COUPLING SPIN RESONANCES WITH SIBERIAN SNAKES*

Nermeen Khalil†, Stony Brook University, Stony Brook, NY 11794, U.S.A.
V. Ptitsyn, Brookhaven National Laboratory, Upton, NY 11973, U.S.A.

Abstract

The polarization of proton beam during the acceleration process in a particle accelerator is affected by the existence of spin resonances. Coupling spin resonances can be excited in the presence of the betatron coupling introduced by rolled quadrupoles and solenoids. A corresponding algorithm has been developed and added to the ASPIRRIN code to include the calculation of the first-order intrinsic resonance harmonics in the case of arbitrary strong betatron coupling and in the presence of Siberian Snakes and spin rotators.The analysis of the coupling resonance harmonics excited in RHIC collider is presented.

INTRODUCTION

Accelerating high energy polarized proton beams in a circular accelerator is complicated with the presence of depolarizing resonances. With the application of the various kind of spin rotating devices (like Snakes and spin rotators) the stable spin axis on the design closed orbit may deviate from the vertical, and an algorithm should be realized which would calculate the spin resonance harmonics with this complex configuration of the magnetic field on the design orbit. Main features of this algorithm for calculating the spin resonance strength were presented in [1]. In the work presented in this paper the algorithm has been extended to include the calculation of horizontal and vertical intrinsic spin resonance amplitudes for the case of strong betatron coupling, caused by rotated quadrupoles and solenoidal fields. We consider, as an example, the betatron coupling in RHIC accelerator which is originated mostly due to rolls of quadrupoles at the RHIC interaction regions and compensated using the local skew quadrupole correctors. In order to calculate the undesired coupling effects on the spin resonance strength an algorithm has been modified in ASPIRRIN [2] code to include the horizontal and vertical intrinsic spin resonance amplitude calculation for arbitrary rotated quadrupole in presence of the Siberian Snakes. In RHIC two Siberian Snakes are installed in each of the two RHIC rings [3].

ALGORITHM DESCRIPTION

In order to describe the spin motion which is initiated by the arbitrary rotated quadrupole , we would need to introduce the calculation of the spin perturbation in general. We consider the spin perturbation due to betatron oscillations as the linear form in the orbit variables $w_j = T_{iq}X_q$. The orbital coordinates and momenta of a particle will be defined as $X^T = (x, p_x, y, p_y)$, where x and y are the horizontal and vertical coordinates of a particle, and p_x and p_y are their conjugate momenta. The linear orbital motion can be defined as $X_q = F_{qr}A_r$ where F_{qr} is the matrix compiled from the complex eigenvectors of the orbital motion, and A_r is the vector of the betatron amplitudes.

Here we are considering the spin perturbation theory in the coordinate system $(\mathbf{l}, \mathbf{m}, \mathbf{n_0})$, where $\mathbf{n_0}$ is the stable spin solution on the design orbit. In this coordinate system a spin vector on the design orbit rotates with the constant spin precession tune ν around $\mathbf{n_0}$. The vector $\mathbf{k} = \mathbf{l}+\mathbf{i}\mathbf{m}$ is related with the spin eigenvector $\mathbf{k_0}$ by $\mathbf{k} = \mathbf{k_0}\cdot e^{i\nu\theta}$, where θ describes the accelerator azimuth. In Eq. 1 the first-order spin resonances are defined as the coefficients of Fourier decomposition of the spin perturbation term

$$\mathbf{w}\cdot\mathbf{k} = \sum_{r,p} \epsilon_{rp} e^{i(p+Q_r)\theta} = \sum_{r,p} A_r \tilde{v}_{rp} e^{i(p+Q_r)\theta} \qquad (1)$$

where p are integer numbers and Q_r present components of the orbital betatron tune vector $(Q_I, -Q_I, Q_{II}, -Q_{II})$. Q_I and Q_{II} are tunes of two betatron oscillation modes. In order to calculate $\tilde{v}_{rp}$ in the program the integral over one turn is taken doing element-by-element integration [1].

$$\tilde{v}_{rp} = \frac{1}{2\pi}\int_0^{2\pi} e^{i\delta_{rp}\theta} \sum_{j,q} T_{jq}\mathcal{F}_{qr}k_{0j}d\theta \qquad (2)$$

where $\delta_{rp} = \nu - (p + Q_r)$ describes detuning of the spin tune from a resonance tune.

In order to obtain $\tilde{v}_{rp}$ for rotated quadrupole we start with considering the following equations of motion of particle in arbitrary rotated quadrupole:

$$x^{''} = -g_c x - \kappa_c y \qquad y^{''} = g_c y - \kappa_c x \qquad (3)$$

where $g_c = g\cdot cos(2\phi)$ and $\kappa_c = g\cdot sin(2\phi)$ are defined by the quadrupole rotation angle ϕ and the quadrupole field gradient g.

From (3) we can get the expressions for the orbital motion eigenvectors which will be used later in the paper :

$$F_{3r} = -AF^{''}_{3r} + BF^{''}_{1r} \qquad F_{1r} = AF^{''}_{1r} + BF^{''}_{3r} \quad (4)$$

Here:

$$A = -cos(2\phi)/g \qquad B = -sin(2\phi)/g \qquad (5)$$

The components of horizontal and vertical components of the spin perturbations in a quadrupole are given by:

$$w_x = (1+\nu_0)y^{''} \qquad w_y = -(1+\nu_0)x^{''} \qquad (6)$$

* Work supported by Brookhaven Science Associates, LLC under Contract No. DE-AC02-98CH10886 with the U.S. DOE
† nermeen.khalil@stonybrook.edu

ISBN 978-3-95450-138-0

where $\nu_0 = G\gamma$, with magnetic moment anomaly G and relativistic factor γ. From (1) and (6) the contribution to a spin resonance harmonic produced by a betatron mode oscillating with the tune Q_r from a quadrupole can be formulated as the following:

$$\tilde{v}_{rp}^{quad} = (1+\nu_0)(k_{0x}I_3 - k_{0y}I_1) \quad (7)$$

where I_1 and I_2 are the integrals defined as

$$I_1 = \int_{\theta_1}^{\theta_2} F_{1r}^{''} e^{i\delta_{rp}\theta} d\theta \quad (8)$$

$$I_3 = \int_{\theta_1}^{\theta_2} F_{3r}^{''} e^{i\delta_{rp}\theta} d\theta \quad (9)$$

and θ_1 and θ_2 correspond to the entrance and the exit of the quadrupole.

To find the integrals (8) and(9) one can apply twice the integration by parts and the expressions (4) to get the set of linear equations for I_1 and I_3. Resolving the equations one gets the solution in the matrix form:

$$\begin{pmatrix} I_1 \\ I_3 \end{pmatrix} = M^{-1} \begin{pmatrix} C_1 \\ C_3 \end{pmatrix} \quad (10)$$

where M, C_1 and C_3 are expressed as

$$M = \begin{pmatrix} 1 + A\delta_{rp}^2 & B\delta_{rp}^2 \\ B\delta_{rp}^2 & 1 - A\delta_{rp}^2 \end{pmatrix} \quad (11)$$

$$C_3 = \left[(F_{3r}^{'} - i\delta_{rp}F_{3r})e^{i\delta_{rp}\theta}\right]_{\theta_1}^{\theta_2} \quad (12)$$

$$C_1 = \left[(F_{1r}^{'} - i\delta_{rp}F_{1r})e^{i\delta_{rp}\theta}\right]_{\theta_1}^{\theta_2} \quad (13)$$

Then the final formula which defines the contribution from an individual rotated quadrupole to the spin resonance harmonic is:

$$\tilde{v}_{rp}^{quad} = Y[k_{0x}(\delta_{rp}^2 \sin(2\phi)C_1 + (g - \delta_{rp}^2\cos(2\phi))C_3) - k_{0y}((g + \delta_{rp}^2\cos(2\phi))C_1 + \delta_{rp}^2 \sin(2\phi)C_3)] \quad (14)$$

where

$$Y = g(1+\nu_0)/(g^2 - \delta_{rp}^4) \quad (15)$$

Summing these contributions from all quadrupoles gives us the full spin resonance harmonic defined by r and p.

Using similar technique the spin resonance harmonics contribution from a solenoid was derived as:

$$\tilde{v}_{rp}^{sol} = \frac{(G-\nu_0)K_s}{\delta^2 - (GK_s)^2}(-i\delta_{rp}[e^{i\delta_{rp}\theta}((F_{2r}+\frac{1}{2}K_sF_{3r})k_{0x} + (F_{4r}-\frac{1}{2}K_sF_{1r})k_{0y})]_{\theta_1}^{\theta_2} + GK_s[e^{i\delta_{rp}\theta}((F_{2r}+\frac{1}{2}K_sF_{3r})k_{0y} - (F_{4r}-\frac{1}{2}K_sF_{1r})k_{0x})]_{\theta_1}^{\theta_2}) + \frac{(1+\nu_0)K_s}{2}\left[e^{i\delta_{rp}\theta}(F_{1r}k_{0x} + F_{3r}k_{0y})\right]_{\theta_1}^{\theta_2} \quad (16)$$

where K_s is the normalized field of the solenoid and the last term describes the contribution from the solenoid ends.

CALCULATIONS FOR THE BETATRON COUPLING AT RHIC

We would show some tests which have been done by ASPIRRIN after modifying it with the new algorithm. The main motivation for our calculation was to test the code ability of calculating the resonance strength amplitude in the general case for the betatron oscillation coupling. The calculations were done for the RHIC lattice with two Siberian Snakes, which had uncoupled tunes $Q_x = 27.685$, $Q_y = 29.673$ with betatron amplitudes of $10\mu m \cdot rad.$ in both planes.

The spin tune in RHIC with the Siberian Snakes is equal to 0.5. because of that in all plots in this section we show the result for resonance harmonics which are closest to 0.5 value (that is with the resonance tunes in $[0-1]$ range). Moreover, we add together the amplitudes of the two spin resonance harmonics, one below and other above 0.5, corresponding to the same betatron tune (Q_I or Q_{II}). That gives the resonance amplitude of the linear resonance harmonic, which is more natural when considering an accelerator with the Siberian Snake [1].

First we started by varying the field strength of different IR skew quadrupoles correctors, which are used to compensate for the coupling caused by the rolled quadrupoles, and observing how the spin resonance harmonics depend on the betatron coupling, which is characterized by minimum betatron tune split ΔQ_{min}. In figure 1 a typical behavior of the vertical and horizontal resonance harmonics are shown by varying the gradient strength of skew-quadrupole (SQ06C2B), the horizontal resonance amplitude would increase non linearly while the vertical amplitude would behave the opposite.

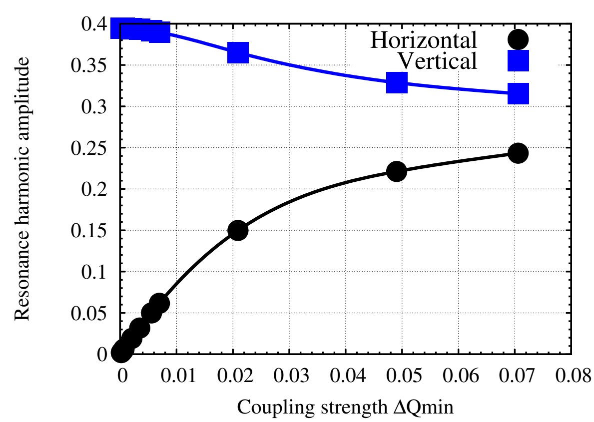

Figure 1: Dependence of the vertical and horizontal resonance harmonic amplitudes on the coupling strength when varying a skew quadrupole corrector. $G\gamma = 422.3$.

Figure 2 demonstrates another typical feature of the resonance harmonic dependence observed during the skew quadrupole variation studies. While the vertical and horizontal harmonics changes the sum of their squares remains approximately constant: $\tilde{v}_{hor}^2 + \tilde{v}_{ver}^2 \approx const.$ This indicates a rotational type transformation between the horizontal and vertical resonance harmonics when the betatron coupling is introduced.

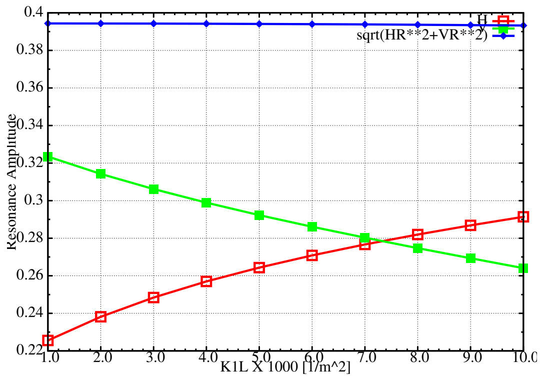

Figure 2: Conservation of the sum of squares of the horizontal and vertical resonance harmonic amplitudes when varying a skew quadrupole corrector(SQ08C2B). $G\gamma = 422.3$.

Next, we considered actual coupling errors present in RHIC, where strong sources of the the betatron coupling are due to the rolls of the IR quadrupoles. These quad rolls are well known from the beam-based and magnetic measurements. Thus, we applied the known rolls and well as local skew quadrupole corrector strengths used at RHIC operation [4]. Figure 3 shows the calculated values of the vertical resonance amplitudes versus $G\gamma$ in a region of strong resonances in RHIC. The figure shows some reduction of the values of vertical harmonics amplitude after applying the quad rolls and skew quad correction.

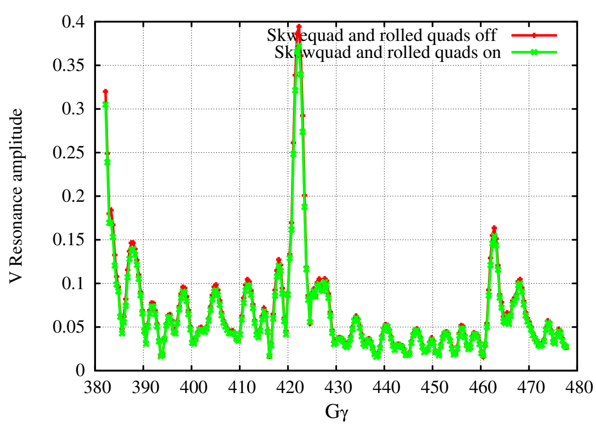

Figure 3: Calculated vertical spin resonance harmonics with and without the actual RHIC IR quadrupole rolls and local IR skew corrections.

Figure 4 shows the calculated values of the the horizontal resonance amplitude versus $G\gamma$ and it is observed that the horizontal resonance amplitudes are excited up to 0.15 when the actual quadrupole rolls and skew quad corrector strengths are used (in this case $\delta Q_{min} \approx 0.01$). After that we used optimized values for certain local skew quadrupole gradients, which reduced the δQ_{min} coupling parameter to 0.001. The horizontal resonance harmonics are clearly reduced in this case. The plot also shows that the horizontal resonance amplitudes are suppressed to zero when the skew quadrupoles are turned off and the quadrupole rolls are 0.

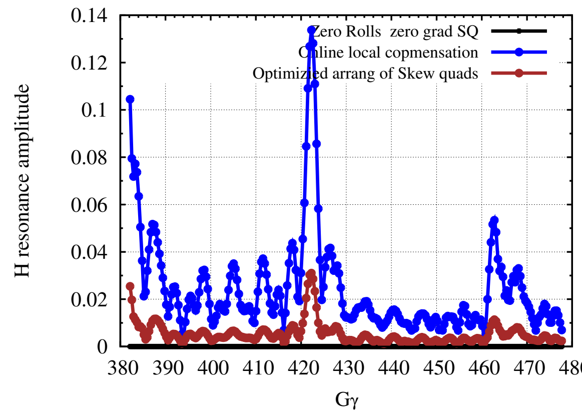

Figure 4: Calculated horizontal spin resonance harmonics with and without the actual RHIC IR quadrupole rolls and local IR skew corrections. The result for optimized corrections is also shown.

SUMMARY

The calculation of spin resonance harmonics with coupled transverse betatron motion was implemented in the ASPIRRIN code. We noticed a rotation-type transformation of the resonance strength of vertical and horizontal harmonics when introducing the betatron coupling. The local coupling correction for actual values of quad rolls and IR skew quad correctors at 6 Interaction regions in RHIC results in exciting the horizontal resonance strength up to 0.15 at $G\gamma$ of value of 422.3. Optimizing the skew quadrupoles arrangement for better compensation of the local coupling helps to reduce the horizontal spin resonance amplitudes even before applying the global coupling correction.

REFERENCES

[1] V. Ptitsyn and N. Khalil, "Calculation of spin resonance harmonics in an accelerator with Snakes' submitted to Proceed. of 20th Internat. Symposium on Spin Physics, Dubna, September 2012.

[2] V. Ptitsyn, S.R. Mane and Yu.M. Shatunov, Nucl. Instr. and Meth. A, V.608 2009 p.225;

[3] I. Alekseev, *et al.*, "Configuration Manual, Polarized Proton Collider at RHIC".

[4] Y. LUO, *et al.* , "REVISIT LOCAL COUPLING CORRECTION IN THE INTERACTION REGIONS OF RHIC", BNL Report C-A-AP-439 (2011)

ISBN 978-3-95450-138-0