STABILITY OF EMITTANCE VS. SPACE-CHARGE DOMINATED BEAMS IN AN ELECTRON RECIRCULATOR *

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Abstract

We report on experiments and simulations of beam lifetime at the University of Maryland Electron Ring (UMER) for an emittance as well as a strongly space-charge dominated beam. We note the presence of expected (and strong) integer resonances for both beam currents and the absence, for moderate envelope mismatch, of some half-integer resonances for the high current beam only. The observations are related to particle-tracking simulations with the matrix code ELEGANT. The simulations employ a simple incoherent space charge model for a continuous beam, as well as lattice and magnet errors.

INTRODUCTION

The operation of all circular accelerators rely on careful selection of the betatron tunes for optimal beam quality and lifetime. In addition, when collective effects are an important factor, systematic study of current-dependent phenomena, e.g. envelope resonance conditions, is required. The basis for our investigation is the University of Maryland Electron Ring (UMER), a low-energy, high-current compact machine dedicated to beam physics research [1, 2]. We discuss the use of a simple computer model and show that the results are consistent with the main features of experimental beam lifetime charts, i.e. charts that display the surviving fraction of injected beam current at a given turn as a function of betatron bare tunes. Of particular interest is to compare the beam lifetimes of emittance-dominated and space-charge dominated beams.

When space charge is a factor, realistic 6D computer simulations of beam evolution over even a few turns is very computer intensive. Customarily, particle-in-cell codes are employed for tracking hundreds of thousands or even million of macroparticles. Other calculations with space charge such as transverse rms envelope matching are performed with matrix codes such as TRACE3D [3] which employ a linear space charge model and do not allow particle tracking. By contrast, the matrix code ELEGANT [4] can be adapted to do particle tracking in the presence of strong transverse space charge including non-linearities. The ELEGANT model of transverse space charge is only approximate but has yielded satisfactory results in envelope as well as dispersion calculations in UMER [5].

EXPERIMENTS

We have conducted experiments with two electron beams, 0.6 mA and 6.0 mA, both at 10 keV. The initial normalized rms emittances are 0.4 μm (0.6 mA) and 1.3 μm (6.0 mA); the bunch duration is 100 ns for both beams. In one set of experiments, all 70 DC magnetic ring quadrupoles are powered with currents varying from 1.65 to 2.10 A in 10 mA steps, while the injection (pulsed) quadrupoles (YQ and QR1) are kept fixed at the nominal operating currents. For each operating set of quad currents, the transmitted peak beam current is measured with a wall-current monitor located roughly half-way around the ring. In the second set of experiments, the value of one of the injection quads (QR1) is also varied so its focal length equals the corresponding one for the ring focusing quadrupoles. The other injection quadrupole (YQ) could not be varied because it plays a critical role for the injection angle into the closed orbit.

Figure 1a displays the results of beam lifetime at the 10th turn as a function of *estimated* bare tunes for the 0.6 mA beam (emittance dominated) when varying the strength of QR1. Deep red in the figures indicates 90-100% transmission while deep blue or purple near complete loss. The overall beam transmission for the 0.6 mA beam at the 10th turn is slightly improved by varying QR1 (the results with fixed QR1 are not shown), particularly far from the nominal operating point (I_F, I_D) = (1.826,1.826)A, indicated in Fig. 1a by "0". "$I_{F,D}$" refer to the currents of focusing and defocussing ring quadrupoles. Also worth noting is that good transmission for 0.6 mA occurs for $\nu_{0x} \gtrsim 6.5$, but it is greatly reduced for $\nu_{0x} \lesssim 6.5$.

Figures 1 b-c illustrate the beam lifetime results for the 6.0 mA beam (space-charge dominated at injection). The results of varying QR1 are more dramatic for the 6.0 mA beam. In this case, the beam losses near the nominal horizontal bare tune ν_{0x} = 7.5 are markedly reduced. Further, significant beam transmission occurs for highly asymmetric focusing, around (ν_{0x}, ν_{0y}) = (5.5, 8.5), unlike the case for 0.6 mA (Fig. 1a.) Finally, beam transmission for 6.0 mA displays an asymmetry around the line defined by $\nu_{0x} + \nu_{0y}$ = 13, corresponding to a sum resonance. Notice that beam transmission, overall, is noticeably better for the 6.0 mA beam than for the 0.6 mA, with e.g. no visible beam loss at ν_{0x} = 6.5 for the former.

The *estimated* bare tunes in Fig. 1 are determined through a transformation of the quadrupole current space, which is square [(1.65, 2.09) A×(1.65, 2.09) A in 10 mA steps], into the trapezoidal tune space using an effective quadrupole peak gradient per amp of g_0 = 3.95 G/cmA and standard matrix formulas (specifically, the inversion of eqs. 18-ab in Ref. [6].) The choice of g_0 corresponds to a calibration to the observed integer-resonance bands for the *low-current* beam (Fig. 1a). Another model of UMER in ELEGANT based on fitting g_0 to match results from response matrix measurements leads to a slightly different tune calibration [7]. We stress the fact that the plots in Fig. 1 are meant mostly as a guide to study beam lifetime

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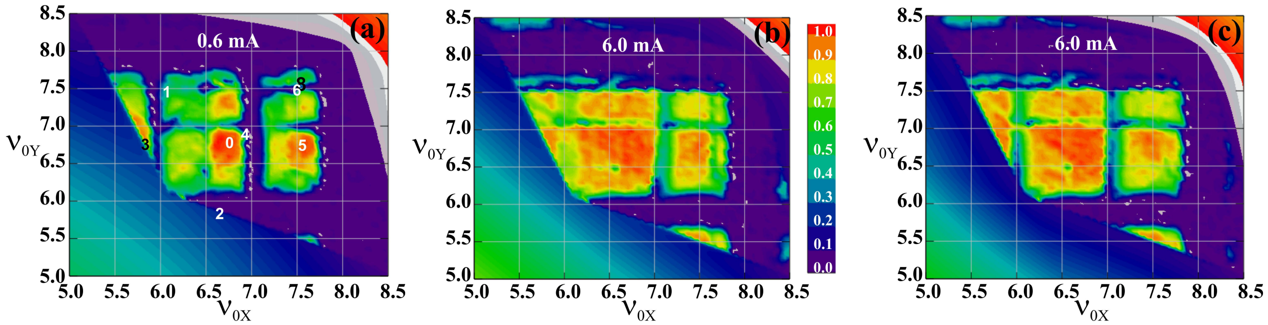

Figure 1: Beam lifetime charts at 10th turn as a function of estimated bare tunes (x: horizontal, y: vertical): (a) 0.6 mA beam (emittance dominated) with varying strength of injection quadrupole QR1 (see text ant Table 1); (b) 6.0 mA beam (space charge dominated) with fixed strength of QR1; and (c) 6.0 mA beam with varying strength of QR1. The green and red areas on opposite corners are artifacts of the graphics.

as a function of tunes, and that a more accurate calculation of tunes requires inclusion of all magnets. In fact, including the injection quadrupoles, and especially a realistic model of the main bending dipoles is important. The dipoles are used also as orbit correctors in UMER; the corrections compensate for the action of the vertical earth's field, the quadrupole radial displacement errors, and the machine imperfections.

Image forces also modify the tunes by values measured to be -0.003 and -0.03 for 0.6 and 6.0 mA beams, respectively, at the default operating point of 1.826 A [8]. The effect of image forces is seen by comparing Figs. 1a and 1b: e.g. the occurrence of the resonance at $\nu_{x0} = 7.0$ is shifted to an *apparently higher* value for the 6.0 mA beam relative to the 0.6 mA beam.

COMPUTER SIMULATIONS

The lattice model in ELEGANT employs hard-edge focusing and bending elements following the model described in [6], 3 injection elements (2 wide-aperture quadrupoles and one dipole), and a uniform 0.4 G earth's *B*-field pointing downward. A small quadrupole focusing term K1 = 3.45 m^{-2} is also included at every dipole; this arises from the presence of a sextupole component and the sagitta effect of particle orbits. Furthermore, *symplectic* quadrupoles and dipoles are employed.

We include magnet alignment and strength errors for the calculations: displacement errors of (rms) amplitude $(DX, DY) =$ (0.3, 0.1) mm; ring quadrupole strength error of amplitude $\Delta K1 = 1.0\ \mathrm{m}^{-2}$; and 2 mrad quadrupole and dipole tilt angle errors. The distribution of errors is Gaussian in all cases, with a cutoff of 2σ. Furthermore, we link the radial (DX) alignment errors of ring quadrupoles in groups of 4 as in the actual ring sections. We also implement a simple model of incoherent *transverse* space charge for a *continuous* beam [5], but image forces and momentum errors are *not* included. Also missing in the model are a number of vertical-steering corrector magnets.

The simulations are run over 10 turns with initial *beam* Courant-Snyder parameters that correspond to near rms-envelope matched conditions, and particle tracking is performed with 10K particles. We limit the simulations to 10 turns mainly because effects from debunching are not included. These effects, which manifest after about 20 turns for the 6.0 mA beam, render ineffective the measurement of the AC component of the beam current which is key for quantifying the beam lifetime as a function of tunes.

Out of the more than 2,000 operating tunes represented in Figs. 1, we focus our attention on a small number for tune measurements and for calculations in ELEGANT. The tunes are obtained by doing sinusoidal fits to 3 sets of difference orbits (for 6.0 mA); the resulting errors are 0.03 to 0.1. We calculate the small-amplitude tunes in ELEGANT by running a simple lattice model, with no magnet errors, over one turn. The Table below summarizes the results for 9 operating points; these are also indicated as numerals in Fig. 1a. The calculated and measured tunes agree in all cases but one to better than 0.08.

Table 1: Calculated (ELE.) and Measured (EXP.) Horizontal (x) and Vertical (y) Betatron *Linear* Tunes. Injection quad YQ is fixed at $K1 = -99.86\ \mathrm{m}^{-2}$ (6.0 mA beam) for most points, while injection quad QR1 is varied (see text and Fig. 1a.)

	Ring Quad Currents I_F**(A)**, I_D**(A)**	ν_{0x} ELE.	ν_{0y} ELE.	ν_{0x} EXP.	ν_{0y} EXP.
0	1.826, 1.826	6.744	6.850	6.677	6.850
1	1.750, 1.920	6.066	7.521	N/A	N/A
2	1.750, 1.650	6.641	5.908	N/A	N/A
3	1.650, 1.750	5.768	6.756	5.787	6.740
4	1.850, 1.850	6.852	6.950	6.727	6.902
5	1.960, 1.860	7.538	6.787	7.555	6.793
6	1.990[1], 1.990[1]	7.453	unst.	7.531	7.632
7	1.990[2], 1.990[2]	7.443	7.567	7.452	7.525
8	2.000[3], 2.000[3]	7.536	7.610	7.570	7.601

[1] Inj. quads: (YQ,QR1) = ($-$99.86,114.43) m^{-2}
[2] Inj. quads: (YQ,QR1) = ($-$114.43,114.43) m^{-2}
[3] Inj. quads: (YQ,QR1) = ($-$115.00,115.00) m^{-2}

At operating points like 0 and 5 in Table 1 and Fig. 1a, beam transmission at the 10th turn is essentially 100%. The magnet and lattice errors assumed (see above) lead in calculations to centroid oscillations of maximum amplitude of close to 6 mm and 10 mm in the horizontal and vertical planes, respectively, at point 0. At point 5, the calculated centroid oscillations are 4 mm and 10 mm, maximum amplitude. The magnitude of the oscillations in the vertical plane is about a factor of 2 larger than observed in beam-position-monitor data and also in results from simple calculations. Also regarding operating point 5, we note the

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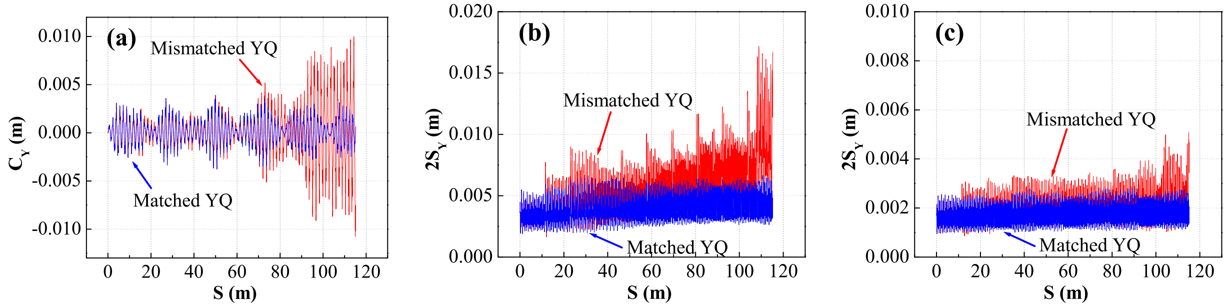

Figure 2: Results of calculations (*with* incoherent transverse space charge) in ELEGANT over 10 turns at (1.99, 1.99)A operating point in UMER: a) Vertical component of centroid motion for either 0.6 or 6.0 mA beams; b) 6.0 mA beam's vertical 2×rms semi-axis; and c) 0.6 mA beam's vertical 2×rms semi-axis. The strength of injection quadrupole YQ is increased by ∼15% to yield near-matched envelope conditions past $\nu_{0y} = 7.5$. The vacuum pipe radius is 0.025 m.

proximity to $\nu_{0x} = 7.5$, but the fact that beam transmission is unaffected at the 10th turn (Fig. 1c). Furthermore, envelope calculations with the "unmatched" original strength of the injection quad QR1, corresponding to Fig. 1b, show a steep increase in the 2×rms horizontal envelope of the 6.0 mA beam to almost 16 mm before the 10th turn, but only a modest increase for the 0.6 mA under the same conditions. Thus, retuning of QR1 in calculations makes a far more noticeable difference for high current than for low current, as in the experiment.

At four operating points (1, 2, 4, and 6 in Table 1), the beam circulates for 5 turns or less, insufficient to allow us to measure the tunes accurately at all these points. In ELEGANT, point 1 corresponds to tunes near (ν_{0x}, ν_{0y}) = (6.0, 7.5), while point 2 leads to tunes near (6.5, 6.0). The large centroid excursions seen in calculations for point 1 can explain the almost complete beam loss after only a few turns. Point 4 has (ν_{0x}, ν_{0y}) near (7.0, 7.0) and leads to the largest centroid oscillations and also to steep envelope growth after only 5 turns. Lastly, point 6 leads to an unstable vertical orbit and corresponding growing vertical centroid oscillations and envelope growth.

In Fig. 2a, we show the results of vertical centroid motion for two values of the strength of the injection quad YQ corresponding to "mismatched" and "matched" focal lengths, i.e. points 6 and 7 in Table 1. Also shown (Fig. 2b) are the vertical envelopes (2×rms semi-axis) for the 6.0 mA beam. These results are consistent with experiment: point 6 yields only 5 turns while point 7 recovers almost 80% beam transmission. A simple calculation using $\Delta\nu_0 = (4\pi)^{-1}\bar{\beta}(\Delta k_0)L$, where $\bar{\beta} = 0.22$ m, $\Delta k_0 = +15$ m^{-2}, $L = 7.45$ cm, shows that retuning YQ, as described above, leads to $\Delta\nu_{0y} = +0.020$, compared with $\Delta\nu_{0x} = -0.010$ from Table 1 (ELE.) However, no conclusive change in tunes can be measured because of the large errors, at least ± 0.05 near $\nu_{0y} = 7.5$.

Figure 2c shows the vertical 2×rms semi-axis of the 0.6 mA beam for "matched" and "mismatched" cases. The difference between the two cases is smaller than for the 6.0 mA beam. This example and results for a few additional operating points are in qualitative agreement with experiment, as little difference is seen in the beam lifetime chart for the 0.6 mA beam if the injection quad QR1 is adjusted to match the ring quads.

Naturally, without image forces in the model, the results for centroid motion do not change for the lower current beam, but those for the rms beam size yield about half the amplitude as for 6.0 mA because of the reduced incoherent space charge. At point 1, about half the peak beam current survives in the experiment at low current (Fig. 1a), unlike the case for 6.0 mA. Smaller envelope excursions, differences in orbit steering corrections and detuning from image forces may explain the different beam lifetimes.

To summarize, calculations of linear tunes with the code ELEGANT using a simple model are in good agreement with measurements in UMER. Further, calculated centroid and beam envelope evolution over 10 turns relate well to observed beam losses at points near integer and half-integer tunes. Other than reasonable assumptions for the magnitudes of magnet and lattice errors, the model has only one adjustable parameter, i.e. the transfer function of ring quadrupoles. The beam lifetime charts from experiments show that the emittance-dominated beam behaves more coherently than the space-charge dominated beam. Additional work for the near future includes more realistic modeling of lattice errors, improved tune measurements, and calculations and measurements of incoherent tunes.

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