5$



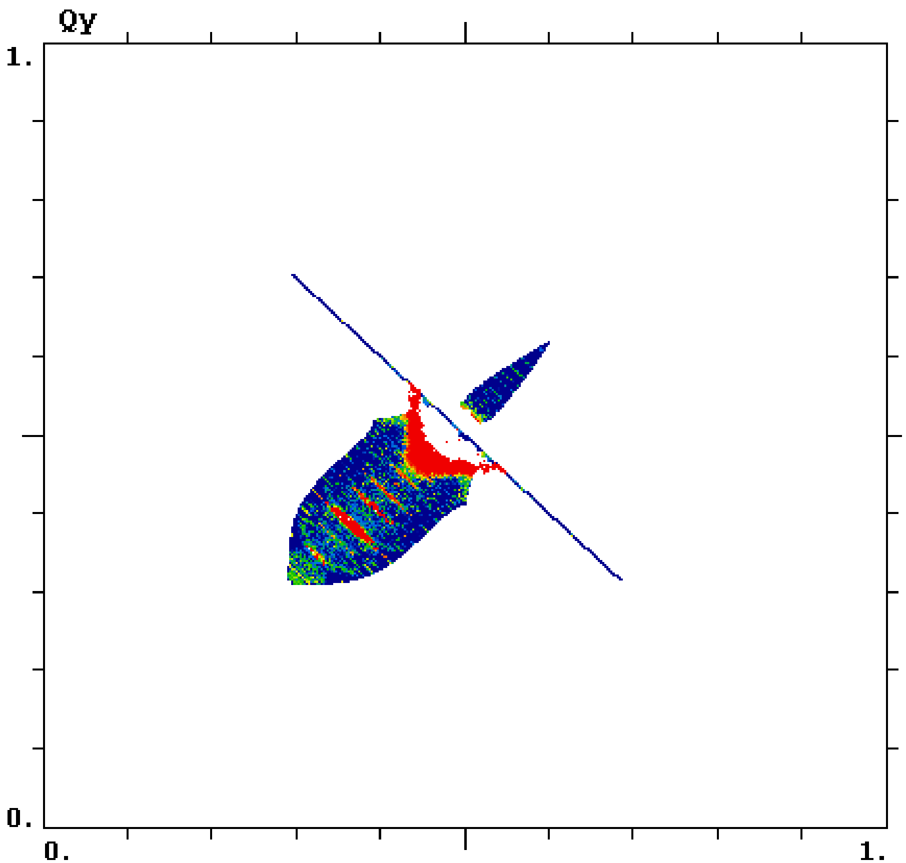


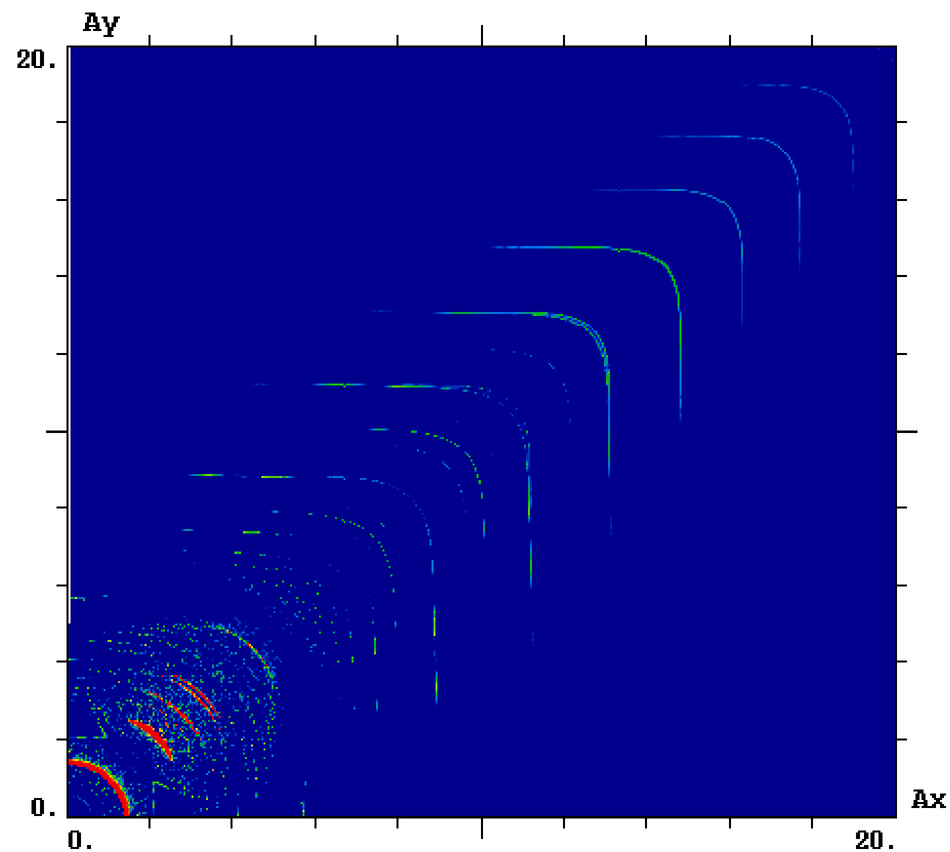
Gaussian distribution $\sigma=\sqrt{2}\beta$, $\xi=1$





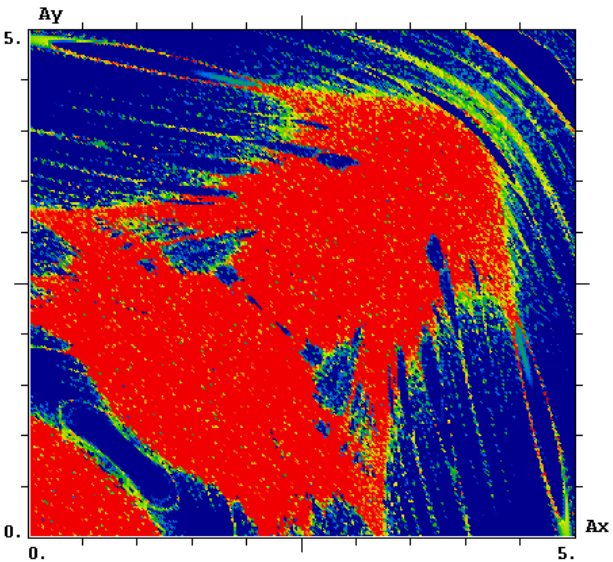
Gaussian distribution $\sigma=\sqrt{2}\beta$, $\xi=0.5$



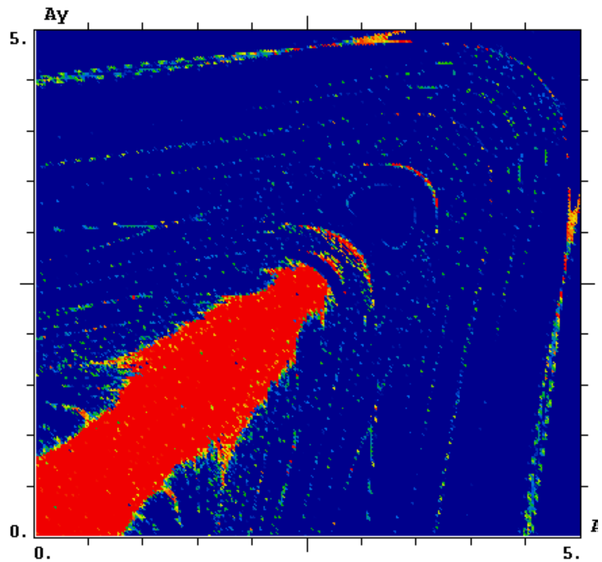


Effect of σ/β ratio, Q=0.4 ξ=0.5

$\sigma/\beta = 0.5$



1.0



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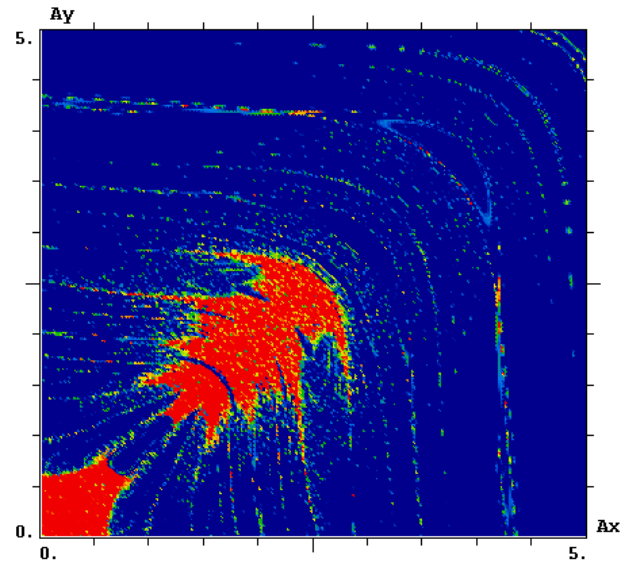
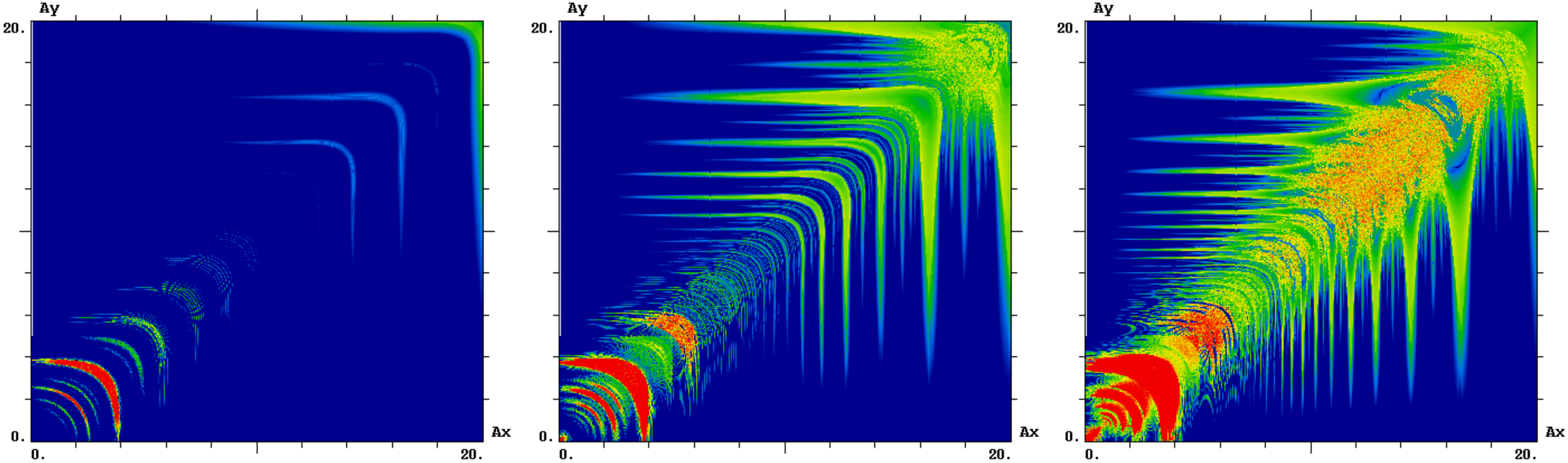


Effect of synchrotron oscillations

$a_s = 0.1$ 0.5 1.0



$\xi = 0.25$, Q=0.3

Summary

- We re-visited approach 2D integrability in Round Colliding Beams based on shaping of bunch longitudinal density
- Proper design of machine optics does not require working point close to integer
- In the 'ideal' case of $\lambda \sim 1/\beta$, large beam-beam parameter could be obtained for on-momentum particles
  - For Gaussian bunches, ξ is limited to ~0.5
  - Modulation for non-zero synchrotron amplitudes significantly disturbs integrability
- Optimization for VEPP-2000 is in progress
  - Proper account of arc nonlinearities
  - Calculations of luminosity and beam lifetime